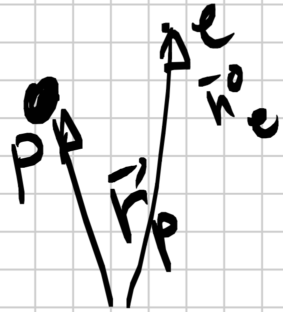
LECTURE #1

Note Title 8/25/2008

- **SYLLABUS**
- HYDROGEN ATOM

Ch 4 GRIFFITH [READING MATERIAL ONLINE]

2-BODY



$$\Psi(\vec{r}_e, \vec{r}_p)$$

$$\hat{H}\,\Psi(\vec{r}_e, \vec{r}_p) = E\,\Psi(\vec{r}_e, \vec{r}_p)$$

$$\hat{H} = -\frac{\hbar^2}{2m_p}\vec{\nabla}_p^2 - \frac{\hbar^2\vec{\nabla}_e^2}{2m_e} - \frac{e^2}{|\vec{r}_e - \vec{r}_p|}$$

cgs $\quad e$ (in esu)

$$\vec{\nabla}_p^2 = \frac{\partial^2}{\partial x_p^2} + \frac{\partial^2}{\partial y_p^2} + \frac{\partial^2}{\partial z_p^2} = \Delta$$

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(1) SEPARATE CENTER OF MASS DEGREES OF FREEDOM FROM

e – PROTON RELATIVE MOTION

$$\vec{R} = \frac{m_p \vec{r}_p + m_e \vec{r}_e}{m_p + m_e} \qquad \vec{r} = \vec{r}_e - \vec{r}_p$$

SEPARATION OF VARIABLE IN THE PARTIAL DIFFERENTIAL EQUATION (SCHRÖDINGER)

$$\Psi(\vec{r}_e, \vec{r}_p) = \Psi(\vec{R}, \vec{r}) = \Phi(\vec{R})\varphi(\vec{r})$$

$$-\frac{\hbar^2 \vec{\nabla}_p^{\,2}}{2m_p} - \frac{\hbar^2 \vec{\nabla}_e^{\,2}}{2m_e} - \frac{e^2}{|\vec{r}_e - \vec{r}_p|}$$

REDUCED MASS

$$\frac{1}{\mu} = \frac{1}{m_e} + \frac{1}{m_p}$$

$$(\vec{r}_e, \vec{r}_p) \longrightarrow (\vec{R}, \vec{r})$$

$$\left[-\frac{\hbar^2 \vec{\nabla}_R^{\,2}}{2(m_p+m_e)} - \frac{\hbar^2 \vec{\nabla}_r^{\,2}}{2\mu} - \frac{e^2}{r}\right]\Phi(\vec{R})\,\varphi(\vec{r}) =$$

$$= E\,\Phi(\vec{R})\,\varphi(\vec{r})$$

CENTER-MASS DYNAMICS

$$-\frac{\hbar^2 \vec{\nabla}_R^2}{2(m_e+m_p)} \Phi(\vec{R}) = E_{cm} \Phi(\vec{R})$$

$$\Phi_K(\vec{R}) = \frac{1}{\sqrt{V}} e^{i\vec{K}\cdot\vec{R}} \qquad E_K = \frac{\hbar^2 K^2}{2(m_e+m_p)}$$

$V \rightarrow$ QUANTIZATION VOLUME

RELATIVE MOTION $E_{TOT} = E_{CM} + E_{REL}$

$$\left(-\frac{\hbar^2 \vec{\nabla}_r^2}{2\cancel{\mu}} - \frac{e^2}{|\vec{r}|} \right) \varphi(\vec{r}) = E_{REL}\, \varphi(\vec{r})$$

c g s

ATOMIC UNITS

$$\left[\frac{e^2}{\ell} = E \right]$$

$\hbar^2$	e^2	μ
$[E^2 \cdot t^2]$	$[E \cdot \ell]$	$[m]$

$\dfrac{\hbar^2}{\mu e^2} \to$ IS A LENGTH

BOHR RADIUS

$a_B \sim 0.5\ 10^{-8}\ \mathrm{cm}$

$$\frac{\mu e^4}{\hbar^2} \rightarrow \text{IS AN ENERGY}$$

HARTREE

(1H 27.2 eV)

I WANT TO USE UNITS OF

BOHR AND HARTREES

$$\hbar^2 = e^2 = \mu = 1$$

$$\left(- \frac{\vec{\nabla}_r^2}{2} - \frac{1}{r} \right) \varphi(\vec{r}) = E \, \varphi(\vec{r}) \qquad (1)$$

MANY DIFFERENT SYSTEMS CAN BE DESCRIBED BY EQ. (1) PROVIDED I CHANGE μ OR e^2

EXAMPLES:

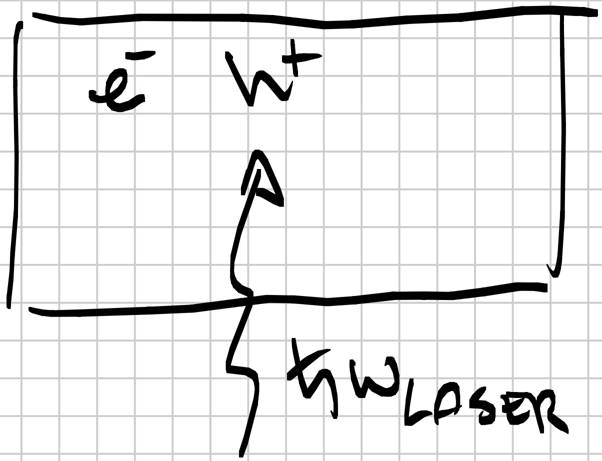
(1) POSITRONIUM

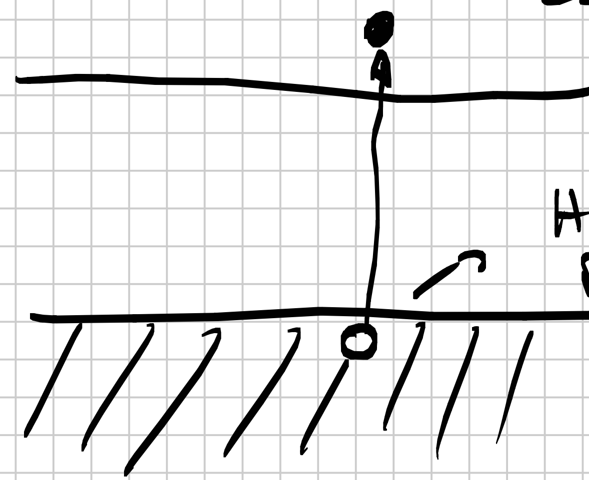
$e^+ \; e^-$

$$m_{e^+} = m_{e^-}$$

$$\mu = \frac{m_e}{2}$$

② EXCITON

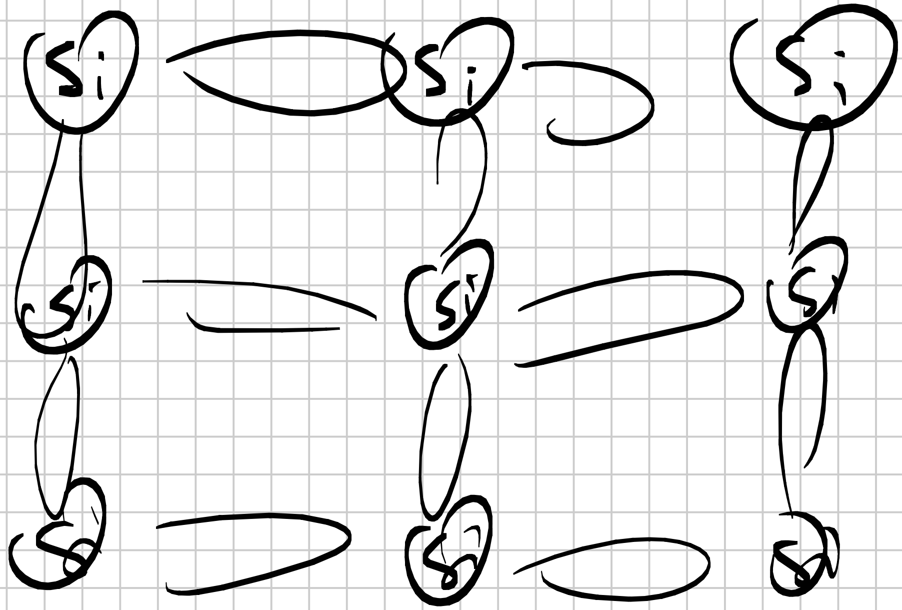




$$\frac{m_h}{m_e} \sim 5$$

$$e^2 \rightarrow \frac{e^2}{\varepsilon}$$ → DIELECTRIC CONSTANT

(3) DONOR



IV V
C
Si P