LECTURE # 22

Note Title

11/24/2008

SPECIAL REVIEW SESSION

6PM - 7PM WED DEC 10TH

ROOM 1308

DEC 9 - 10 OFFICE HRS

FINAL IS IN ROOM 1420

DEC 11th 12:45 - 2:45 PM

WED 26

CLASS ?

YES !

WE WILL WORK ON THE HW5 AND HW6 IN CLASS

CHAPT 22

p 422 - 437 → STUDY

p 437 - 443 → READ ONLY

CHAP 23 ALL STUDY

CLASSICAL LIMIT $(\hbar \to 0)$

$$E(T) = E_0 + \langle \overset{\overset{\text{KINETC}}{\uparrow}}{K} \rangle + \langle \overset{\overset{\text{POTENTIAL}}{\uparrow}}{V} \rangle$$

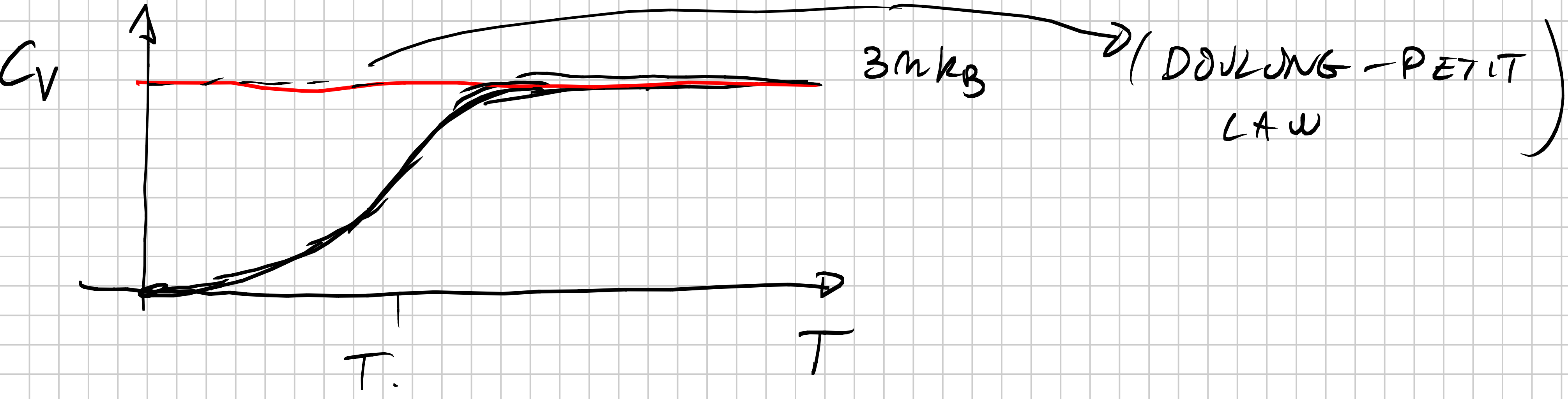
LATTICE WITH

N ATOMS IN 3D ⟹ 3N DEGREES OF FREEDOM

$$\langle K \rangle = \left\langle \frac{p^2}{2m} \right\rangle = 3N \cdot \frac{K_B T}{2}$$

$$\langle V \rangle = 3N \frac{K_B T}{2} \qquad \left(\begin{array}{l} \text{VIRIAL THEOREM} \\ V \sim r^2 \Rightarrow 2\langle K \rangle = 2\langle V \rangle \end{array} \right)$$

$$C_V = \frac{1}{V}\left(\frac{\partial E(T)}{\partial T}\right)_V = 3\frac{N}{V} K_B = 3 n K_B$$



WE NEED $\hbar \neq 0$!

MODES OF OSCILLATIONS WITH A GIVEN $\vec{q}$

EACH $\vec{q}$ CORRESPONDS TO AN HARMONIC OSCILLATOR WITH FREQUENCY $\omega_\lambda(\vec{q})$

CONSIDER THE OSCILLATORS AS QUANTUM OSCILLATORS

$$\Longrightarrow$$

$$\underline{\underline{H}} = \sum_{\vec{q} \in BZ} \hbar\omega_\lambda(\vec{q}) \left(a^{+}_{q\lambda} a_{q\lambda} + \frac{1}{2} \right)$$

$\lambda \in \{$ LONGITUDINAL/TRANSVERSAL BRANCHES

ACOUSTIC/OPTICAL BRANCHES

$a^{\dagger}_{q\lambda}$, $a_{q,\lambda}$ CREATE AND DESTROY QUANTA OF OSCILLATION AT A GIVEN MODE WITH $\vec{q}$ ON THE "λ" BRANCH.

PHONONS

$$E(T) = \langle H \rangle = \sum_{q,\lambda} \frac{\hbar\omega_{\lambda}(\vec{q})}{2} +$$

$$\sum_{q\lambda} \langle a^{\dagger}_{q\lambda} a_{q\lambda} \rangle \hbar\omega_{\lambda}(\vec{q})$$

$$\langle a^{\dagger}_{q\lambda} a_{q\lambda} \rangle = \langle n_{q\lambda} \rangle = \frac{1}{e^{\frac{\hbar\omega_{\lambda}(q)}{k_B T}} - 1}$$

$$E(T) = \sum_{\substack{q \in BZ \\ \lambda}} \frac{\hbar \omega_\lambda(\vec{q})}{e^{\frac{\hbar \omega(q)}{k_B T}} - 1}$$

EINSTEIN (1905 / 1907)

EACH ATOM INDEPENDENT

$$\Rightarrow \quad \hbar \omega_\lambda(\vec{q}) = \hbar \omega_E$$ (INDEPENDENT OF q, λ)

TRANVERSAL BRANCHES

$$E(T) = N \cdot (\overset{\uparrow}{2} + \underset{\downarrow}{1}) \frac{\hbar \omega_E}{e^{\frac{\hbar \omega_E}{k_B T}} - 1}$$

LONGITUDINAL

① LIMIT $k_BT >> \hbar\omega_E$

$e^x \sim 1 + x$ IF $\boxed{x << 1}$

$$e^{\boxed{\frac{\hbar\omega_E}{k_BT}} << 1} \sim 1 + \frac{\hbar\omega_E}{k_BT}$$

$$\Rightarrow E(T) = 3N \cancel{\hbar\omega_E} \frac{k_BT}{\cancel{\hbar\omega_E}}$$

SAME AS BEFORE

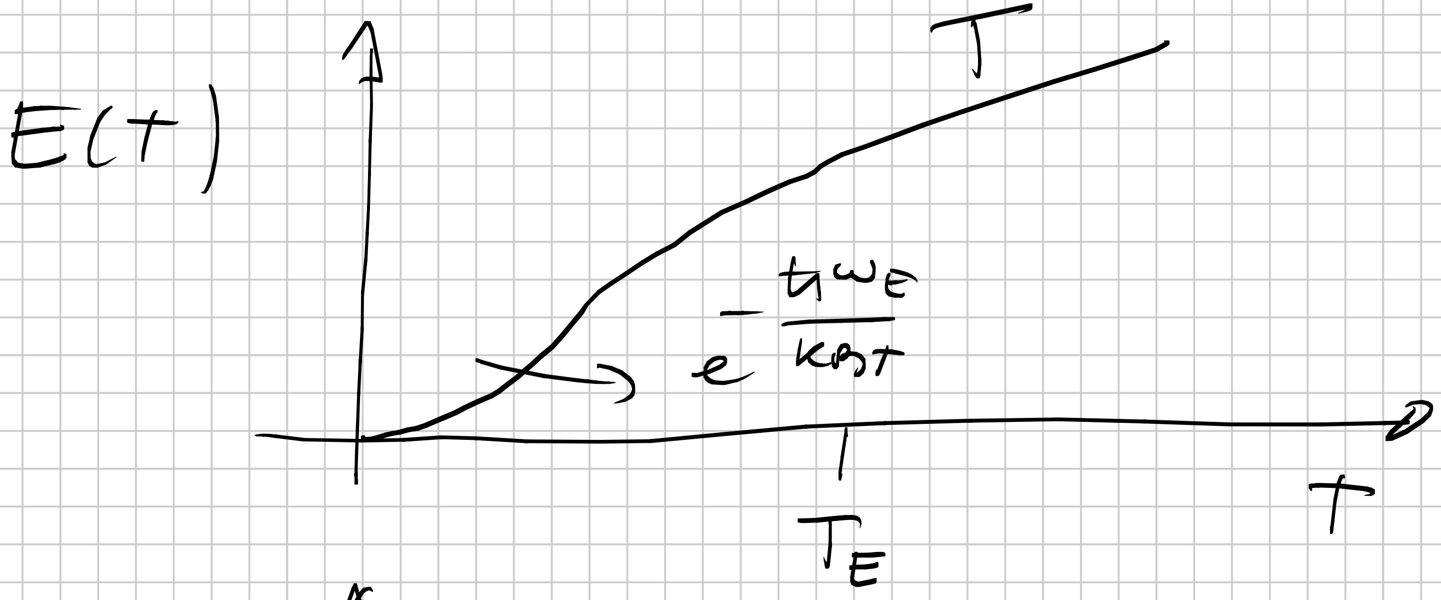
$$C_V = \frac{1}{V}\frac{dE(T)}{dT} = 3nk_B$$

② LIMIT $k_BT << \hbar\omega_E$

$\frac{\hbar\omega_E}{k_BT} >> 1$

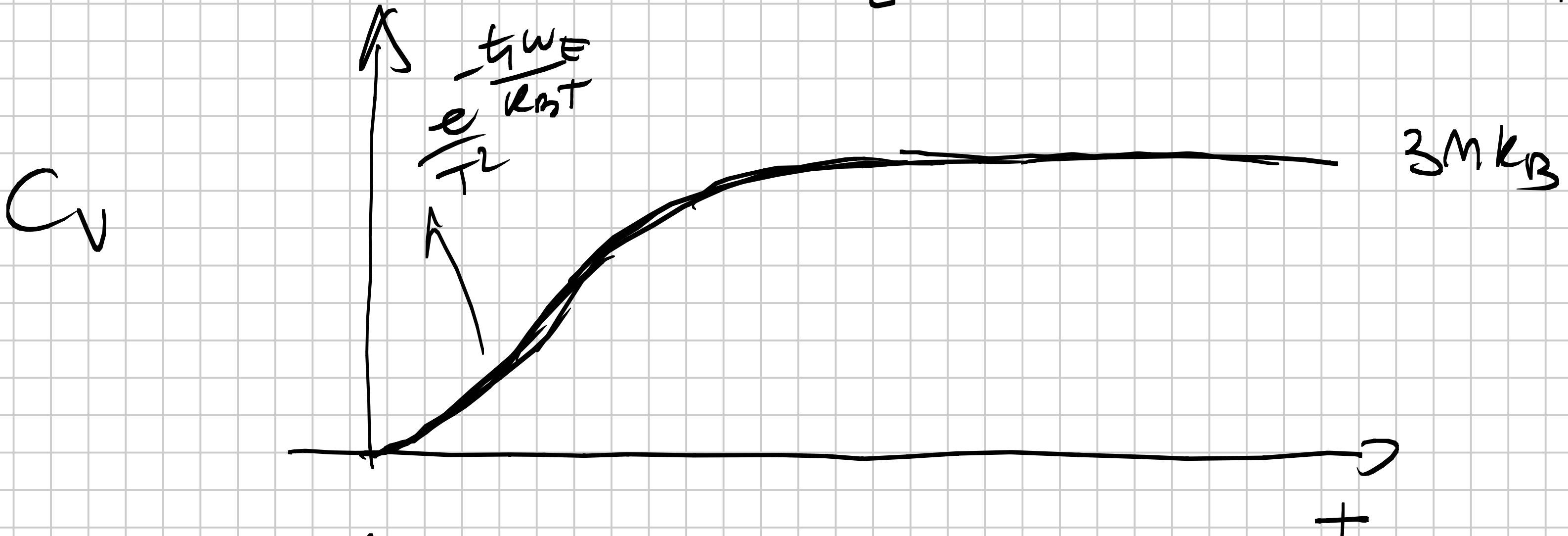
$$e^{\frac{\hbar\omega_E}{k_BT}} - 1 \sim e^{\frac{\hbar\omega_E}{k_BT}}$$

$$E \sim 3N\hbar\omega_E \, e^{-\frac{\hbar\omega_E}{k_BT}}$$



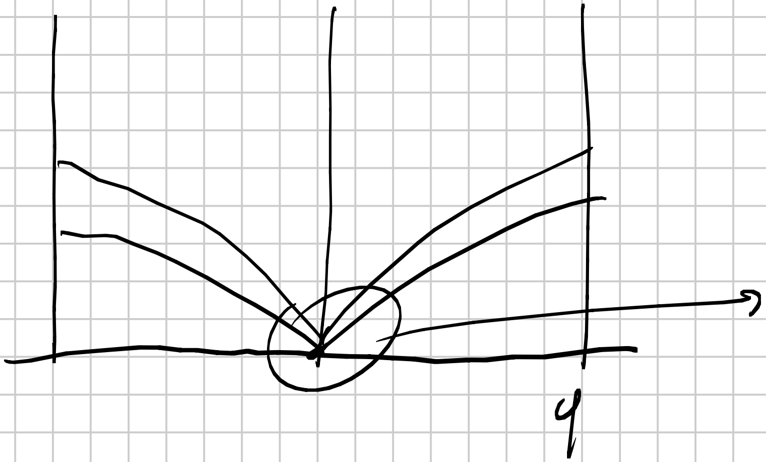
EINSTEIN TEMPERATURE

$T_E \sim \frac{\hbar\omega_E}{k_B}$



EINSTEIN MODEL

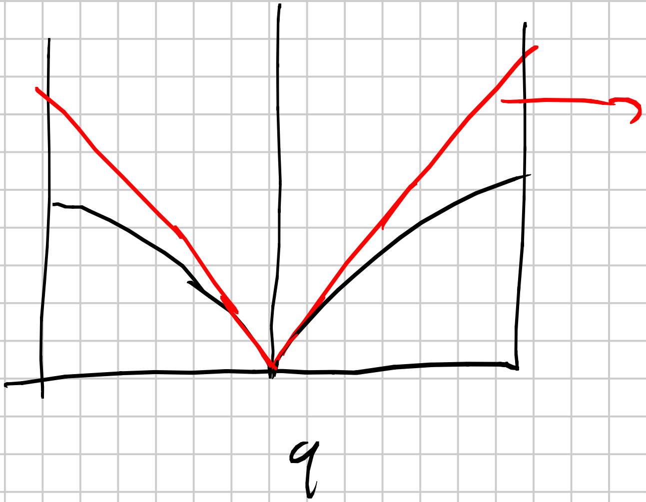
EXPERIMENT



LOW ENERGY MODES DEPEND STRONGLY ON q
0

DEBYE MODEL (1912)

①



SLOPE = SPEED OF SOUND

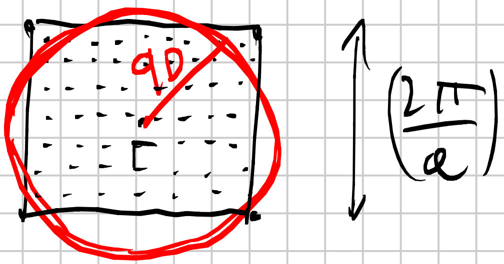
v = SPEED OF SOUND IN THE SOLID

$$\hbar\omega(q) \sim \hbar v |\vec{q}|$$

②

$$\sum_{\vec{q} \in BZ}$$

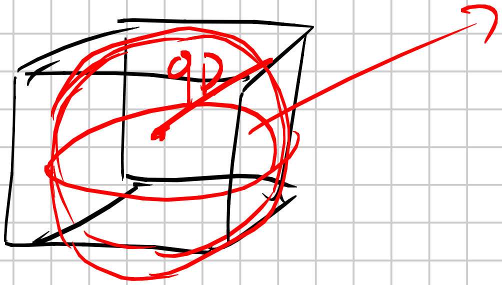
2D CRYSTAL (SQUARE LATTICE)



SAME AREA FOR DEBYE DISK AND REAL BRILLOUIN ZONE

$$\pi q_D^2 = \left(\frac{2\pi}{a}\right)^2$$

3D CRYSTALS



DEBYE SPHERE

$$\frac{4}{3}\pi q_D^3 = \left(\frac{2\pi}{a}\right)^3$$

$$q_D^3 = 6\pi^2 n$$

$$\hbar\omega(\vec{q}) \sim \hbar v|\vec{q}|$$

$$E(T) = 3\sum_{q\in BZ} \frac{\hbar v|\vec{q}|}{e^{\frac{\hbar v|\vec{q}|}{k_B T}} - 1}$$

$$\sum_{q\in SPHERE} \to \int \frac{d^3\vec{q}}{\left[\frac{(2\pi)^3}{V}\right]} \to \overset{*}{4\pi}\cdot\frac{V}{(2\pi)^3}\int_0^{q_D} q^2\,dq$$

$$E = 3V\int_0^{q_D} \frac{4\pi}{(2\pi)^3}\,\frac{q^2\,\hbar v q}{e^{\frac{\hbar v q}{k_B T}} - 1}\,dq$$

$$\frac{\hbar v q}{k_B T} = x \qquad\qquad T_D = \frac{\hbar v q_D}{k_B}$$

$$E = 3V \frac{(4\pi)}{(\hbar v)^3} \frac{(k_B T)^4}{(2\pi)^3} \int_0^{\frac{T_D}{T}} \frac{x^3}{e^x - 1}\, dx$$

USE:

$$q_D^3 = 6\pi^2 \frac{N}{V}$$

$$E(T) = 9N k_B T \left(\frac{T}{T_D}\right)^3 \int_0^{\frac{T_D}{T}} \frac{x^3}{e^x - 1}\, dx \qquad ⊛$$

LIMIT ① CLASSICAL: $T \gg T_D$

$$\Rightarrow \quad \frac{T_D}{T} \ll 1$$

$$\int_0^{\varepsilon \to \text{SMALL}} \quad \Rightarrow \quad x \text{ IS SMALL}$$

$$e^x - 1 \sim x$$

⊛ $$\int_0^{\frac{T_D}{T}} \frac{x^3}{x}\, dx \sim \frac{x^3}{3}\Big|_0^{T_D/T} = \frac{1}{3}\left(\frac{T_D}{T}\right)^3$$

$$E(T) \sim 9 N k_B T \cancel{\left(\frac{T}{T_D}\right)^3} \cdot \cancel{\left(\frac{T_D}{T}\right)^3} \cdot \frac{1}{3} \rightarrow \text{CLASSICAL RESULT}$$

$$\rightarrow 3 N k_B T$$

② QUANTUM LIMIT: $T \ll T_{DEBYE}$

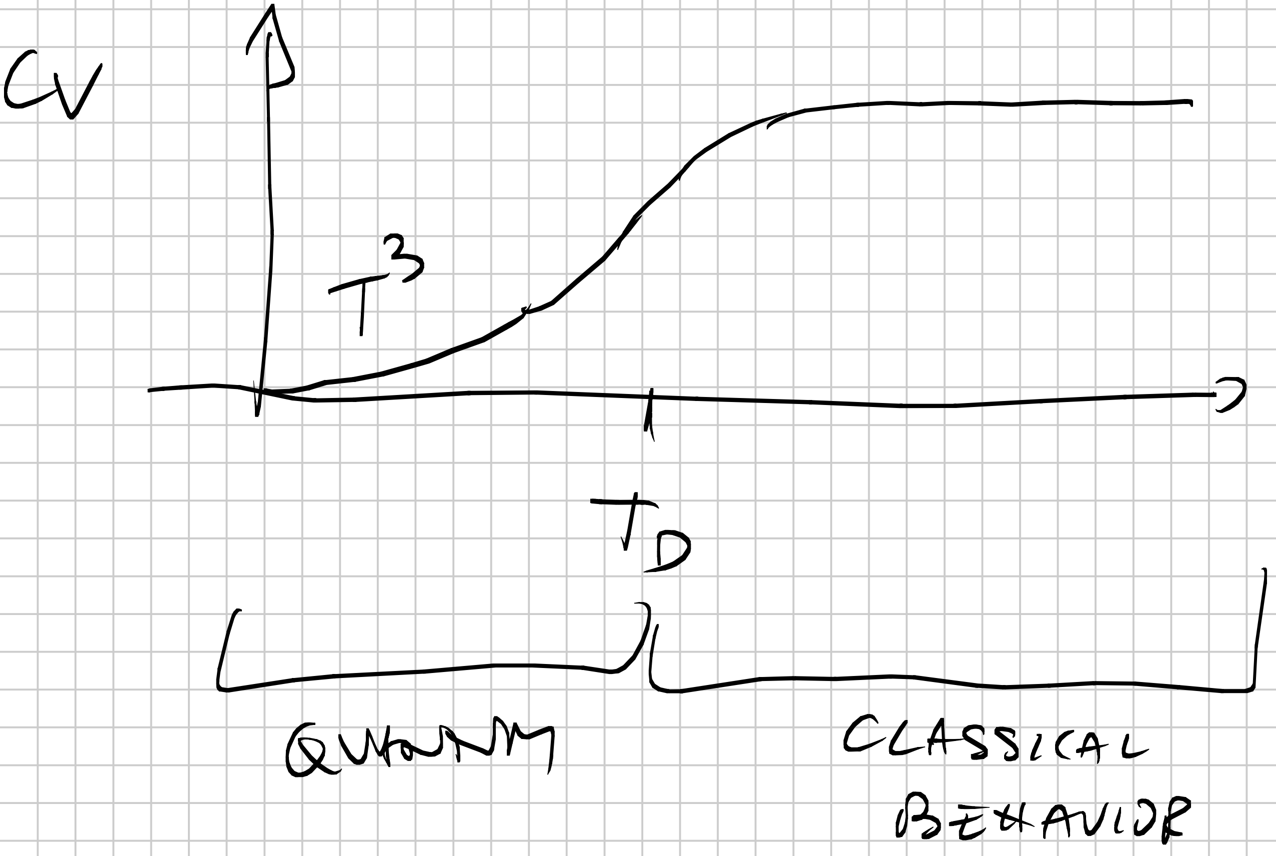
$$\int_0^{\frac{T_D}{T}} \qquad\qquad \frac{T_D}{T} \to \infty$$

$$= \frac{\pi^4}{15}$$

$$\Rightarrow \int_0^{\frac{T_D}{T}} \frac{x^3}{e^x-1}\, dx \to \boxed{\int_0^{\infty} \frac{x^3}{e^x-1}} dx$$

$$E = 9N k_B T \left(\frac{T}{T_D}\right)^3 \frac{\pi^4}{15} \Rightarrow E \propto T^4$$

$$C_V = \frac{1}{V}\left(\frac{\partial E}{\partial T}\right)_V \sim \frac{12}{5}\pi^4 n k_B \left(\frac{T}{T_D}\right)^3$$



FOR ELECTRONS

ε_F FERMI ENERGY

ELECTRON OF SURFACE OF FERMI SPHERE

$$\underbrace{\downarrow}^{T_F}$$

$T \gg \frac{\varepsilon_F}{k_B}$ → CLASSICAL

$T \ll T_F$ → QUANTUM

FOR PHONONS

$T >> T_D = \frac{\hbar\omega_D}{k_B}$ CLASSICAL

$\hbar\omega_D \sim \hbar v(q_D)$

PHONON ON SURFACE
OF DEBYE SPHERE

$T << T_D$ QUANTUM

$\hbar\omega_D \sim .1\ eV \Rightarrow T_D \sim 10^2 \sim 10^3\ K$

$\varepsilon_F \sim 10\ eV \Rightarrow T_F \sim 10^4 \sim 10^5\ K$