## Scott Pratt

## Do not open exam until instructed to do so.

## Quadratic Formula

$a x^{2}+b x+c=0$,
$x=\left[-b \pm \sqrt{b^{2}-4 a c}\right] /(2 a)$

## Geometry

Circle: circumference $=2 \pi R$, area $=\pi R^{2}$
Sphere: area $=4 \pi R^{2}$, volume $=4 \pi R^{3} / 3$
Trigonometry


$$
\begin{gathered}
\sin \alpha=\frac{A}{C}, \quad \cos \alpha=\frac{B}{C} \\
\tan \alpha=\frac{A}{B}
\end{gathered}
$$

B

$$
\frac{\sin \alpha}{A}=\frac{\sin \beta}{B}=\frac{\sin \gamma}{C}
$$

Polar Coordinates
$x=r \cos \theta, \quad y=r \sin \theta, r=\sqrt{x^{2}+y^{2}}, \quad \tan \theta=y / x$
SI Units and Constants

| quantity | unit | abbreviation |
| :---: | :---: | :---: |
| Mass $m$ | kilograms | kg |
| Distance $x$ | meters | m |
| Time $t$ | seconds | s |
| Force $F$ | Newtons | $\mathrm{N}=\mathrm{kg} \mathrm{m} / \mathrm{s}^{2}$ |
| Energy $E$ | Joules | $\mathrm{J}=\mathrm{N} \mathrm{m}$ |
| Power $P$ | Watts | $\mathrm{W}=\mathrm{J} / \mathrm{s}$ |
| Temperature $T$ | ${ }^{\circ} \mathrm{C},{ }^{\circ} \mathrm{K}$ or ${ }^{\circ} \mathrm{F}$ | $T_{\circ} F=32+(9 / 5) T_{\circ}{ }^{\circ} \mathrm{C}$ |
| Pressure $P$ | Pascals | $\mathrm{Pa}=\mathrm{N} / \mathrm{m}^{2}$ |

1 cal=4.1868 J, $1 \mathrm{hp}=745.7 \mathrm{~W}, 1$ liter $=10^{-3} \mathrm{~m}^{3}$
$g=9.81 \mathrm{~m} / \mathrm{s}^{2}, \mathrm{G}=6.67 \times 10^{-11} \mathrm{Nm}^{2} / \mathrm{kg}^{2}$
$1 \mathrm{~atm}=1.013 \times 10^{5} \mathrm{~Pa}, 0^{\circ} \mathrm{C}=273.15^{\circ} \mathrm{K}, N_{A}=6.023 \times 10^{23}$
$R=8.31 \mathrm{~J} /\left(\mathrm{mol}^{\circ} \mathrm{K}\right)=0.0821 \mathrm{~L} \mathrm{~atm} /(\mathrm{mol} \mathrm{K})$,
$k_{B}=R / N_{A}=1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}, \sigma=5.67 \times 10^{-8} \mathrm{~W} /\left(\mathrm{m}^{2} \mathrm{~K}^{4}\right)$
$v_{\text {sound }}=331 \sqrt{T / 273} \mathrm{~m} / \mathrm{s}$
$\mathrm{H}_{2} 0: c_{\text {ice,liq.,steam }}=\{0.5,1.0,0.48\} \mathrm{cal} / \mathrm{g}^{\circ} \mathrm{C}$
$L_{F, V}=\{79.7,540\} \mathrm{cal} / \mathrm{g}, \rho=1000 \mathrm{~kg} / \mathrm{m}^{3}$.
1-d motion, constant $a$
$\Delta x=(1 / 2)\left(v_{0}+v_{f}\right) t$
$v_{f}=v_{0}+a t$
$\Delta x=v_{0} t+(1 / 2) a t^{2}$
$\Delta x=v_{f} t-(1 / 2) a t^{2}$
$(1 / 2) v_{f}^{2}-(1 / 2) v_{0}^{2}=a \Delta x$
Range: $R=\left(v_{0}^{2} / g\right) \sin 2 \theta$
Forces, Work, Energy, Power, Momentum \& Impulse
$F=m a$, Gravity: $F=m g, P E=m g h$
Friction: $f=\mu N$, Spring: $F=-k x, P E=(1 / 2) k x^{2}$
$W=F x \cos \theta, K E=(1 / 2) m v^{2}, P=\Delta E / \Delta t=F v$
$p=m v, I=F \Delta t=\Delta p$
$X_{c m}=\left(m_{1} x_{1}+m_{2} x_{2}+\cdots\right) /\left(m_{1}+m_{2}+\cdots\right)$
Elastic coll.s: $v_{1}^{\prime}-v_{2}^{\prime}=-\left(v_{1}-v_{2}\right)$

## Rotational Motion

$\Delta \theta=(1 / 2)\left(\omega_{0}+\omega_{f}\right) t, \omega_{f}=\omega_{0}+\alpha t$
$\Delta \theta=\omega_{0} t+(1 / 2) \alpha t^{2}=\omega_{f} t-(1 / 2) \alpha t^{2}$
$\alpha \Delta \theta=(1 / 2) \omega_{f}^{2}-(1 / 2) \omega_{0}^{2}$
$\omega=2 \pi / T=2 \pi f, f=1 / T$
Rolling: $a=\alpha r, v=\omega r$
$a_{c}=v^{2} / r=\omega v=\omega^{2} r$
$\tau=r F \sin \theta=I \alpha, I_{\text {point }}=m R^{2}$
$I_{\text {cyl.shell }}=M R^{2}, I_{\text {sphere }}=(2 / 5) M R^{2}$
$I_{\text {solid cyl. }}=(1 / 2) M R^{2}, I_{\text {sph. shell }}=(2 / 3) M R^{2}$
$L=I \omega=m v r \sin \theta,(\theta=$ angle between v and r$)$
$K E=(1 / 2) I \omega^{2}=L^{2} /(2 I), W=\tau \Delta \theta$
Gravity and circular orbits
$P E=-G \frac{M m}{r}, \Delta P E=m g h($ small $h)$

$$
F=G \frac{M m}{r^{2}}, \quad \frac{G M}{4 \pi^{2}}=\frac{R^{3}}{T^{2}}
$$

Gases, liquids and solids
$P=F / A, P V=n R T, \Delta P=\rho g h$
$\left\langle(1 / 2) m v^{2}\right\rangle=(3 / 2) k_{B} T$
ideal monotonic gas: $U=(3 / 2) n R T=(3 / 2) P V$
$F_{\text {bouyant }}=\rho_{\text {displaced liq. }} . V_{\text {displaced liq. }} g$
Stress $=F / A$, Strain $=\Delta L / L, Y=$ Stress $/$ Strain
$\frac{\Delta L}{L}=\frac{F / A}{Y}, \frac{\Delta V}{V}=\frac{-\Delta P}{B}, Y=3 B$
Continuity: $\rho_{1} A_{1} v_{1}=\rho_{2} A_{2} v_{2}$
Bernoulli: $P_{a}+\frac{1}{2} \rho_{a} v_{a}^{2}+\rho_{a} g h_{a}=P_{b}+\frac{1}{2} \rho_{b} v_{b}^{2}+\rho_{b} g h_{b}$
Thermal
$\Delta L / L=\alpha \Delta T, \Delta V / V=\beta \Delta T, \beta=3 \alpha$
$Q=m C_{v} \Delta T+m L$ (if phase trans.)

## Conduction and Radiation

$P=k A\left(T_{b}-T_{a}\right) / L=A\left(T_{b}-T_{a}\right) / R, R \equiv L / k$
$P=e \sigma A T^{4}$
Thermodynamics
$\Delta U=Q+W, \quad W=-P \Delta V, Q=T \Delta S, \Delta S>0$
Engines: $W=\left|Q_{H}\right|-\left|Q_{L}\right|$
$\epsilon=W / Q_{H}<\left(T_{H}-T_{L}\right) / T_{H}<1$
Refrigerators and heat pumps: $W=\left|Q_{H}\right|-\left|Q_{L}\right|$
$\epsilon=Q_{L} / W<T_{L} /\left(T_{H}-T_{L}\right)$
Simple Harmonic Motion and Waves
$f=1 / T, \omega=2 \pi f$
$x(t)=A \cos (\omega t-\phi), v=-\omega A \sin (\omega t-\phi)$
$a=-\omega^{2} A \cos (\omega t-\phi)$
Spring: $\omega=\sqrt{k / m}$
Pendulum: $T=2 \pi \sqrt{L / g}$
Waves: $y(x, t)=A \sin [2 \pi(f t-x / \lambda)+\delta], v=f \lambda$
$I=\mathrm{const} A^{2} f^{2}, I_{2} / I_{1}=R_{1}^{2} / R_{2}^{2}$
Standing waves: $\lambda_{n}=2 L / n$
Strings: $v=\sqrt{T / \mu}$
Solid/Liquid: $v=\sqrt{B / \rho}$
Sound: $I=$ Power $/ A=I_{0} 10^{\beta / 10}, I_{0} \equiv 10^{-12} \mathrm{~W} / \mathrm{m}^{2}$
Decibels: $\beta=10 \log _{10}\left(I / I_{0}\right)$
Beat freq. $=\left|f_{1}-f_{2}\right|$
Doppler: $f_{\text {obs }}=f_{\text {source }}\left(V_{\text {sound }} \pm v_{\text {obs }}\right) /\left(V_{\text {sound }} \pm v_{\text {source }}\right)$
Pipes: same at both ends: $L=\lambda / 2, \lambda, 3 \lambda / 2$
Pipes: open at only one end: $L=\lambda / 4,3 \lambda / 4,5 \lambda / 4 \cdots$


The two beautifully drawn astronauts reside in a space station which is rotating with angular frequency $\omega_{0}$ in deep space. The astronauts reside inside the rotating tube, whose inner and outer radius are pictured.
$1 p t$
$\triangleright$ If the space station is rotating counter-clockwise, which astronaut(s) are able to maintain their position due to the simulated gravity?

$\triangleright$ An astronaut standing on a scale measures her weight to be $W_{0}$ when the frequency is $\omega_{0}$. If the frequency is tripled to $\omega=3 \omega_{0}$, the astronaut now measures her weight to be $W=\ldots-\ldots-\quad W_{0}$
2. $\mathbf{A} \bigcirc 1$ (no increase) $\quad \mathbf{B} \bigcirc \operatorname{sqrt}(3) \quad \mathbf{C} \bigcirc 3$ $\mathbf{D} \bigcirc 1 /$ sqrt (3) $\quad \mathbf{E} \bigcirc 9 \quad \mathbf{F} \bigcirc 1 / 9$
$2 p t$ Some intellectually curious physics students hold a rolling race by rolling items down a steep hill. Contestant A is a heavy wagon supported by four heavy wagon wheels. Contestant B is a single isolated wagon wheel, identical to what was used by the wagon. The weight of the wagon, including the wheels, is 3200 lbs , half of which is in the wheels. The weight of the isolated wheel is 400 lbs. Assume that the objects roll without slipping and that air resistance and rolling resistance are negligible.
$\triangleright$ At the finish line, the kinetic energy (both rotational and linear) of the wagon (Contestant A) is 8 times larger than the kinetic energy of the isolated wheel (Contestant B).
3. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$\triangleright$ The wagon wins the race.
4. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$2 p t$ A boulder rests on the bottom of Lake Michigan. The weight of the boulder out of the water is $W$, the contact force between the lake bottom and the boulder is $N$, and the buoyant force acting on the boulder is $B$.
$\triangleright$ Defining upward as positive, the net force acting on the boulder (buoyant + gravitational + contact) is zero.
5. $\mathbf{A} \bigcirc$ CANNOT BE DETERMINED
$\mathbf{B} \bigcirc$ greater than $\mathbf{C} \bigcirc$ less than $\mathbf{D} \bigcirc$ equal to
$\triangleright$ The magnitude of $B$ is $\qquad$ the magnitude of $W$.
6. A $\bigcirc$ CANNOT BE DETERMINED
$\mathbf{B} \bigcirc$ greater than $\mathbf{C} \bigcirc$ less than $\mathbf{D} \bigcirc$ equal to
$2 p t$ A fixed number of moles of an ideal gas are kept in a container of volume V and an absolute temperature T .
$\triangleright$ If T doubles while V is held constant, the new net internal energy of the gas will be $\qquad$ times the original internal energy of the gas.
7. $\mathbf{A} \bigcirc 1 / 4 \quad \mathbf{B} \bigcirc 1 / 2 \quad \mathbf{C} \bigcirc 1 / \operatorname{sqrt}(2)$ $\mathbf{D} \bigcirc$ equal to $\mathbf{E} \bigcirc \operatorname{sqrt}(2) \quad \mathbf{F} \bigcirc 2 \quad \mathbf{G} \bigcirc 4$
$\triangleright$ If T and V are both doubled, the new r.m.s. velocity of the molecules in the gas will be $\qquad$ times the original r.m.s. molecular velocity.

$$
\begin{array}{ll}
\text { 8. } & \mathbf{A} \bigcirc 1 / 4 \quad \mathbf{B} \bigcirc 1 / 2 \quad \mathbf{C} \bigcirc 1 / \operatorname{sqrt}(2) \\
\mathbf{D} \bigcirc \text { equal to } \mathbf{E} \bigcirc \operatorname{sqrt}(2) & \mathbf{F} \bigcirc 2 \quad \mathbf{G} \bigcirc 4
\end{array}
$$

$2 p t$ A cylinder filled with helium is divided into two halves, A and B , by a copper piston which does not allow the molecules to pass from A to B. Initially, each half has the same amount of gas at the same pressure and temperature. Then, suddenly, side A is heated to a higher temperature without changing the amount of gas or volume in either side. The piston is then allowed to move freely. The cylinder + piston system is well insulated from the outside world.
$\triangleright$ After A and B eventually equilibrate with one another, the two sides will once again have equal volumes.
9. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$\triangleright$ After the heating, energy will flow from A to B due to convection.
10. $\mathbf{A} \bigcirc$ True $\mathbf{B} \bigcirc$ False
$1 p t$ A mass of 44 kg is suspended from a steel wire of diameter 1.9 mm and length 0.8 m . How much does the wire stretch in mm? Young's modulus for steel is $2.0 \times 10^{11} \mathrm{~N} / \mathrm{m}^{2}$.

| $\mathbf{1 1 . A} \bigcirc 0.389$ | $\mathbf{B} \bigcirc$ | 0.487 | $\mathbf{C} \bigcirc 0.608$ | $\mathbf{D} \bigcirc$ |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 0.950$ | $\mathbf{F} \bigcirc 1.188$ | $\mathbf{G} \bigcirc 1.485$ | $\mathbf{H} \bigcirc 1.856$ |  |

1 pt In order to facilitate cleaning, water is to be pumped to the top of a giant new Sparty statue, which will be 315 m high. What gauge pressure (the difference between the absolute pressure and the pressure of the surrounding atmosphere) is needed in the water line at the base of the building to raise the water to this height? (in Pa )

$$
\begin{array}{rlll}
\text { 12. } \mathbf{A} \bigcirc 1.27 \times 10^{6} & \mathbf{B} \bigcirc 1.58 \times 10^{6} & \mathbf{C} \bigcirc 1.98 \times 10^{6} \\
\mathbf{D} \bigcirc 2.47 \times 10^{6} & \mathbf{E} \bigcirc 3.09 \times 10^{6} & \mathbf{F} \bigcirc & 3.86 \times 10^{6} \\
\mathbf{G} \bigcirc 4.83 \times 10^{6} & \mathbf{H} \bigcirc 6.04 \times 10^{6} & &
\end{array}
$$

1 pt A pipe of diameter 6.5 cm carries water without turbulence. The velocity of the water is $15.3 \mathrm{~cm} / \mathrm{s}$. Downstream from this section, the pipe narrows to a diameter of 3.4 cm . What is the velocity of the water at this point? (in $\mathrm{cm} / \mathrm{s}$ )

| $\mathbf{1 3 . A} \bigcirc 35.8$ | $\mathbf{B} \bigcirc 44.7$ | $\mathbf{C} \bigcirc 55.9$ | $\mathbf{D} \bigcirc 69.9$ |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc 87.4$ | $\mathbf{F} \bigcirc 109.2$ | $\mathbf{G} \bigcirc 136.5$ | $\mathbf{H} \bigcirc 170.7$ |

$1 p t$ An immersion heater has a power rating of 1300 watts. It is used to heat water for coffee. How many liters of water can be brought from room temperature $\left(20^{\circ} \mathrm{C}\right)$ to $90^{\circ} \mathrm{C}$ in 60 minutes?

| $\mathbf{1 4 . A} \bigcirc 4.19$ | $\mathbf{B} \bigcirc$ | 5.23 | $\mathbf{C} \bigcirc 6.54$ | $\mathbf{D} \bigcirc$ | 8.18 |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | 10.22 | $\mathbf{F} \bigcirc$ | 12.78 | $\mathbf{G} \bigcirc$ | 15.97 |
| $\mathbf{1} \bigcirc$ | $\mathbf{H} \bigcirc$ | 19.96 |  |  |  |

$1 p t$ An ideal gas at fixed volume is originally at a temperature $\mathrm{T}=85$ degrees C when the pressure is measured at 1.9 atm . The temperature is then dropped to -55 degrees C . What is the new pressure? (in atm)

| $\mathbf{1 5 . A} \bigcirc$ | 0.26 | $\mathbf{B} \bigcirc$ | 0.38 | $\mathbf{C} \bigcirc$ | 0.55 |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | $\mathbf{D} .80$ |  |  |  |  |
| $\mathbf{E} \bigcirc$ | 1.16 | $\mathbf{F} \bigcirc$ | 1.68 | $\mathbf{G} \bigcirc$ | 2.43 |
| $\mathbf{H} \bigcirc$ | 3.53 |  |  |  |  |

$1 p t$ A car is designed to get its energy from a rotating flywheel with a radius of 1.85 m and a mass of 575 kg . The flywheel is shaped like a pancake and can be considered as a uniform cylinder. Before a trip, the flywheel is attached to an electric motor, which brings the flywheel's rotational speed up to $4600 \mathrm{rev} / \mathrm{min}$. If the flywheel is to supply energy to the car as would a 9500 Watt motor, find the time (in minutes) the car could run before the flywheel would have to be brought back up to speed.

| 16.A $\bigcirc 200$ | $\mathbf{B} \bigcirc 290$ | $\mathbf{C} \bigcirc 421$ | $\mathbf{D} \bigcirc 611$ |  |
| ---: | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | 885 | $\mathbf{F} \bigcirc$ | 1284 | $\mathbf{G} \bigcirc$ |
| 1861 | $\mathbf{H} \bigcirc$ | 2699 |  |  |

1 pt The surface temperature of Star Y is 8100 K and the power output of the star is $5.30 \mathrm{E}+27 \mathrm{~W}$. If the temperature increases by a factor of 1.19 while keeping the size of the star fixed, what will the new power output be? (in W)

| $\mathbf{1 7 . A} \bigcirc 2.64 \times 10^{27}$ | $\mathbf{B} \bigcirc 3.15 \times 10^{27}$ |
| ---: | :--- |
| $\mathbf{C} \bigcirc 3.74 \times 10^{27}$ | $\mathbf{D} \bigcirc 4.45 \times 10^{27}$ |
| $\mathbf{E} \bigcirc 7.51 \times 10^{27}$ | $\mathbf{F} \bigcirc 8.93 \times 10^{27}$ |
| $\mathbf{G} \bigcirc 1.06 \times 10^{28}$ | $\mathbf{H} \bigcirc 1.06 \times 10^{28}$ |

1 pt A large steam pipe is covered with $1.3-\mathrm{cm}$-thick insulating material of thermal conductivity $0.29 \mathrm{~J} /\left(\mathrm{s} \mathrm{m}{ }^{\circ} \mathrm{C}\right)$. How much energy (in J ) is lost every second when the steam is at $220{ }^{\circ} \mathrm{C}$ and the outside of the pipe has a temperature of $20^{\circ} \mathrm{C}$ ? The pipe has a circumference of 7.25 m and a length of 50 m . Neglect losses through the ends of the pipe.

| $\mathbf{1 8 . A} \bigcirc 6.87 \times 10^{5}$ | $\mathbf{B} \bigcirc 9.14 \times 10^{5}$ | $\mathbf{C} \bigcirc 1.22 \times 10^{6}$ |  |
| ---: | :--- | :--- | :--- |
| $\mathbf{D} \bigcirc 1.62 \times 10^{6}$ | $\mathbf{E} \bigcirc 2.15 \times 10^{6}$ | $\mathbf{F} \bigcirc 2.86 \times 10^{6}$ |  |
| $\mathbf{G} \bigcirc 3.80 \times 10^{6}$ | $\mathbf{H} \bigcirc 5.06 \times 10^{6}$ |  |  |



A movable piston having a mass of 95 kg and a cross-sectional area of $0.006 \mathrm{~m}^{2}$ traps 0.19 mol of helium (an ideal gas) in a vertical cylinder. The piston slides without friction in the cylinder. The outer side of the piston is at atmospheric pressure, $1.013 \times 10^{5} \mathrm{~Pa}$.
$p t$
What is the pressure in the cylinder? (in Pa )

| $\mathbf{1 9 . A} \bigcirc 5.81 \times 10^{4}$ | $\mathbf{B} \bigcirc 8.42 \times 10^{4}$ | $\mathbf{C} \bigcirc 1.22 \times 10^{5}$ |  |
| ---: | :--- | :--- | :--- |
| $\mathbf{D} \bigcirc 1.77 \times 10^{5}$ | $\mathbf{E} \bigcirc 2.57 \times 10^{5}$ | $\mathbf{F} \bigcirc 3.72 \times 10^{5}$ |  |
| $\mathbf{G} \bigcirc 5.40 \times 10^{5}$ | $\mathbf{H} \bigcirc 7.82 \times 10^{5}$ |  |  |

$1 p t$ The density of a large tree is $270 \mathrm{~kg} / \mathrm{m}^{3}$, and that of sea water is $1030 \mathrm{~kg} / \mathrm{m}^{3}$. What fraction of the total volume of a tree floating in the sea water is exposed?

| $\mathbf{2 0 . A} \bigcirc$ | 0.417 | $\mathbf{B} \bigcirc$ | 0.555 | $\mathbf{C} \bigcirc$ | 0.738 |
| ---: | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{E} \bigcirc$ | $\mathbf{D} \bigcirc$ | 0.981 |  |  |  |
| 1.305 | $\mathbf{F} \bigcirc$ | 1.736 | $\mathbf{G} \bigcirc 2.309$ | $\mathbf{H} \bigcirc$ | 3.071 |

