Physics 471
More in-class Discussion Questions

I am grateful to Michael Dubson of the University of Colorado for the vast majority of these questions.

QM1-33. A vector can be written as a column of its components in a basis ($\hat{x}, \hat{y}, \hat{z}$); likewise a vector in Hilbert space (a wave function) can be written as an infinite column of its components in a basis of the ψ_n 's :

$$\vec{A} = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \qquad \Psi = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ \vdots \end{pmatrix}$$

The dot product of two vectors **A** and **B** is given by

$$\vec{A}\cdot\vec{B} = \sum_{i=x,y,z} A_i B_i \;.$$

The inner product $\int dx\, \Psi^* \Phi$ of wavefunctions $\Psi = \sum_n d_n \psi_n$ and $\Phi = \sum_n c_n \psi_n$, is given by

A) $\sum_n d_n^* c_n$ B) $\sum_n |d_n||c_n|$

C) $\sum_n |d_n|^2 |c_n|^2$ D) $\sum_n \left(|d_n|^2 + |c_n|^2\right)$ E) zero

Answer: A

QM1-34. If f(x) and g(x) are wavefunctions, and c is a constant, then $\langle c \cdot f \mid g \rangle = ?$

A) $c\langle f \mid g \rangle$ B) $c^{*}\langle f \mid g \rangle$ C) $|c|\langle f \mid g \rangle$

D) $c\langle f^{*} \mid g \rangle$ E) None of these

Answer: B

QM1-35. True (A) or False (B): If f(x) is a wavefunction,

then $\left(\frac{1}{i}\frac{df}{dx}\right)^* = -\frac{1}{i}\frac{df^*}{dx}$

Answer: A

QM1-36. What is the difference between these two expressions?

1. $\langle \psi_1 | \psi_2 \rangle$ 2. $|\psi_1\rangle\langle\psi_2|$

A. They are both operators, but different.

B. They are both numbers, but different.

C. 1 is an operator, while 2 is a number.

D. 1 is a number, while 2 is an operator.

E. There is no difference.

Answer: D

QM1-37. What is $f(x_1, x_2) = \int dx\, \delta(x - x_1)\delta(x - x_2)$?

A) zero B) 1 C) 2 D) $\delta(0)$ E) $\delta(x_2 - x_1)$

Answer: E

QM1-38. Do the set of delta-functions $\delta(x-x_0)$ (all values of x_0) form a complete set? That is, can ***any*** function f(x) in the Hilbert Space be written as a linear combination of the delta-function like so:

$$f(x) = \int_{x0=-\infty}^{x0=+\infty} F(x_0)\,\delta(x-x_0)\,dx_0$$

A) Yes B) No

[If you answer Yes, you should be able to construct the function $F(x_0)$.]

Answer: A. The function is $F(x_0) = f(x_0)$. The resulting equation is just the definition of the delta-function.

QM1-39. The momentum operator $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$ is Hermitian, meaning $\langle f \,|\, \hat{p}\, g \rangle = \langle \hat{p}\, f \,|\, g \rangle$. Is $\hat{p}^2$ Hermitian?

A) Yes B) No

Answer: A. There are several ways to see this. The easiest is to consider that the Hamiltonian contains the term $p^2/2m$, so p^2 must be Hermitian.

QM1-40. Suppose a state Ψ(x,t) is known to be an energy eigenstate (state n) : $\Psi(x,t) = \psi_n(x)\exp(-iE_n\, t/\hbar)$. Can that energy eigenstate be written as

$$\Psi(x,t) = \int dp\, \Phi(p,t)\, f_p(x) \text{ where } f_p(x) = (1/\sqrt{2\pi\hbar})\exp(ip\, x/\hbar)$$

?

A) Yes B) No C) Maybe

Answer: A. In fact, any state can be written in that form, because $f_p(x)$ is an eigenstate of momentum, and we know that the momentum eigenstates form a complete basis for Hilbert space.

QM1-41. Can the wavefunction $\Psi(x,t)$ describing an **arbitrary** physical state **always** be written in the form $\Psi(x,t) = \psi_n(x)\exp(-iE_n t/\hbar)$, where $\psi_n(x)$ and E_n are solutions of $\hat{H}\psi_n(x) = E_n \psi_n(x)$?

A) Yes B) No

Answer: B. The stationary states are special, but they are not the most general state. However, any arbitrary physical state CAN be written as a linear superposition of such stationary states, because they form a complete basis for Hilbert space.

QM1-42. A system (described by PE = V(x)) is in state $\Psi(x,t)$ when a measure of the energy is made. The probability that the measured energy will be the nth eigenvalue E_n is $\left|\left\langle \psi_n \mid \Psi(x,t) \right\rangle\right|^2 = \left|c_n \exp(-iE_n t/\hbar)\right|^2$.
Does the probabilility of finding the energy = E_n when the system is in state $\Psi(x,t)$ depend on the time t of the measurement?

A) Yes B) No

Answer: B. The expression above simplifies to $\left|c_n\right|^2$.

QM1-43. A system is in state that is a linear combination of the n = 1 and n =3 energy eigenstates:

$$\Psi(x,t) = \frac{1}{\sqrt{2}} e^{-i\omega_1 t}\,\psi_1(x) + \frac{1}{\sqrt{2}} e^{-i\omega_3 t}\,\psi_3(x)$$

What is the probability that a measurement of energy will yield energy E_1?

A) 1/2 B) $1/\sqrt{2}$ C) 1/4 D) $\frac{1}{\sqrt{2}}\exp(-i2\omega_1 t)$ E) zero

Answer: A. See the previous problem.

QM1-44. An isolated system evolves with time according to the TDSE with V = V(x). The wavefunction $\Psi = \Psi(x,t)$ depends on time. Does the expectation value of the energy $\langle \hat{H} \rangle$ depend on time?

A) Yes always B) No, never
C) Sometimes, depending on initial conditions

Answer: B. As long as V doesn't depend on time, then energy is conserved.

QM1-45. Suppose the state function Ψ is known to be the eigenstate Ψ_1 of operator $\hat{A}$ with eigenvalue a_1 : $\hat{A}\,\Psi_1 = a_1\,\Psi_1$

What is the standard deviation $\sigma_A = \sqrt{\left\langle \Psi_1 \,|\, \left(\hat{A} - \langle A \rangle\right)^2 \Psi_1 \right\rangle}$?

A) zero always B) non-zero always

C) zero or non-zero depending on details of the eigenfunction Ψ_1.

Answer: A.

QM1-46. Suppose two observable operators commute:

$[\hat{A}, \hat{B}] = 0$

Is $\sigma_A \, \sigma_B$ zero or non-zero?

A) zero always B) non-zero always

C) zero or non-zero depending on details of the state function Ψ used to compute $\sigma_A \, \sigma_B$.

Answer: C. If A and B commute, there is no uncertainty principle governing their uncertainties. But σ_a and σ_b can still be large if somebody gives you an awful wavefunction.

QM1-47. If you have a ***single*** physical system with an unknown wavefunction Ψ, can you determine $\langle E \rangle = \langle \Psi \,|\, \hat{H}\Psi \rangle$ experimentally?

A) Yes B) No

Answer: B. The expectation value represents the average value you would get if you performed many measurements on identical physical systems. If you have only one such system, the act of measuring it will change it, so you can't repeat the measurement with the same initial (unknown) wavefunction.

QM1-48. If you have a system initially with some state function Ψ, and then you make a measurement of the energy and find energy E, how long will it take, after the energy measurement, for the expectation value of the position to change significantly?

A) forever, $\langle x \rangle$ = constant B) $\hbar / E$ C) neither of these

Recall that $\Delta E \cdot \Delta t \geq \hbar / 2$

Answer: A. After you measure the energy, the system collapses into an energy eigenstate, i.e. a stationary state. All expectation values are independent of time in stationary states (hence their name).

Observable A: $\hat{A}\psi = a\psi$ normalized eigenstates ψ_1 , ψ_2, eigenvalues a_1 , a_2.
Observable B: $\hat{B}\phi = b\phi$ normalized eigenstates ϕ_1 , ϕ_2, eigenvalues b_1 , b_2.

The eigenstates are related by

$$\psi_1 = (2\varphi_1 + 3\phi_2)/\sqrt{13} \qquad \psi_2 = (3\varphi_1 - 2\phi_2)/\sqrt{13}$$

QM1-49: Observable A is measured, and the value a_1 is found. What is the system's state immediately after measurement?

A) ψ_1 B) ψ_2 C) $c_1\psi_1 + c_2\psi_2$ (c's non-zero)
D) ϕ_1 E) ϕ_2

Answer: A

QM1-50: Immediately after the measurement of A, the observable B is measured. What is the probability that b_1 will be found?
A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

Answer: E

QM1-51: If the grad student failed to measure B, but instead measured A for a second time, what is the probability that the second measurement will yield a_1?

A) 0 B) 1 C) 0.5 D) $2/\sqrt{13}$ E) 4/13

Answer: B

QM1-52. Consider three functions $f(x)$, $g(y)$, and $h(z)$. $f(x)$ is a function of x only, $g(y)$ is a function of y only, and $h(z)$ is a function of z only. They obey the equation $f(x) + g(y) + h(z) = C = \text{constant}$. What can you say about f, g, and h?

A) f, g, and h must all be constants.
B) One of f, g, and h, must be a constant. The other two can be functions of their respective variables.
C) Two of f, g, and h must be constants. The remaining function can be a function of its variable.

Answer A.

QM1-53. For the particle in a 3D box, is the state $(n_x , n_y , n_z) =$ (1, 0, 1) allowed? A) Yes B) No

Answer: B. The value n_y=0 will result in $\Psi(x, y, z) = 0$.

QM1-54. The ground state energy of the particle in a 3D box is $(1^2 + 1^2 + 1^2)\dfrac{\hbar^2\pi^2}{2ma^2} = 1\varepsilon$. What is the energy of the 1st excited state?

A) 2ε B) 3ε C) 4ε D) 5ε E) 6ε

Answer: A. (If ε were defined in the logical way, the answer would be E. But check the definition.)

QM1-55. What is the degeneracy of the state $(n_x , n_y , n_z) =$ (1, 2, 3)?
A) 1 B) 3 C) 4 D) 6 E) 9

Answer: D.

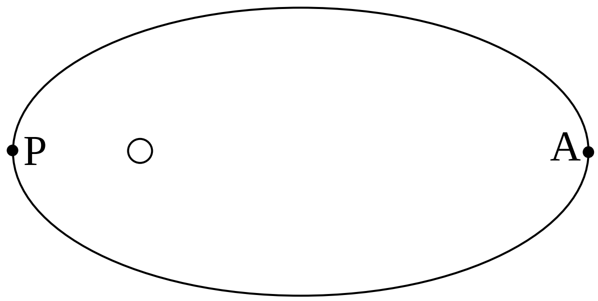
QM1-56. Is the 3D wavefunction

$$\psi(x, y, z) = \left(\frac{2}{a}\right)^{3/2} \sin\left(\frac{n_x \pi x}{a}\right)\sin\left(\frac{n_y \pi y}{a}\right)\sin\left(\frac{n_z \pi z}{a}\right)$$

an eigenfunction of $\hat{H}_x = \dfrac{-\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$?
A) Yes B) No

Answer: A.

QM1-57. A planet is in elliptical orbit about the sun.



The torque $\vec{\tau} = \vec{r} \times \vec{F}$ on the planet about the sun is
A) zero always. B) Non-zero always
C) zero at some points, non-zero at others

Answer: A. The force vector is antiparallel to the position vector.

QM1-58. The magnitude of the angular momentum of the planet about the sun $\vec{L} = \vec{r} \times \vec{p}$ is

A) greatest at perihelion (point P)
B) greatest at aphelion (point A)
C) constant everywhere in the orbit

Answer: C. The angular momentum is conserved because there is no torque.

QM1-59. The commutator $[\hat{y}\,\hat{p}_z,\ \hat{x}\,\hat{p}_z]$ is

A) zero B) none-zero C) sometimes zero, sometimes non-zero

Answer: A.

QM1-60. The commutator $[L_z^{\,2},\ L_z]$ is

A) zero B) none-zero C) sometimes zero, sometimes non-zero

Answer: A.

QM1-61. In Cartesian coordinates, the volume element is dx dy dz . In spherical coordinates, the volume element is

A) $r^2 \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$ B) $\sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

C) $r^2 \cos\theta \sin\varphi \, dr \, d\theta \, d\varphi$ D) $r \sin\theta \cos\varphi \, dr \, d\theta \, d\varphi$

E) None of these

Answer: E. It looks like A without the $\cos(\varphi)$ term.

QM1-62. In Cartesian coordinates the normalization condition is $\int_{-\infty}^{+\infty} dx \int_{-\infty}^{+\infty} dy \int_{-\infty}^{+\infty} dz \, |\Psi|^2 = 1$. In spherical coordinates, the normalization integral has limits of integration:

A) $\int_{0}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$ B) $\int_{-\infty}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$

C) $\int_{0}^{+\infty} dr \int_{0}^{2\pi} d\theta \int_{0}^{2\pi} d\varphi \ldots$ D) $\int_{-\infty}^{+\infty} dr \int_{0}^{\pi} d\theta \int_{0}^{\pi} d\varphi \ldots$

E) None of these

Answer: E. A is the closest, but the π and 2π should be exchanged.

QM1-63. Recall that an operator $\hat{Q}$ is Hermitian if $\langle f \mid \hat{Q} g \rangle = \langle \hat{Q} f \mid g \rangle$ for all normalizable functions f and g. The operator $\hat{L}_z$ is Hermitian, since it corresponds to an observable. Is the operator $i\hat{L}_z$ Hermitian?

A) Yes B) No

Answer: B. The Hermitian conjugate of i is –i.

QM1-64. $[L^2, L_+] = [L^2, L_x + i L_y]$ Does this commutator equal zero?

A) Yes, $[L^2, L_+] = 0$ B) No $[L^2, L_+] \neq 0$

Answer: A. The raising operator increases the eigenvalue of L_z, but it keeps the state in the same L^2 ladder.

QM1-65. The operator for (angular momentum)^2 is
$L^2 = {L_x}^2 + {L_y}^2 + {L_z}^2$.

Is it true that $\langle L^2 \rangle = \langle {L_x}^2 \rangle + \langle {L_y}^2 \rangle + \langle {L_z}^2 \rangle$?

A) Yes, always B) No, never
C) Sometimes yes, sometimes no, depending on the state function Ψ used to compute the expectation value.

Answer: A.

QM1-66. In spherical coordinates,

$$\nabla f = \hat{r}\frac{\partial f}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial f}{\partial \theta} + \hat{\varphi}\frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}$$

, and in QM, the angular momentum operator is $\hat{L} = \frac{\hbar}{i}\vec{r}\times\nabla = \frac{\hbar}{i}r\hat{r}\times\nabla$,

the $\hat{r}$ component of $\hat{L}$ is ?

A) 0

B) non-zero but dependent on θ, φ only (independent of r)

C) non-zero but dependent on r, θ, and φ

Answer: A. $\hat{r}\times\vec{A}$ has no $\hat{r}$ component, for any $\vec{A}$.

QM1-67. In QM, the operator $L^2 = \hat{L}\cdot\hat{L}$

A) depends on θ, and φ only (independent of r)

B) depends on r, θ, and φ

C) depends on θ only (independent of r, φ)

Answer: A. There are two ways to see this. First, you can look at the differential form of the L^2 operator. Second, you can realize that any wavefunction of the form $\Psi(r,\theta,\phi) = R(r)Y_l^m(\theta,\phi)$ is an eigenstate of L_2 with eigenvalue $\hbar^2 l(l+1)$, for any arbitrary *R(r)*. So L^2 can't depend on *r*.

QM1-68. Ignoring spin, what is the angular momentum of the ground state of an electron in a hydrogen atom, in units of h-bar?

A) 0 B) 1/2 C) 1 D) 3/2 E) I don’t know

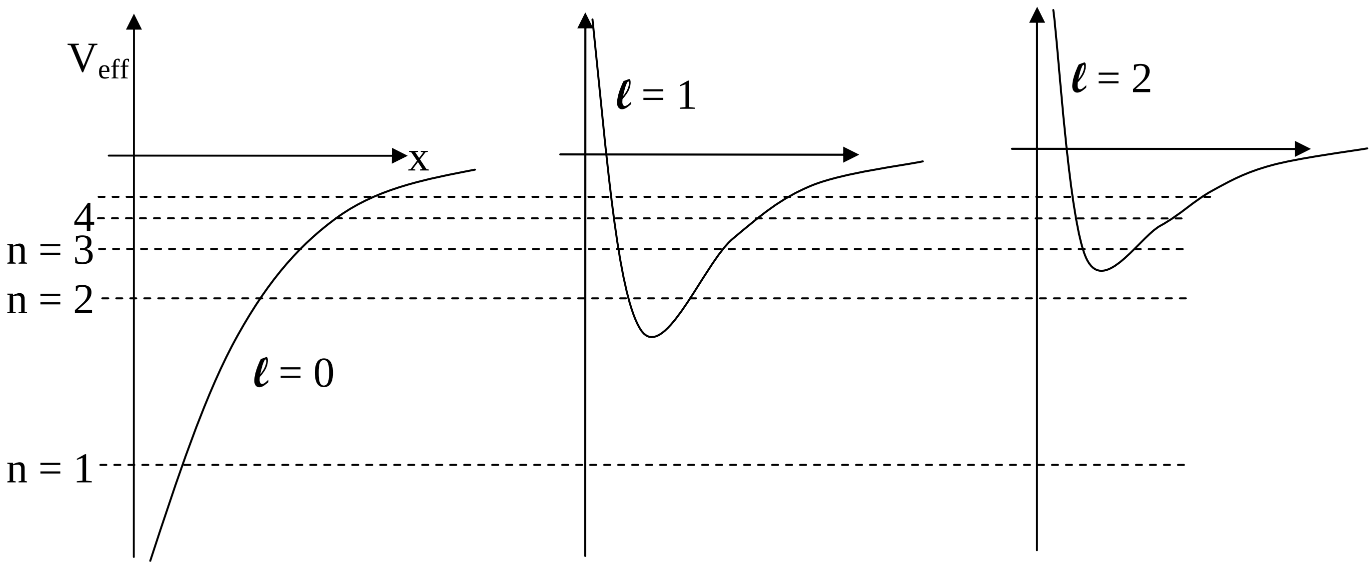
Answer: A. The ground state is spherically symmetric.

QM1-69. In classical mechanics, the translational KE of a particle is $\frac{p^2}{2m}$. What is the formula for rotational KE (where I is moment-of-inertia)?

A) $\frac{1}{2} I\, L^2$ B) $\frac{L^2}{2I}$ C) $I\omega$ D) $2\, I\, L^2$

Answer: B.

QM1-70. The effective potential is shown for $\ell = 0$, 1, and 2. The first several allowed energy levels are shown.

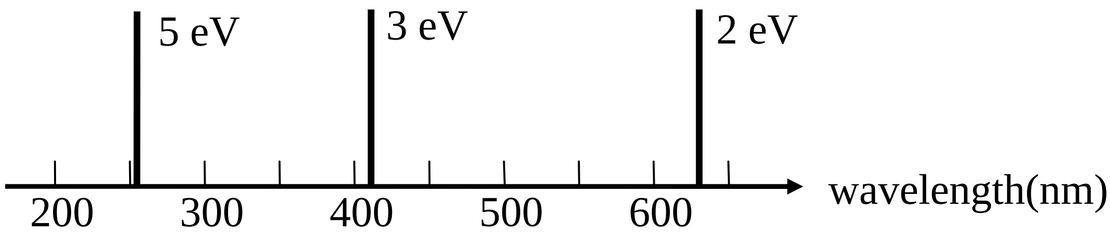


As indicated in the figure, the n = 2, $\ell = 0$ state and the n = 2, $\ell = 1$ state happen to have the same energy (given by $E_{n=2} = E_1/2^2$). Do these states have the same radial wavefunction R(r) ?

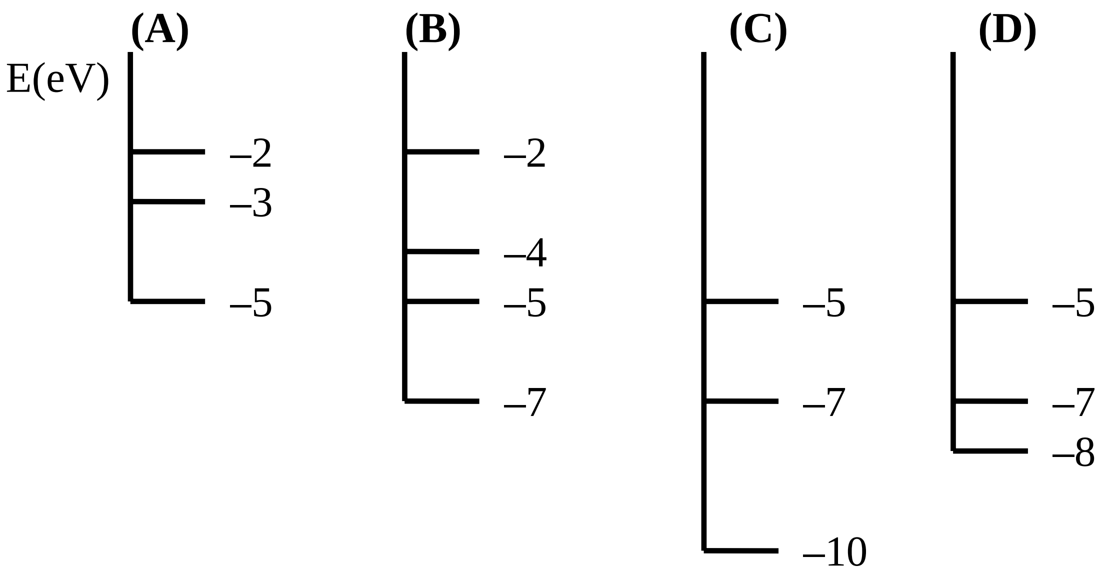
A) Yes B) No

Answer: B. See the table in Griffiths.

QM1-71. The spectrum of "Perkonium" has 3 emission lines



Which energy level structure is consistent with the spectrum?



Answer: C. The spectrum given at the top of the page consists of energy differences between pairs of levels.

QM1-72. If $\exp(+i\,m\,2\pi) = 1$, then it must be true that

A) m = 0, 1, 2, … B) m = 0, 1/2, 1, 3/2, 2, …

C) m = 0, ±1, ±2, … D) m = 2πn where n = 0, ±1, ±2, …

E) None of these

Answer: C.

QM1-73. Apart from normalization, the spherical harmonic $Y_\ell^\ell(\theta,\phi) = (\sin\theta)^\ell \exp(i\,\ell\,\phi)$. The zero-angular momentum state Y_0^0 ..

A) has no θ, ϕ dependence: it is a constant

B) depends on θ only; it has no ϕ dependence

C) depends on ϕ only; it has no θ dependence

D) depends on both θ and ϕ

Answer: A.

QM1-74. Normalization $\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta \left|Y_0^0\right|^2 = 1$ requires that $Y_0^0 =$

A) 1 B) 4π C) $\frac{1}{4\pi}$ D) $\frac{1}{\sqrt{4\pi}}$

E) None of these

Answer: D.

QM1-75. True (A) or False (B) ?
Any arbitrary physical state of an electron bound in the H-atom potential can always be written as

$$\psi_{n\ell m}(r, \theta, \phi) = R_{n\ell}(r)\, Y_\ell^m(\theta, \phi)$$

with suitable choice of n , $\boldsymbol{\ell}$, and m.

Answer: B. Only two people got this one correct in class! The statement most of you were thinking about was, "Any arbitrary physical state of an electron bound in the H-atom can be written as a linear superposition of the energy eigenstates."

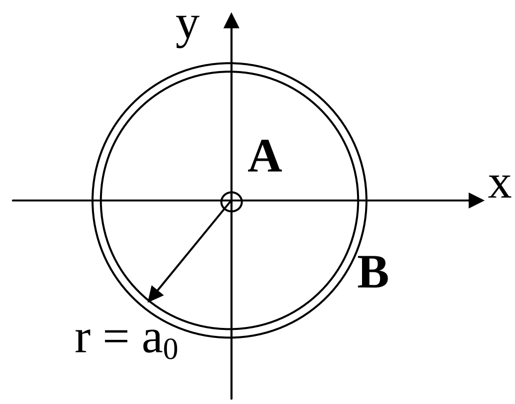
QM1-76. A particle in a 1D Harmonic oscillator is in the state $\Psi(x) = \sum_n c_n u_n(x)$ where $u_n(x)$ is the n^{th} energy eigenstate $\hat{H} u_n = E_n u_n$. A measurement of the energy is made. What is the probability that result of the measurement is the value E_m ?

A) $\langle c_m | \Psi(x) \rangle$ B) $\left| \langle c_m | \Psi(x) \rangle \right|^2$

C) $\left| \langle u_m | \Psi(x) \rangle \right|^2$ D) $\langle u_m | \Psi(x) \rangle$ E) c_m

Answer: C

QM1-77. Consider an electron in the ground state of an H-atom: The wavefunction is $\psi(r) = A\exp(-r/a_0)$
Where is the electron more likely to be found?
A) Within dr of the origin (r = 0)
B) Within dr of a distance $r = a_0$ from the origin?



Answer: B. Remember, the volume element contains the factor r^2.

QM1-78. Consider the object formed by placing a ket to the left of a bra like so: $|f\rangle\langle g|$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: D.

QM1-79. Consider the object formed by placing a bra to the left of a operator like so: $\langle g|\hat{Q}$. This thing is best described as...

A) nonsense. This is a meaningless combination.
B) a functional (transforms a function or ket into a number)
C) a function (transforms a number into a number)
D) an operator (transforms a function or ket into another function or ket).
E) None of these.

Answer: B.

QM1-80. Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$. What is $\hat{P}_2|\psi\rangle$, where $\hat{P}_2 = |2\rangle\langle 2|$ is the projection operator for the state $|2\rangle$?

A) c_2 B) $|2\rangle$ C) $c_2|2\rangle$ D) $c_2{}^*\langle 2|$ E) 0

Answer: C.

QM1-81. Consider the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$.

What is $\hat{P}_{12}|\psi\rangle$, where $\hat{P}_{12} = |1\rangle\langle 1| + |2\rangle\langle 2|$?

A) $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$ B) $|1\rangle + |2\rangle$ C) 0

D) $\langle\psi| = c_1^*\langle 1| + c_2^*\langle 2|$ E) None of these

Answer: A.

QM1-82. If the state $|\psi\rangle = c_1|1\rangle + c_2|2\rangle$, as well as the basis states $|1\rangle$ and $|2\rangle$ are normalized, then the state $\hat{P}_1|\psi\rangle = |1\rangle\langle 1|\psi\rangle = c_1|1\rangle$ is

A) normalized

B) not normalized.

Answer: B (unless c_1=1 and c_2=0.)

QM1-83. Consider two kets and their corresponding column vectors:

$$|\Psi\rangle = \begin{pmatrix} 1 \\ 1 \\ \sqrt{2} \end{pmatrix} \qquad |\phi\rangle = \begin{pmatrix} 1 \\ 1 \\ -\sqrt{2} \end{pmatrix}$$

Are these two state orthogonal? Is $\langle\psi|\phi\rangle = 0$?

A) Yes B) No

Answer: A.

Are these states normalized? A) Yes B) No

Answer: B.

QM1-84. Consider a Hilbert space spanned by three energy eigenstates:
$\hat{H}|n\rangle = E_n|n\rangle, \quad n = 1, 2, 3$. In this space, what is the matrix corresponding to the Hamiltonian?

A) $\begin{pmatrix} E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \\ E_1 & E_2 & E_3 \end{pmatrix}$ B) $\begin{pmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

D) $\begin{pmatrix} E_1 & E_1 & E_1 \\ E_2 & E_2 & E_2 \\ E_3 & E_3 & E_3 \end{pmatrix}$ E) None of these

Answer: B. This was the first question that everybody got right!