Physics 472 – Spring 2008

Homework #9, due Friday, March 28

(Point values are in parentheses.)

1. [5] (a) Griffiths problem 9.1. To see which of the four matrix elements are zero, you are welcome to use Tables 4.3 and 4.7 in Griffiths. But there is an easier and more elegant way. When you analyzed the Stark Effect in the last problem on Homework #7, you used the following two symmetries: rotational symmetry: $\left[\hat{z}, \hat{L}_z\right] = 0$ implies $(m_l' - m_l)\langle n', l', m_l' | z | n, l, m_l \rangle = 0$. The parity transformation $\hat{\Pi}\hat{z}\hat{\Pi} = -\hat{z}$ implies $(-1)^{l'+l}\langle n', l', m_l' | z | n, l, m_l \rangle = -\langle n', l', m_l' | z | n, l, m_l \rangle$.

   [3] (b) The electron in a hydrogen atom is initially in the ground state. At time $t = 0$, an electric field is applied in the z-direction. The field strength then decreases with time as: $\vec{E}(t) = \varepsilon_0 \hat{z} e^{-t/\tau}$. After a long time ($t >> \tau$), what is the probability that the electron has made a transition to an excited state?

2. [6] Griffiths problem 9.2. There are two ways to solve a pair of coupled first-order differential equations. The first method is to write the two equations as a single matrix equation, using a known Ansatz for the form of the solution. The second method is to combine the two equations into a single, second-order differential equation. I suggest you use the second method. Start by differentiating the equation for $\dot{c}_b$, then plug in the expression for $\dot{c}_a$ from the first equation. You should get a second-order linear differential equation for $\dot{c}_b$, with constant coefficients. To simplify the algebra, define $\alpha^2 = \frac{|H'_{ab}|^2}{\hbar^2}$. Plug in a solution of the form $c_b(t) = e^{\lambda t}$, and you'll get a quadratic equation for λ with solutions λ_1 and λ_2. If you haven't made any mistakes, your solutions should be $\lambda = \frac{i}{2}(\omega_0 \pm \omega)$, where $\omega = \sqrt{\omega_0^{\,2} + 4\alpha^2}$. The general solution is then $c_b(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$. So far everything you have done is straightforward. Now here comes a trick to save you time and effort. Given the relationship between the two frequencies in your general solution, you can re-write the solution as $c_b(t) = e^{i\omega_0 t/2}\left[C\cos(\omega t/2) + D\sin(\omega t/2)\right]$. Now plug in the initial conditions for both $c_b(0)$ and $c_a(0)$ to find *C* and *D*.

3. [3] A particle of mass *m* is in a one-dimensional infinite square-well potential lying in the range $0 < x < a$. Suddenly at time t = 0, the right wall of the square well is moved to $x = 2a$. What is the probability that an energy measurement will give the result $E_0 = \frac{\pi^2\hbar^2}{2m(2a)^2}$? If, instead, the wall were moved extremely slowly, what would be the probability to get the same result?

4. [3] Griffiths problem 9.14 part (a) only.