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## Do not open exam until instructed to do so.

## Quadratic Formula

$a x^{2}+b x+c=0$,
$x=\left[-b \pm \sqrt{b^{2}-4 a c}\right] /(2 a)$
Geometry
Circle: circumference $=2 \pi R$, area $=\pi R^{2}$
Sphere: area $=4 \pi R^{2}$, volume $=4 \pi R^{3} / 3$
Trigonometry


$$
\begin{gathered}
\sin \alpha=\frac{A}{C}, \quad \cos \alpha=\frac{B}{C} \\
\tan \alpha=\frac{A}{B}
\end{gathered}
$$



$$
\frac{\sin \alpha}{A}=\frac{\sin \beta}{B}=\frac{\sin \gamma}{C}
$$

$$
A^{2}+B^{2}-2 A B \cos \gamma=C^{2}
$$

Polar Coordinates
$x=r \cos \theta, \quad y=r \sin \theta, r=\sqrt{x^{2}+y^{2}}, \quad \tan \theta=y / x$
SI Units and Constants

| quantity | unit | abbreviation |
| :---: | :---: | :---: |
| Mass $m$ | kilograms | kg |
| Distance $x$ | meters | m |
| Time $t$ | seconds | s |
| Force $F$ | Newtons | $\mathrm{N}=\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ |
| Energy $E$ | Joules | $\mathrm{J}=\mathrm{N} \mathrm{m}$ |
| Power $P$ | Watts | $\mathrm{W}=\mathrm{J} / \mathrm{s}$ |
| Temperature $T$ | ${ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{K}$ or ${ }^{\circ} \mathrm{F}$ | $T_{\circ}{ }_{\mathrm{F}}=32+(9 / 5) T_{\circ}{ }^{\circ} \mathrm{C}$ |
| Pressure $P$ | Pascals | $\mathrm{Pa}=\mathrm{N} / \mathrm{m}^{2}$ |

$1 \mathrm{cal}=4.1868 \mathrm{~J}, 1 \mathrm{hp}=745.7 \mathrm{~W}, 1$ liter $=10^{-3} \mathrm{~m}^{3}$
$g=9.81 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{G}=6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
$1 \mathrm{~atm}=1.013 \times 10^{5} \mathrm{~Pa}, 0^{\circ} \mathrm{C}=273.15^{\circ} \mathrm{K}, N_{A}=6.023 \times 10^{23}$
$R=8.31 \mathrm{~J} /\left(\mathrm{mol}^{\circ} \mathrm{K}\right), k_{B}=R / N_{A}=1.38 \times 10^{-23} \mathrm{~J} /{ }^{\circ} \mathrm{K}$
$\sigma=5.67 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} \mathrm{~K}^{4}\right)$
$v_{\text {sound }}=331 \sqrt{T / 273} \mathrm{~m} / \mathrm{s}$
$\mathrm{H}_{2} 0: c_{\text {ice, liq.,steam }}=\{0.5,1.0,0.48\} \mathrm{cal} / \mathrm{g}^{\circ} \mathrm{C}$
$L_{F, V}=\{79.7,540\} \mathrm{cal} / \mathrm{g}, \rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$.
1-d motion, constant $a$
$\Delta x=(1 / 2)\left(v_{0}+v_{f}\right) t$
$v_{f}=v_{0}+a t$
$\Delta x=v_{0} t+(1 / 2) a t^{2}$
$\Delta x=v_{f} t-(1 / 2) a t^{2}$
$(1 / 2) v_{f}^{2}-(1 / 2) v_{0}^{2}=a \Delta x$
Range: $R=\left(v_{0}^{2} / g\right) \sin 2 \theta$
Forces, Work, Energy, Power, Momentum \& Impulse
$F=m a$, Gravity: $F=m g, P E=m g h$
Friction: $f=\mu N$, Spring: $F=-k x, P E=(1 / 2) k x^{2}$
$W=F x \cos \theta, K E=(1 / 2) m v^{2}, P=\Delta E / \Delta t=F v$
$p=m v, I=F \Delta t=\Delta p$
$X_{c m}=\left(m_{1} x_{1}+m_{2} x_{2}+\cdots\right) /\left(m_{1}+m_{2}+\cdots\right)$
Elastic coll.s: $v_{1}^{\prime}-v_{2}^{\prime}=-\left(v_{1}-v_{2}\right)$

## Rotational Motion

$\Delta \theta=(1 / 2)\left(\omega_{0}+\omega_{f}\right) t, \omega_{f}=\omega_{0}+\alpha t$
$\Delta \theta=\omega_{0} t+(1 / 2) \alpha t^{2}=\omega_{f} t-(1 / 2) \alpha t^{2}$
$\alpha \Delta \theta=(1 / 2) \omega_{f}^{2}-(1 / 2) \omega_{0}^{2}$
$\omega=2 \pi / T=2 \pi f, f=1 / T$
Rolling: $a=\alpha r, v=\omega r$
$a_{c}=v^{2} / r=\omega v=\omega^{2} r$
$\tau=r F \sin \theta=I \alpha, I_{\text {point }}=m R^{2}$
$I_{\text {cyl.shell }}=M R^{2}, I_{\text {sphere }}=(2 / 5) M R^{2}$
$I_{\text {solid cyl. }}=(1 / 2) M R^{2}, I_{\text {sph. shell }}=(2 / 3) M R^{2}$
$L=I \omega=m v r \sin \theta,(\theta=$ angle between v and r$)$
$K E=(1 / 2) I \omega^{2}=L^{2} /(2 I), W=\tau \Delta \theta$
Gravity and circular orbits
$P E=-G \frac{M m}{r}, \Delta P E=m g h($ small $h)$

$$
F=G \frac{M m}{r^{2}}, \quad \frac{G M}{4 \pi^{2}}=\frac{R^{3}}{T^{2}}
$$

Gases, liquids and solids
$P=F / A, P V=n R T, \Delta P=\rho g h$
$\left\langle(1 / 2) m v^{2}\right\rangle=(3 / 2) k_{B} T$
ideal monotonic gas: $U=(3 / 2) n R T=(3 / 2) P V$
$F_{\text {bouyant }}=\rho_{\text {displaced liq. }} . V_{\text {displaced liq. }} g$
Stress $=F / A$, Strain $=\Delta L / L, Y=$ Stress $/$ Strain
$\frac{\Delta L}{L}=\frac{F / A}{Y}, \frac{\Delta V}{V}=\frac{-\Delta P}{B}, Y=3 B$
Continuity: $\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$
Bernoulli: $P_{a}+\frac{1}{2} \rho_{a} v_{a}^{2}+\rho_{a} g h_{a}=P_{b}+\frac{1}{2} \rho_{b} v_{b}^{2}+\rho_{b} g h_{b}$
Heat
$\Delta L / L=\alpha \Delta T, \Delta V / V=3 \alpha \Delta T$
$Q=m C_{v} \Delta T+m L$ (if phase trans.)

## Conduction and Radiation

$P=k A\left(T_{b}-T_{a}\right) / L=A\left(T_{b}-T_{a}\right) / R, R \equiv L / k$
$P=e \sigma A T^{4}$

## Thermodynamics

$\Delta U=Q+W, \quad W=-P \Delta V, Q=T \Delta S, \Delta S>0$
Engines: $W=\left|Q_{H}\right|-\left|Q_{L}\right|$
$\epsilon=W / Q_{H}<\left(T_{H}-T_{L}\right) / T_{H}<1$
Refrigerators and heat pumps: $W=\left|Q_{H}\right|-\left|Q_{L}\right|$
$\epsilon=Q_{L} / W<T_{L} /\left(T_{H}-T_{L}\right)$
Simple Harmonic Motion and Waves
$f=1 / T, \omega=2 \pi f$
$x(t)=A \cos (\omega t-\phi), v=-\omega A \sin (\omega t-\phi)$
$a=-\omega^{2} A \cos (\omega t-\phi)$
Spring: $\omega=\sqrt{k / m}$
Pendulum: $T=2 \pi \sqrt{L / g}$
Waves: $y(x, t)=A \sin [2 \pi(f t-x / \lambda+\delta)], v=f \lambda$
$I=\mathrm{const} A^{2} f^{2}, I_{2} / I_{1}=R_{1}^{2} / R_{2}^{2}$
Standing waves: $\lambda_{n}=2 L / n$
Strings: $v=\sqrt{T / \mu}$
Solid/Liquid: $v=\sqrt{B / \rho}$
Sound: $I=$ Power $/ A=I_{0} 10^{\beta / 10}, I_{0} \equiv 10^{-12} \mathrm{~W} / \mathrm{m}^{2}$
Decibels: $\beta=10 \log _{10}\left(I / I_{0}\right)$
Beat freq. $=\left|f_{1}-f_{2}\right|$
Doppler: $f_{\text {obs }}=f_{\text {source }}\left(V_{\text {sound }} \pm v_{\text {obs }}\right) /\left(V_{\text {sound }} \pm v_{\text {source }}\right)$
Pipes: same at both ends: $L=\lambda / 2, \lambda, 3 \lambda / 2$
Pipes: open at only one end: $L=\lambda / 4,3 \lambda / 4,5 \lambda / 4 \cdots$

2 pt Consider $\mathrm{A}=42 \mathrm{~m}$ and $\mathrm{B}=3 \mathrm{~s}^{2}$. Please identify if the operations below are possible or not, and if they are, whether or not the answer given is correct.
$\triangleright$ You can divide A by $\mathrm{B}(\mathrm{A} / \mathrm{B})$, and get $14 \mathrm{~m} / \mathrm{s}^{2}$.

1. $\mathbf{A} \bigcirc$ This is impossible

B This is possible, but the answer is false
$\mathbf{C} \bigcirc$ This is correct
$\triangleright$ You can subtract $B$ from $A(A-B)$, and get 39 m .
2. $\mathbf{A} \bigcirc$ This is impossible
$\mathbf{B} \bigcirc$ This is possible, but the answer is false
$\mathbf{C} \bigcirc$ This is correct


Consider the five identical massless springs shown above supporting the three identical masses as shown. The presence of the mass M in "a" stretched the spring by an amount $\mathrm{x}_{\mathrm{a}}$. Similarly, the UPPER spring in "b" was stretched by $\mathrm{x}_{\mathrm{b}}$ and the LEFT-SIDE spring in "c" was stretched by an amount $\mathrm{x}_{\mathrm{c}}$.

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\triangleright x _ { c } \text { is _-------- } x _ { a }
    3. A}\bigcirc\mathrm{ one fourth }\mathbf{B}\bigcirc\mathrm{ one half
    C\bigcirc equal to }\mathbf{D}\bigcirc\mathrm{ twice }\mathbf{E}\bigcirc4\mathrm{ times
\triangleright x _ { b } \text { is _--------- }
    4. A}\bigcirc\mathrm{ one fourth }\mathbf{B}\bigcirc\mathrm{ one half
    C\bigcirc equal to }\mathbf{D}\bigcirc\mathrm{ twice }\mathbf{E}\bigcirc4\mathrm{ times
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3 pt An automobile slides across an icy parking lot at a speed of $30 \mathrm{~km} / \mathrm{hr}$ and collides with a parked car of equal mass. Immediately after the collision the cars lock and slide together. Assuming the icy surface can be treated as frictionless,
$\triangleright$ The total kinetic energy (KE of car $1+\mathrm{KE}$ of car 2 ) is conserved in the collision.
5. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$\triangleright$ The collision is perfectly elastic.
6. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$\triangleright$ Just after the collision, the cars move with a velocity equal to one half the initial velocity of the incoming car.
7. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False


The two beautifully drawn astronauts reside in a space station which is rotating with angular frequency $\omega$ in deep space. The astronauts reside inside the rotating tube, whose inner and outer radius are pictured.
$\triangleright$ If the space station is rotating clockwise, which astronaut(s) are able to maintain their position due to the simulated gravity?
8. $\mathbf{A} \bigcirc \mathrm{A} \quad \mathrm{B} \bigcirc \mathrm{B} \quad \mathrm{C} \bigcirc \mathrm{A}$ and B
$\triangleright$ If the space station is rotating counter-clockwise, which astronaut(s) are able to maintain their position due to the simulated gravity?
9. $\mathbf{A} \bigcirc \mathrm{A} \quad \mathbf{B} \bigcirc \mathrm{B} \quad \mathbf{C} \bigcirc \mathrm{A}$ and B
$\triangleright$ If the rotational frequency is doubled to $2 \omega$, the astronauts feel an increase of their simulated weight by a factor of $\qquad$
$\begin{array}{llll}\text { 10. } & \mathbf{A} \bigcirc 1 \text { (no increase) } & \mathbf{B} \bigcirc \operatorname{sqrt}(2) & \mathbf{C} \bigcirc 2 \\ \mathbf{D} \bigcirc \operatorname{sqrt}(8) & \mathbf{E} \bigcirc 4 & \mathbf{F} \bigcirc 8\end{array}$
$1 p t$ A 3.5 kg object is suspended from a spring of spring constant $725 \mathrm{~N} / \mathrm{m}$. Starting from the equilibrium position, the object has an initial speed of $7.9 \mathrm{~m} / \mathrm{s}$. What is the maximum displacement of the object during its subsequent motion? (in cm)

| 11.A $\bigcirc 34.3$ | $\mathbf{B} \bigcirc 40.1$ | $\mathbf{C} \bigcirc 46.9$ | $\mathbf{D} \bigcirc 54.9$ |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 64.2$ | $\mathbf{F} \bigcirc 75.1$ | $\mathbf{G} \bigcirc 87.9$ | $\mathbf{H} \bigcirc 102.9$ |

1 pt A 135 W light bulb is left on for 3 hours. What energy is used by the bulb? (in J)

$$
\begin{array}{rlll}
\text { 12. } \mathbf{A} \bigcirc 7.00 \times 10^{5} & \mathbf{B} \bigcirc 7.91 \times 10^{5} & \mathbf{C} \bigcirc & 8.94 \times 10^{5} \\
\mathbf{D} \bigcirc 1.01 \times 10^{6} & \mathbf{E} \bigcirc 1.14 \times 10^{6} & \mathbf{F} \bigcirc & 1.29 \times 10^{6} \\
\mathbf{G} \bigcirc 1.46 \times 10^{6} & \mathbf{H} \bigcirc 1.65 \times 10^{6} & &
\end{array}
$$

1 pt The diameter of the main rotor of a single-engine helicopter is 13.8 m . Find the number of revolutions per second (in Hz or $1 / \mathrm{s}$ ) necessary for the tip of the rotor to reach the speed of sound, $v_{\text {sound }}=343 \mathrm{~m} / \mathrm{s}$.

| $\mathbf{1 3 . A} \bigcirc 7.91$ | $\mathbf{B} \bigcirc$ | 8.94 | $\mathbf{C} \bigcirc 10.10$ | $\mathbf{D} \bigcirc$ | 11.42 |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 12.90$ | $\mathbf{F} \bigcirc$ | 14.58 | $\mathbf{G} \bigcirc 16.47$ | $\mathbf{H} \bigcirc$ | 18.61 |

$1 p t$ The launching mechanism of a toy gun consists of a spring whose spring is compressed a distance 4.3 cm before launching. If the maximum height to which the gun can launch a $25-\mathrm{g}$ projectile is 33 m , what is the spring constant? (in $\mathrm{N} / \mathrm{m}$ )

| $\mathbf{1 4 . A} \bigcirc$ | 4482 | $\mathbf{B} \bigcirc$ | 5603 | $\mathbf{C} \bigcirc$ | 7003 | $\mathbf{D} \bigcirc$ |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | 10943 | $\mathbf{F} \bigcirc$ | 13678 | $\mathbf{G} \bigcirc$ | 17098 | $\mathbf{H} \bigcirc$ |



The height of the block in the picture is $\mathrm{b}=3.2 \mathrm{~m}$, and the length of the base is $\mathrm{a}=4.8 \mathrm{~m}$. The coefficient of friction is chosen so that the box slides down the plane at constant velocity. If a 30 kg box slides down the entire length of the plane, what is the thermal energy generated by friction? (in J)

| $\mathbf{1 5 . A} \bigcirc 833.4$ | $\mathbf{B} \bigcirc 941.8$ | $\mathbf{C} \bigcirc 1064.2$ |  |
| ---: | :--- | :--- | :--- |
| $\mathbf{D} \bigcirc 1202.5$ | $\mathbf{E} \bigcirc 1358.9$ | $\mathbf{F} \bigcirc 1535.5$ |  |
| $\mathbf{G} \bigcirc 1735.1$ | $\mathbf{H} \bigcirc 1960.7$ |  |  |

1 pt A railroad car of mass 18000 kg is released from rest in a railway switchyard and rolls to the bottom of a slope $\mathrm{H}_{\mathrm{i}}$ $=21 \mathrm{~m}$ below its original height. At the low point, it collides with and sticks to another car of identical mass. The two cars roll together up another slope and climb up to a height $H_{f}$ above the low point, where they come to a stop before rolling back. Ignore the effects of friction and calculate $\mathrm{H}_{\mathrm{f}}$. (in m)

| $\mathbf{1 6 . A} \bigcirc 0.95$ | $\mathbf{B} \bigcirc 1.26$ | $\mathbf{C} \bigcirc 1.68$ | $\mathbf{D} \bigcirc 2.23$ |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 2.97$ | $\mathbf{F} \bigcirc 3.95$ | $\mathbf{G} \bigcirc 5.25$ | $\mathbf{H} \bigcirc 6.98$ |

$1 p t$ A rock is dropped from outer space (initial velocity=0) at a radius $R=1.65 \mathrm{E}+7 \mathrm{~m}$ from Earth's center. What is it's speed when it strikes the surface of the Earth? (in $\mathrm{m} / \mathrm{s}$ ) (Ignore the air resistance)
DATA: $R_{\text {earth }}=6.38 \times 10^{6} \mathrm{~m}, M_{\text {earth }}=5.98 \times 10^{24} \mathrm{~kg}$.

| $\mathbf{1 7} . \mathbf{A} \bigcirc 1.37 \times 10^{3}$ | $\mathbf{B} \bigcirc 1.98 \times 10^{3}$ | $\mathbf{C} \bigcirc 2.87 \times 10^{3}$ |  |
| ---: | :--- | :--- | :--- |
| $\mathbf{D} \bigcirc 4.17 \times 10^{3}$ | $\mathbf{E} \bigcirc 6.04 \times 10^{3}$ | $\mathbf{F} \bigcirc 8.76 \times 10^{3}$ |  |
| $\mathbf{G} \bigcirc 1.27 \times 10^{4}$ | $\mathbf{H} \bigcirc 1.84 \times 10^{4}$ |  |  |



A hungry 980-N bear walks out on a beam in an attempt to retrieve some "goodies" hanging at the end as shown above. The beam is uniform, weighs 280 N , and is 5 m long. The goodies weigh 98 N .

$$
p t
$$

When the bear is at $\mathrm{x}=0.75 \mathrm{~m}$, what is the tension in the blue support wire? (in N )

| $\mathbf{1 8 . A} \bigcirc$ | 267 | $\mathbf{B} \bigcirc$ | 302 | $\mathbf{C} \bigcirc 341$ | $\mathbf{D} \bigcirc 385$ |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | 435 | $\mathbf{F} \bigcirc$ | 492 | $\mathbf{G} \bigcirc$ | 556 |
| $\mathbf{H} \bigcirc$ | 628 |  |  |  |  |

## $p t$

If the maximum tension that the wire can sustain is 784 N , what is the maximum distance $x_{\text {max }}$ that the bear may walk before the wire breaks? (in m)

| $\mathbf{1 9 . A} \bigcirc 1.18$ | $\mathbf{B} \bigcirc 1.57$ | $\mathbf{C} \bigcirc 2.09$ | $\mathbf{D} \bigcirc 2.79$ |  |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 3.71$ | $\mathbf{F} \bigcirc 4.93$ | $\mathbf{G} \bigcirc 6.55$ | $\mathbf{H} \bigcirc$ | 8.72 |

1 pt Supergirl, who has a weight of 105 lbs , claims that at top speed she has the same momentum as a 5 ton truck moving at 55 mph . What is Supergirl's top speed in mph? (One ton $=2000 \mathrm{lbs}$ )

| $\mathbf{2 0 . A} \bigcirc$ | 3352 | $\mathbf{B} \bigcirc 4190$ | $\mathbf{C} \bigcirc$ | 5238 | $\mathbf{D} \bigcirc 6548$ |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | 8185 | $\mathbf{F} \bigcirc$ | 10231 | $\mathbf{G} \bigcirc$ | 12788 |
| $\mathbf{H} \bigcirc$ | 15985 |  |  |  |  |

