Solutions to Midterm #2

1. [12 points] A particle of mass = 0.6 kg is attached to a spring with force constant k = 3.5 N/m.

(a) [3 points] What is the natural frequency ν_0 of this system?

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{3.5}{0.6}} = 2.42\ s^{-1}$$

$$\nu_0 = \frac{\omega_0}{2\pi} = \frac{2.42}{2\pi} = \boxed{0.384\ Hz}$$

(b) [5 points] The system is immersed in a viscous fluid such that any oscillation becomes critically damped. If the particle is given an initial displacement of $x_0 = 0.3$ m and an initial velocity of $v_0 = 1.4$ m/s at t = 0, derive an equation for the subsequent time dependence of the displacement x(t).

Critically damped $\Rightarrow \beta = \omega_0 = 2.42$

$$x(t) = (A + Bt)e^{-\beta t}$$

when $t=0$ $\quad x = x_0 = A \Rightarrow \boxed{A = 0.3\ m}$

$$\dot{x}(t) = -\beta A e^{-\beta t} + Be^{-\beta t} - \beta B t e^{-\beta t}$$

when $t=0$ $\quad \dot{x} = v_0 = -\beta A + B$

$$\Rightarrow B = v_0 + \beta A$$

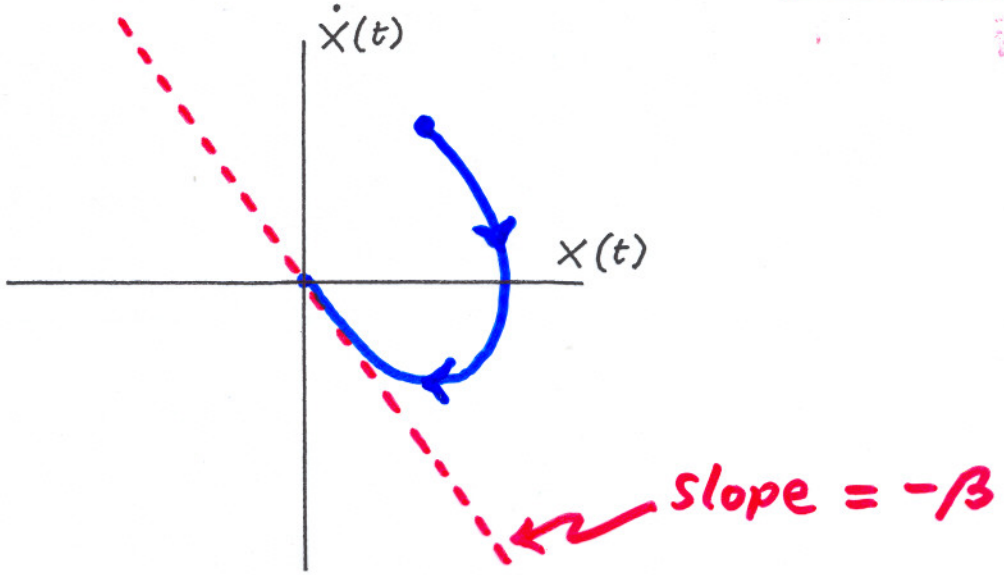
(c) [4 points] Sketch this behavior on a phase diagram of $\dot{x}(t)$ vs x(t). Show the asymptotic (i.e. t --> ∞) behavior of the ratio $\dot{x}(t)/x(t)$.

$$\therefore B = 1.4 + 2.42 \cdot 0.3$$

$$\Rightarrow \boxed{B = 2.12\ m/s}$$

$$\frac{\dot{x}(t)}{x(t)} = -\beta + \frac{B}{A+Bt}$$

$\Rightarrow -\beta$ as $t \to \infty$



2. [8 points] A bungee jumper with a mass 65.0 kg jumps from a high bridge. After reaching her lowest point, she oscillates up and down, hitting a low point eight more times in 38.0 seconds. She finally comes to rest 25.0 m below the level of the bridge.

(a) [4 points] Calculate the stiffness constant of the bungee cord.

$\nu_0 = \frac{8}{38} = 0.211$ $\quad \omega_0 = 2\pi\nu_0 = 1.32\ s^{-1}$

$\omega_0^2 = \frac{k}{m} \Rightarrow k = m\omega_0^2 = 65.0 \cdot 1.32^2$

$k = 113.7\ N/m$

(b) [4 points] What is the unstretched length of the bungee cord?

In final position $kx = mg$

$\therefore x = \frac{mg}{k} = \frac{65.0 \cdot 9.81}{113.7} = 5.61\ m$

25 m = unstretched length + 5.61

$\Rightarrow$ unstretched length, $l_0 = 25 - 5.61$

19.4 m

3. [5 points] A particle moves in one dimension with a potential energy plotted below. Make a sketch of the trajectories in phase space corresponding to the five total energies shown in the figure.