Physics 472 – Spring 2008

Homework #3, due Friday, February 1

1. Griffiths problem 4.34. We did this in class, but it's a good warm-up for the later problems.

2. a) Consider an electron in a hydrogen atom state with l=1. The quantum number j that represents the total electron angular momentum $\vec{J} = \vec{L} + \vec{S}$ can take the values j=3/2 or j=½. In class we showed how to use the lowering operator to express the eigenstates of J^2 and J_z, labeled $\left| j, m_j \right\rangle$, in terms of linear combinations of the states $\left| l, m_l \right\rangle \otimes \left| s, m_s \right\rangle$. Follow the same procedure here to construct all four states of the j=3/2 ladder and both states of the j=1/2 ladder. Check your results using the Clebsch-Gordan coefficients in Table 4.8.

b) Do Griffiths problem 4.55, parts (e) and (f). Note that you have to invert the answers you got to part (a), i.e. you need to express the states $\left| l, m_l \right\rangle \otimes \left| s, m_s \right\rangle$ in terms of the states $\left| j, m_j \right\rangle$.

3. a) Consider an electron in a state with arbitrary orbital angular momentum l. Follow the same procedure you used in the previous problem to construct the eigenstates of J^2 and J_z, $\left| j, m_j \right\rangle$, as linear combinations of the states $\left| l, m_l \right\rangle \otimes \left| s, m_s \right\rangle$. Do this only for the top two states of the highest j ladder and the top state of the next j ladder.

   b) Do Griffiths problem 4.36. For part (a) you may use Table 4.8. For part (b), use the results you derived in problem 2 above.

4. Define the total angular momentum as $\vec{J} = \vec{L} + \vec{S}$. Using only the canonical commutation relations for $\vec{L}$ and $\vec{S}$ and the fact that $\left[\vec{L}, \vec{S}\right] = 0$, show that the spin-orbit interaction commutes with the total angular momentum: $\left[\vec{L} \cdot \vec{S}, \vec{J}\right] = 0$. Do this in the following two ways:

   a) First, show that $\vec{L} \cdot \vec{S}$ commutes with one component of $\vec{J}$, i.e. show that $\left[\vec{L} \cdot \vec{S}, J_z\right] = 0$. It follows by rotational symmetry that $\vec{L} \cdot \vec{S}$ commutes with the other two components of $\vec{J}$.

   b) Second, write $\vec{L} \cdot \vec{S} = \frac{1}{2}\left(J^2 - L^2 - S^2\right)$ and show that $\left[L^2, \vec{J}\right] = \left[S^2, \vec{J}\right] = \left[J^2, \vec{J}\right] = 0$.

   It is because $\left[\vec{L} \cdot \vec{S}, \vec{J}\right] = 0$ that we can diagonalize the spin-orbit Hamiltonian using eigenstates of J^2 and J_z.