LECTURE #3

Note Title 9/3/2008

- HYDROGENIC SYSTEMS

  H, POSITRONIUM, EXCITON, DONORS

GRIFFITHS ch 4.3 ch 4.4 + R.M

- SEPARATION OF VARIABLES

$$\vec{r} = (r, \vartheta, \phi) \qquad \varphi(\vec{r}) = R(r)\, Y(\vartheta, \phi)$$

① $$\hat{l}^2 Y(\vartheta, \phi) = c\, Y(\vartheta, \phi)$$

STURM–LIOUVILLE PROBLEM

② $$\left[-\frac{1}{2r^2} \frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} - \frac{1}{r} + \frac{c}{r^2} \right] R(r) = E\, R(r)$$

$$\vec{l} = \vec{r} \times \vec{p}$$

$$\vec{p} \to \hat{p} = -i\vec{\nabla}$$

$$(p_x, p_y, p_z) \qquad \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

$$\vec{l} = -i\left(\hat{u}_\varphi \frac{\partial}{\partial\vartheta} - \hat{u}_\vartheta \frac{1}{\sin\vartheta}\frac{\partial}{\partial\varphi}\right)$$

$$\hat{l}^2 = l_x^2 + l_y^2 + l_z^2 = \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\phi^2}$$

$Y_l(\vartheta,\varphi)$ ARE EIGENFUNCTIONS OF l^2

C CAN ONLY BE = 0, 2, 6, 12, ...

$C = l(l+1)$ WITH $l = 0, 1, 2, 3, \ldots$

$Y(\theta, \phi)$ ARE ALSO EIGENFUNCTIONS OF $\hat{l}_z = -i \frac{\partial}{\partial \phi}$

$Y_l^m(\theta, \phi) \propto P(\cos\theta)\, e^{\pm i m \phi}$

$$\left[-\frac{1}{2r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} R - \frac{1}{r} + \frac{l(l+1)}{r^2}\right] R(r) = E R(r)$$

$$\boxed{n = \frac{1}{\sqrt{-2E}}} \qquad \rho = \frac{2r}{n}$$

$$R(\rho) = \rho^l e^{-\rho/2} W_l(\rho)$$

$\Downarrow$

EQUATION FOR $W(\rho)$

$$\rho \frac{d^2 w}{\partial \rho^2} + (2\ell + 2 - \rho)\frac{\partial w}{\partial \rho} + (n - \ell - 1)\, w = 0$$

$w \sim e^{\rho}$ NOT ACCEPTABLE

PROB TO HAVE ELECTRON AT $\rho = \infty$ DIVERGES

FOR WHICH VALUES OF n, ℓ DO I HAVE A MEANINGFUL $w(\rho)$?

$$\begin{cases} n - \ell & \text{INTEGER} \geq 1 \\ n & \text{INTEGER} \geq 1 \end{cases} \Rightarrow \quad E = -\frac{1}{2n^2}$$

n PRINCIPAL QUANTUM NUMBER

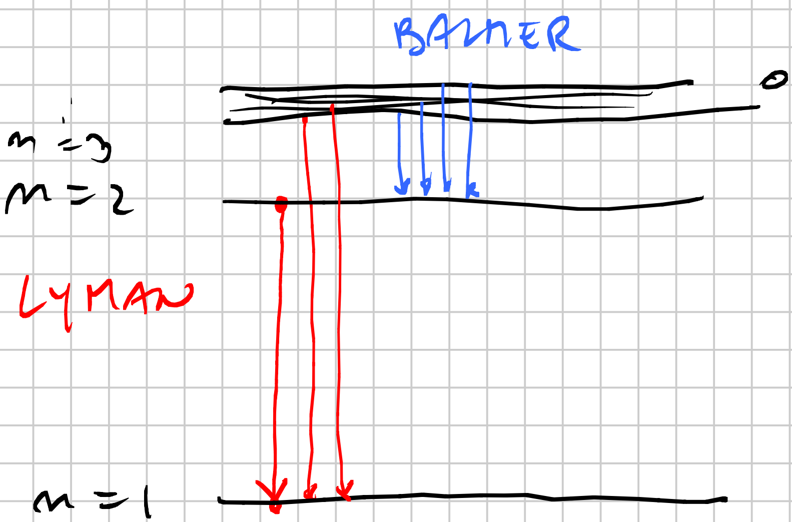
ℓ ANGULAR MOMENTUM

$n = 1$ $\boxed{\ell = 0}$ S - ORBITAL

$n = 2$ $\ell = 0$ $\boxed{\ell = 1}$ → P - ORBITAL

$n = 3$ $\ell = 0$ $\ell = 1$ $\boxed{\ell = 2}$ d - ORB

f, g, h ...



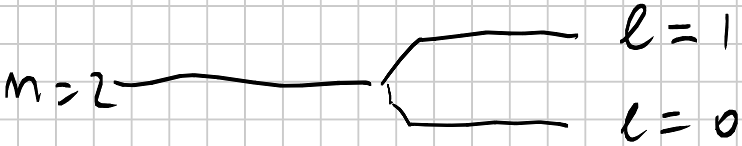
$$E = -\frac{1}{8}$$

$$E = -\frac{1}{2} = -Ry = -13.6\ eV$$

$$E = -\frac{1}{2n^2}$$

DOES NOT DEPEND ON ℓ

SPECIAL PROPERTY (SYMMETRY) OF COULOMB POTENTIAL.



SPLITTING DUE TO SPIN - ORBIT

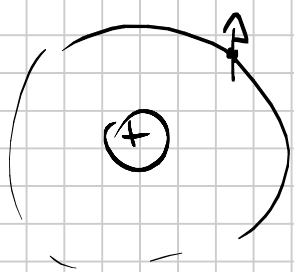
FINE STRUCTURE OF H

GRIF 6.3 + 6.5

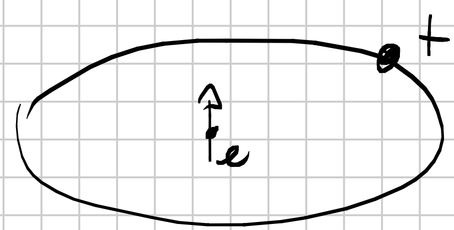
RELATIVISTIC EFFECT

DIRAC EQUATION → GIVE CORRECT STRENGTH

PHYSICAL ORIGIN:



IN THE e^- FRAME



"MOVING" PROTON ⇒ CURRENT ⇒ MAGNETIC FIELD $\vec{B}_{eff}$

ELECTRON MAGNETIC MOMENT

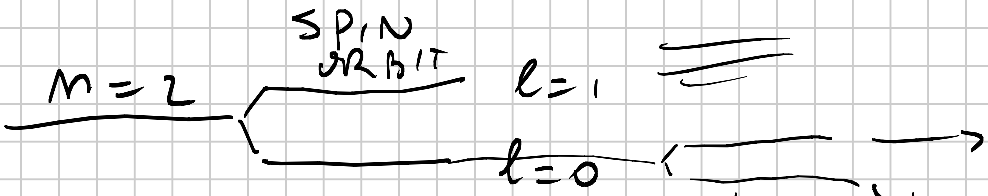
$$H = -\vec{\mu}_e \cdot \vec{B}_{eff}$$

$$\vec{\mu}_e \propto \vec{S}_e$$

$$|\vec{B}_{eff}| \propto \text{CURRENT} \propto |\vec{\ell}|$$

$$H_{SO} = \alpha \, \vec{S}_e \cdot \vec{l}$$

HYPERFINE STRUCTURE



↑↑ TRIPLET

$S_e S_p$ IN SINGLET ↑↓ − ↓↑

DUE TO COUPLING OF $\vec{S}_e$ WITH $\vec{S}_P$

$\vec{S}_p$ = SPIN PROTON

$$H_{HYPERFINE} = \beta \, \vec{S}_e \cdot \vec{S}_p$$

MANY ELECTRON ATOMS

TYPO QUESTION 6 -

$$m = \left| \frac{\partial y(\theta)}{\partial \theta} \right| \Big/ y(\theta)$$