Name (please print): SOLUTIONS PID:

Midterm Exam #1 PHY 321 Spring 2008 Feb. 6 2008

Total points = **25**. Show all of your work!

**1. [6 points]** A football player punts the football so that it will have a "hang time" (time of flight) of 4.5 sec and land 46 m away. The ball leaves the player's foot 1.5 m above the ground. Neglect air resistance and take g = 9.81 m/s$^2$.

$x - x_0 = \text{Range} = V_0 \cos\theta\, t$

$\Rightarrow \quad 46 = V_0 \cos\theta \cdot 4.5 \quad \ldots\ldots (1)$

$y = y_0 + V_0 \sin\theta\, t - \frac{1}{2} g t^2$

$\Rightarrow 0 = 1.5 + V_0 \sin\theta \cdot 4.5 - \frac{1}{2} \cdot 9.81 \cdot t^2 \quad \ldots\ldots (2)$

(a) What must be the magnitude of the ball's initial velocity?

$(1) \Rightarrow V_0 \cos\theta = \frac{46}{4.5} = 10.22$

$(2) \Rightarrow V_0 \sin\theta = \frac{1}{4.5}\left(\frac{1}{2} \cdot 9.81 \cdot 4.5^2 - 1.5\right) = 21.74$

Square and add $\Rightarrow V_0^2 = V_0^2 \cos^2\theta + V_0^2 \sin^2\theta$

$\therefore V_0^2 = 10.22^2 + 21.74^2 = 577.1$

$\Rightarrow \boxed{V_0 = 24.0 \text{ m/s}}$

(b) What is its initial direction (angle relative to the horizontal)?

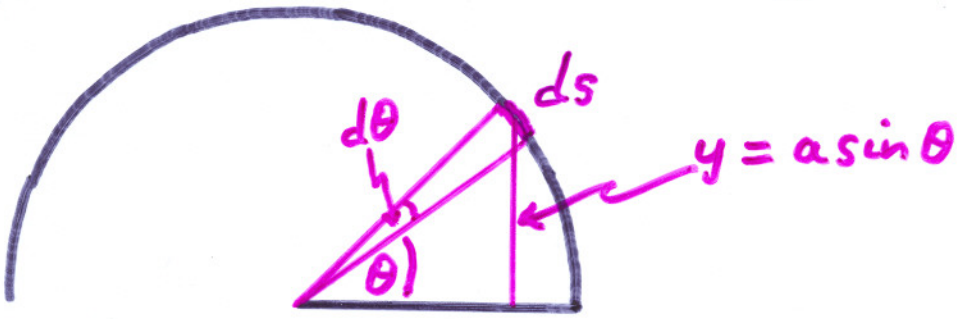
Divide (2) by (1) $\Rightarrow \frac{V_0 \sin\theta}{V_0 \cos\theta} = \tan\theta = \frac{21.74}{10.22} = 2.13$

$\Rightarrow \boxed{\theta = 64.8^\circ}$

**2. [7 points]** A thin length, L, of copper wire is bent into the shape of a semicircle as shown.

$L = \frac{1}{2}$ circumference of circle $\Rightarrow L = \pi a$

$\therefore a =$ radius of circle $= \frac{L}{\pi}$




What are the coordinates of its center of mass (CM)?

Let linear density of wire $= \lambda \Rightarrow M = \lambda L$

Increment of length $ds = a\,d\theta$

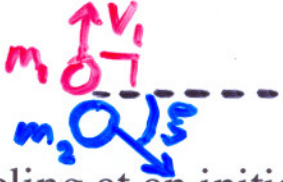
Mass of this is $dm = \lambda ds = \lambda a\,d\theta$

$\boxed{x_{cm} = 0}$ by symmetry

$$y_{cm} = \frac{1}{M}\int_0^{\pi} y\,dm = \frac{1}{M}\int_0^{\pi} a\sin\theta \cdot \lambda a\,d\theta$$

$$\Rightarrow y_{cm} = \frac{\lambda a^2}{M}\int_0^{\pi} \sin\theta\,d\theta = \frac{\lambda a^2}{M}\left[-\cos\theta\right]_0^{\pi}$$

$$\Rightarrow y_{cm} = \frac{\lambda a^2}{M}\cdot 2 = \frac{2\lambda}{\lambda L}\cdot\left(\frac{L^2}{\pi^2}\right) = \boxed{\frac{2L}{\pi^2}}$$


**3. [12 points]** A particle of mass $m_1 = 5.0$ kg traveling at an initial velocity $u_1 = 12$ m/s in the positive x direction collides with a stationary particle of mass $m_2 = 15.0$ kg. After the collision, the first particle is traveling with speed $v_1 = 4$ m/s at an angle of $\psi = 90°$ with respect to the x-axis (i.e. it is moving in the y direction). The collision is not elastic.

(a) [3 points] What is the final speed, $v_2$, of the second particle?

Conservation of momentum in X direction $\Rightarrow$

$$m_1 u_1 + 0 = 0 + m_2 v_2 \cos\xi \quad \cdots\cdots (1)$$

Conservation of momentum in y direction $\Rightarrow$

$$0 = m_1 v_1 - m_2 v_2 \sin\xi \quad \cdots\cdots (2)$$

(b) [3 points] What is the final direction of the second particle? (i.e. find angle $\zeta$)

$$(1) \Rightarrow m_2 v_2 \cos\xi = m_1 u_1 \qquad (2) \Rightarrow m_2 v_2 \sin\xi = m_1 v_1$$

$$\frac{(2)}{(1)} \Rightarrow \frac{m_2 v_2 \sin\xi}{m_2 v_2 \cos\xi} = \tan\xi = \frac{m_1 v_1}{m_1 u_1} = \frac{4}{12} \Rightarrow \boxed{\xi = 18.4°}$$

$$\text{subst. into (1)} \Rightarrow v_2 = \frac{m_1 u_1}{m_2 \cos\xi} = \frac{5 \cdot 12}{15 \cdot \cos 18.4} = \boxed{4.22 \text{ m/s}}$$

(c) [3 points] How much kinetic energy is lost in the collision?

$$\text{Initial KE} = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} \cdot 5.0 \cdot 12^2 = 360 \text{ J}$$

$$\text{Final KE} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} \cdot 5 \cdot 4^2 + \frac{1}{2} \cdot 15 \cdot 4.22^2$$

$$\Rightarrow KE_{LOST} = 360 - 40 - 134 = \boxed{186.0 \text{ J}}$$

(d) [3 points] What is the velocity of the CM frame.

$V_{cm}$ given by $m_1 u_1 + 0 = (m_1 + m_2) V_{cm}$

$$\Rightarrow V_{cm} = \frac{m_1 u_1}{m_1 + m_2} = \frac{5 \cdot 12}{5 + 15} = \frac{60}{20} = \boxed{3.0 \text{ m/s}}$$

# Thoughts on Midterm #1

1. Write down the projectile equations
   You can't put $V_y = 0$ at $y = 0$ !
   There's no acceleration in the x direction
   You seem confused by 2 equations, 2 unknowns

2. You were given $L \Rightarrow$ answer in terms of $L$
   Relate $r$ and $L$, write $M$ in terms of $L$
   Homework problem $\Rightarrow$ integrate over $\theta$
   use dimensional arguments !

3. Collision is not elastic
   $\Rightarrow$ can't use conservation of KE

# MOMENTUM IS A VECTOR !!!