This document replaces everything from Experimental Apparatus on page 57 of the lab manual through page 62. Pages 55 through the top of page 57 remain unchanged.

An introduction and a conclusion are not required for this experiment.

## Experimental Apparatus



In our case, the rigid body consists of two cylinders, which are placed on a metallic rod at varying radii from the axis of rotation. The cylinders and rod are supported by a rotating platform attached to a central pulley and nearly frictionless air bearings. A side view of the apparatus is shown in Figure 3a and a top view of the central pulley is shown in Figure 3 b . The rotating disk has alternating black and white bars on its circumference which are detected by a photo-diode detector as they sweep past it. The computer then uses a program written in the LabVIEW programming language to create a data table containing the angular velocity of the rotating disk and the corresponding times. You can then open the data table, copy it and paste its contents into Kaleidagraph. Recall the
slope of the angular velocity versus time plot is the angular acceleration ( $\alpha$ ) of the rotating disk.

In this experiment, we will change the moment of inertia of the rotating body by changing how the mass is distributed on the rotating body. We will place the two cylindrical masses at four different radii such that $\mathrm{r}=\mathrm{r}_{1}=\mathrm{r}_{2}$ in each of the four cases. We will then use our measurements to calculate the moment of inertia (I) for each of the four radial positions of the cylindrical masses ( r ). The sum of the two cylindrical masses $\left(m_{1}+m_{2}\right)$ can then be found from a graph of $I$ versus $r^{2}$.

To set up your rigid body, wrap the string around the central pulley (axle) and run it over the side pulley to a known weight as shown in Figure 3a.

Consider the following steps:
If we release the weight from rest, the tension in the string will exert a torque on the rigid body causing it to rotate with a constant angular acceleration $\alpha$. The angular acceleration of the rigid body is related to the linear acceleration of the falling mass by:

$$
\begin{gather*}
\alpha=\frac{\text { Linear_acceleration }}{\text { radius_of_axle }}=\frac{a}{R} \\
\text { or } \\
a=R \alpha \tag{2}
\end{gather*}
$$

From Figure 3a, the tension in the string is:

$$
\begin{equation*}
T=M g-M a \tag{3}
\end{equation*}
$$

The tension in the string causes a net torque on the rigid body. Since torque $=($ Lever arm) (Force), the net torque on the rigid body is given by:

$$
\begin{equation*}
\tau=R \times T \tag{4}
\end{equation*}
$$

The moment of inertia of the rigid body is then found from equation $1(\tau=I \alpha)$.

## PROCEDURE

## DO NOT WRITE AN INTRODUCTION OR CONCLUSION FOR THIS EXPERIMENT

The apparatus is delicate. Do not rotate the rotating table or side pulley unless air is flowing through the bearings.

Measure and record the masses of the hanging mass (M) and the two cylinders ( $\mathrm{m}_{1}$ and $\mathrm{m}_{2}$ ).

Place the cylinders on the horizontal rod such that the axes of the cylinders are along the horizontal rod. Make sure the thumbscrew on each cylinder is tightened. The center of mass of each cylinder must be the same distance ( $r$ ) from the axis of rotation (i.e. $r_{1}=r_{2}$ in Figure 3a). Estimate the uncertainty in $r$ called $\delta r$. This should include both the uncertainty in reading your ruler and the uncertainty in locating the cylinder's center of mass.

Open the LabVIEW program located in the inertia folder.
With the air supply on, attach the hanging mass ( $M$ ) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Hold the hanging mass stationary. Then release the hanging mass and immediately click on the START button in LabVIEW. The START button is the white arrow at the far left hand edge of the toolbar at the top of the screen. LabVIEW will display an angular velocity versus time plot on the monitor. In addition, it will construct a data table with the values of the angular velocity and time from its plot. LabVIEW will ask you to assign your data file a name. Transfer your data file into Kaleidagraph. To do this left click on the data file and drag it over the Kaleidagraph icon on the computer's desktop. Make a plot of angular velocity versus time. Make sure you only graph data that correspond to when the hanging mass was accelerating downward. Fit your data with a best fit line and have Kaleidagraph calculate the slope and its uncertainty ( $\alpha \pm \delta \alpha$ ) and record these values in your Excel spreadsheet. In the spreadsheet, use equations 1 through 4 to calculate the linear acceleration of the falling mass, the tension in the string, the torque on the rigid body and the moment of inertia of the rigid body. Also, calculate the uncertainty in the moment of inertia. The radius of the central pulley is $(\mathrm{R}=1.27 \pm 0.01 \mathrm{~cm})$. Be careful not to confuse the radius of the central pulley (axle) " R " with the locations of the cylindrical masses " $r$ ". In this experiment you will create four angular velocity versus time graphs -- you only need to write comments on ONE of these four graphs.

The uncertainty in I is given by:

$$
\delta I=I\left[\frac{\delta M}{M}+\frac{\delta R}{R}+\frac{\delta \alpha}{\alpha}\right]
$$

The uncertainty in $r^{2}$ is given by:

$$
\delta\left(r^{2}\right)=2 r \delta r
$$

Repeat this procedure for two additional (non-zero) values of r .
We would like to place the two cylinders at $\mathrm{r}=0$. To do this, we will use the vertical bar on the support. When you place the cylinders on the vertical bar, make sure the are
oriented the same way as in your previous trials, i.e. with the axes of the two cylinders perpendicular to the vertical bar. As before, make sure to tighten the thumbscrews on the cylinder. Measure the moment of inertia of the body with the two cylinders at $\mathrm{r}=0$. Include this data in your data table.

Transfer your data into Kaleidagraph and make a plot of I vs. $r^{2}$. Your data points should have both horizontal and vertical error bars. Also, fit your data with a best fit line, display its equation with the uncertainties in the slope and intercept.

When the two cylinders are placed on the axis of rotation, the measured moment of inertia $\mathrm{I}_{0}$ is the moment of inertia of the rotating apparatus alone plus the moment of inertia of each of the two cylinders about an axis through their own centers of mass.

$$
\begin{equation*}
I=I_{0} \tag{5}
\end{equation*}
$$

If the two masses are now each placed a distance $r$ from the axis of rotation then equation 5 becomes:

$$
\begin{equation*}
I=\left(m_{1}+m_{2}\right) r^{2}+I_{0} \tag{6}
\end{equation*}
$$

If you compare equation 6 to the form of an equation for a straight line:

$$
y=m x+b
$$

You can see that a plot of I vs. $\mathrm{r}^{2}$ should be a straight line. The slope of this line is the sum of the masses $\left(\mathrm{m}_{1}+\mathrm{m}_{2}\right)$ and the intercept is $\mathrm{I}_{0}$.

## QUESTIONS

1) The moment of inertia of a body depends not only on its mass, but also on how the mass is distributed. Do your data support this? Why or why not?
2) In your plot of I vs. $r^{2}$, why did you use $r^{2}$ and not $r$ in the plot? What are the units of the slope of I vs. $\mathrm{r}^{2}$ ?
3) Discuss the consistency of the slope of your I vs. $r^{2}$ with the value you measured for $\mathrm{m}_{1}+\mathrm{m}_{2}$.
