LECTURE #2

Note Title 8/27/2008

HYDROGEN ATOM

GRIFFITHS Ch 4 [4.1 & 4.2]

RM 1
RM 2 } LANDAU ONLINE

Ⓐ $\boxed{\text{SEPARATION OF VARIABLE}}$

$$\Psi(\vec{r}_e, \vec{r}_p) \to \Psi(\underset{CM}{\vec{R}}, \underset{\vec{r}_e - \vec{r}_n}{\vec{r}}) \longrightarrow \Phi(\vec{R})\,\varphi(\vec{r})$$

$$\cancel{\varphi_1(r_e)\,\varphi_2(r_p)}$$

$$\hat{H} \to \hat{H}_{cm} + \hat{H}_{rel}$$

(1) $\Phi_k(R) = \frac{1}{\sqrt{V}} e^{iK \cdot R}$ CENTER OF MASS (PLANE WAVE FREE ATOM)

(2) $\varphi(\vec{r}) \to$ RELATIVE $e-p$ MOTION

(3) ATOMIC UNITS

$$\left(-\frac{\hbar^2 \vec{\nabla}_r^2}{2\mu} - \frac{e^2}{|\vec{r}|} \right) \varphi(\vec{r}) = E\, \varphi(\vec{r})$$

$\hbar = e = \mu = 1$ THIS IS EQUIVALENT TO

$[l]$ IS MEASURED IN UNITS OF $\frac{\hbar^2}{\mu e^2} = a_B$ BOHR (BOHR RADIUS H)

[E] IS MEASURED IN

UNITS OF $\frac{\mu e^4}{\hbar^2} = \frac{e^2}{a_B}$ HARTREE

GROUND STATE

$$\varphi_{1s}(\vec{r}) = \frac{1}{\sqrt{\pi a_B}} e^{-r/a_B} \qquad E_{1s} = -\frac{\mu e^4}{2\hbar^2} = -13.6\,eV$$

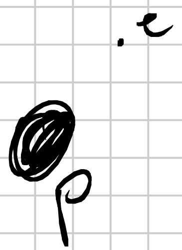
$a_B = .5\,\mathring{A}$

RYDBERG

IN AU

$$\boxed{\varphi_{1s}(r) = \frac{1}{\sqrt{\pi}} e^{-r} \qquad E_{1s} = -\frac{1}{2}}$$

H



$\mu^H \sim m_e$

POSITRONIUM

e^+ e^-

$$\mu^P = \frac{m_e}{2}$$

$$\frac{a_B^P}{a_B^H} = \frac{\cancel{\hbar^2}}{\mu^P \cancel{e^2}} \cdot \frac{\mu^H \cancel{e^2}}{\cancel{\hbar^2}} \sim \frac{\mu_H}{\mu_p} = 2$$

$a_B^P \sim 1 \text{ Å}$

$$\frac{E_{1s}^P}{E_{1s}^H} \sim \frac{1}{2}$$

$E^P \sim -6.7 \text{ eV}$

$$\left(-\frac{1}{2}\vec{\nabla}_r^{\,2} - \frac{1}{|\vec{r}|}\right)\varphi(\vec{r}) = E\,\varphi(\vec{r})$$

$$\varphi(r,\vartheta,\varphi) \xrightarrow{\text{SEPARATION OF VARIABLES}} R(r)\,Y(\vartheta,\phi)$$

$$\left[-\frac{1}{2}\left(\underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}}_{\vec{\nabla}_{\vec{r}}^{\,2}} + \frac{\boxed{\hat{\ell}^2}}{r^2}\right) - \frac{1}{r}\right]R(r)\,Y(\vartheta,\phi) = E\,R\,Y \quad (1)$$

$$\hat{\ell}^2 = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\phi^2}$$

MULTIPLY BY $-2r^2$ DIVIDE BY $R(r)\,Y(\vartheta,\phi)$

EQ (1)

$$\boxed{\frac{1}{R(r)}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}R(r) + 2r^2\left[\frac{1}{r}+E\right]} + \boxed{\frac{1}{Y(\theta\phi)}\hat{\ell}^2 Y(\theta,\phi)} = 0$$

DEPENDS ONLY ON r

DEPENDS ON $\theta\,\phi$ ONLY

$$F(r) + G(\theta,\phi) = 0$$

$f(x)$ $g(y)$ x, y INDEPENDENT

$f(x) + g(y) = 0 \rightarrow$ $f(x) = c$ (CONSTANT)

$g(y) = -c$

2 PROBLEMS

$c = \ell(\ell+1)$ $\ell = 0, 1, 2 \ldots$

(1) $\hat{\ell}^2 Y(\theta,\phi) = -c\, Y(\theta,\phi) \rightarrow Y_\ell^m(\theta,\phi)$ ARE SPHERICAL HARMONICS

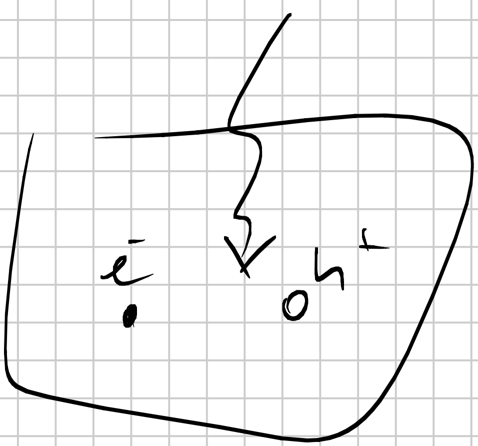
(2) $$\left[-\frac{1}{2r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r}-\frac{1}{r}+\frac{c}{r^2}\right]R(r)=E\,R(r)$$

$$E=-\frac{1}{n^2} \qquad n=1,2,3\ldots$$

$$R(r)=r^{l}e^{-r/2}L_{n+l}^{2l+1}(r)$$

LAGUERRE POLYNOMIALS

EXCITONS



$m_e = .1\, m_0$

$m_h = .2\, m_0$

$$\frac{e^2}{r} \rightarrow \frac{e^2}{\varepsilon r}$$

IN 2 DIMENSIONS

$\varepsilon \sim 10$