Physics 472 – Spring 2008

Homework #8, due Friday, March 21

(Point values are in parentheses.)

1. [5] Griffiths problem 7.1. For this problem and the next one, you will repeatedly use the Gaussian integrals on the inside back cover of Griffiths. I suggest you first convert the integral he has to the following form:

$$\int_0^\infty x^{2n} e^{-ax^2} = \sqrt{\frac{\pi}{a}} \frac{(2n)!}{n!} \frac{1}{a^n 2^{2n+1}}$$

2. [5] Griffiths problem 7.13. In class, Prof. Moore showed you a shortcut to calculate the expectation value of the three-dimensional kinetic energy for a Gaussian trial wavefunction, using Griffiths' result for the one-dimensional Gaussian wavefunction. I would like you to calculate the three-dimensional kinetic energy from scratch. For a spherically-symmetric wavefunction, the Laplacian in spherical coordinates includes only the first term in Griffiths Eq. [4.13].

3. [6] Griffiths problem 7.15.
Hints: For part (a), write $H = H^0 + H'$ as a 2 × 2 matrix. Then finding the eigenvalues is easy. In part (c), the condition you obtain to minimize $\langle \Psi | H | \Psi \rangle$ should be: $\tan(2\phi) = \frac{-2h}{E_b - E_a}$. But your expression for $\langle \Psi | H | \Psi \rangle$ contains trigonometric functions of ϕ rather than 2ϕ. Here are some suggestions to help you keep from getting lost in the algebra: Define two new variables, $\bar{E} = \frac{E_b + E_a}{2}$ and $\Delta = \frac{E_b - E_a}{2}$. Re-write E_a and E_b in terms of $\bar{E}$ and Δ, then use your double-angle formulas to express $\langle \Psi | H | \Psi \rangle$ in terms of 2ϕ. Notice that since $\tan(2\phi) < 0$, the angle 2ϕ lies either in the second or fourth quadrant. One of those possibilities leads to the minimum value for $\langle \Psi | H | \Psi \rangle$ while the other leads to the maximum value! In part (d), expand the exact answer in a Taylor series for small h, and compare with your answer for part (b).

   If you find all of these algebraic tricks annoying, consider this: the two-level system is one of the most common problems in physics. Learning how to solve it will serve you well in the future.

4. [4] Griffiths problem 7.16
Notice that the up state has higher energy than the down state. That is because the electron magnetic moment points in the opposite direction to the spin.
For part (c), you can use either your result from part (a) or part (c) of the previous problem, since they both give the exact answer. Simplify your result and give its physical interpretation. In retrospect, could you have just written down the exact answer without doing any work?