EXPERIMENT 10
The Pendulum

Objectives

- To investigate the functional dependence of the period (τ) of a pendulum on its length (L), the mass of its bob (m), and the starting angle (θ_O). The Greek letter tau (τ) is typically used to denote a time period or time interval
- use a pendulum to measure g, the acceleration due to gravity

Apparatus

Point masses and string, a digital timer, period gate, and meter stick will be used.

Theory

Where there exists a constant net force (F), Newton's Law F = ma tells us that the acceleration (a) is a constant and therefore the position of the object can be written as

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

This is the formula we tested in the Free Fall I lab.

In the analysis of the motion of a pendulum we should realize that

1) The motion is part of a circle so angular acceleration (α) is a useful variable

2) The angular acceleration will not be a constant throughout the motion

Consider the pendulum shown in Figure 1. The acceleration of the bob **tangent** to the arc "drawn" by the pendulum as it swings, a_t, is determined by $\mathbf{F_t}$, the force tangent to the arc. Since the tension in the string (T) always acts along the **radius**, it doesn't contribute to $\mathbf{F_t}$. Decomposing the gravitational force mg into components perpendicular and parallel to the string as shown in the diagram below, we find that

$$F_t = mg\sin\theta$$

Therefore the acceleration tangent to the circle is given by:

$$a_t = \frac{F_t}{m} = g\sin\theta$$

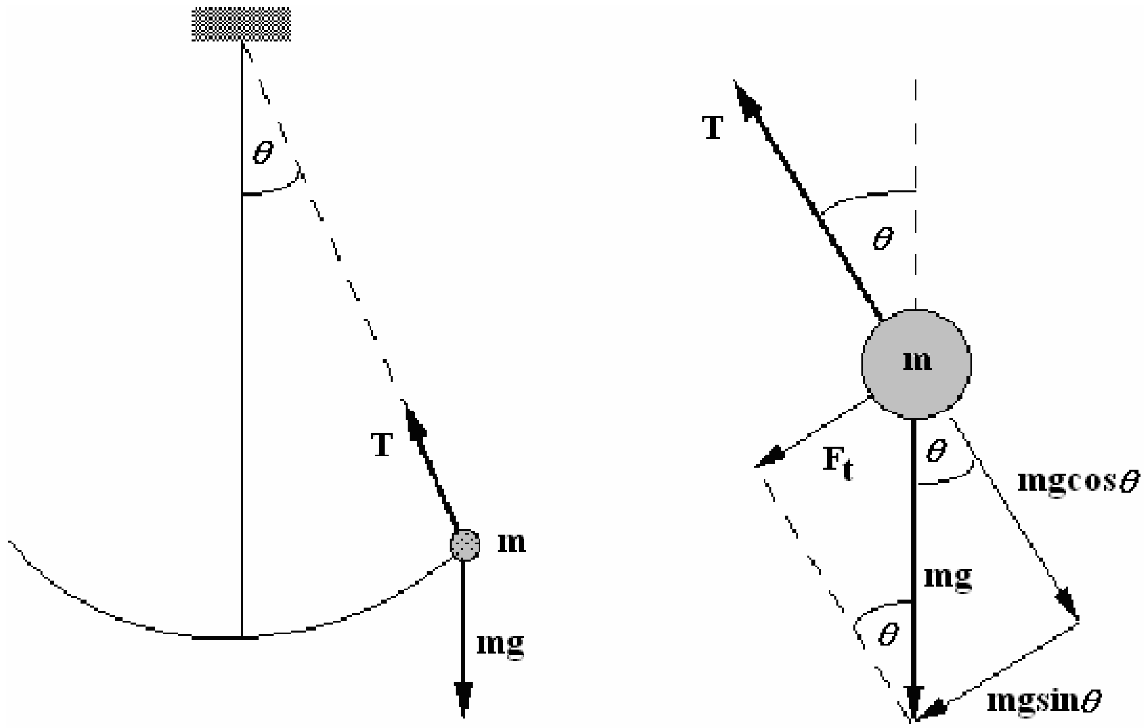

Figure 1

The angular acceleration α is then found by the relationship for circular motion

$$\alpha = -\frac{a_t}{r} = -\frac{g}{L}\sin\theta$$

Thus, as we have suggested, the angular acceleration α is not a constant but varies as the sine of the displacement angle of the pendulum.

For small angles (about $\theta < 0.5$ radian) angular accelerations can be shown (with a little calculus which we will skip) to lead to an oscillation of the angle θ by

$$\theta = \theta_0 \cos\frac{2\pi \ t}{\tau}$$

where θ_O is the angle at time $t = 0$ (when we release the pendulum), and τ is the period of the motion. The period is the time it takes to complete one full cycle of the motion.

The period (τ) of a pendulum depends only on its length (L) and the acceleration due to gravity (g). The period (τ) is independent of the mass of the bob (m) and the starting angle (θ_0). The period of a simple pendulum is given by:

$$\tau = 2\pi\sqrt{\frac{L}{g}}$$

or

$$\tau = \frac{2\pi}{\sqrt{g}}\sqrt{L}$$

This equation has the same form as the equation of a straight line y = mx + b, with an intercept of zero (i.e. b = 0). Notice in this equation, the period (τ) corresponds to y and $\sqrt{L}$ corresponds to x.

Procedure

The three parameters of the system are L, the length of the pendulum from the support to the center of mass of the "point mass;" the mass m; and θ_o, the angle through which you displace the mass as you pull the pendulum back to start it. The quantity you measure is the period τ.

Use the PEND setting of the gate and measure the period several times for several different combinations of L, m and θ_o. The PEND setting of the gate uses light and a photodetector in the following way: when the light beam is interrupted the first time the timer is started, the timer continues to count when the beam is interrupted a second time but stops on the third interruption of the beam. This measures the time taken for one complete oscillation of the pendulum, or in other words the period.

You should keep two parameters fixed while you vary the third. When you vary L, do it over the largest range of values that you can with your apparatus. Always keep the starting angle θ_o less than 0.5 radian (or about 30°).

Prepare the following graphs:

I. τ vs. m (for fixed θ_o and L)

II. τ vs. θ_o (for fixed m and L)

III. τ vs. $\sqrt{L}$ (for fixed m and θ_o)

In all three graphs, use the same scale for the axis displaying τ and determine the scale by the smallest and largest value obtained in I, II and III.

Questions

Comment on how the period depends on each of the three parameters. If any of the graphs should be a straight line, have the computer fit it with a best-fit line and get the equation of the line.

1) Comment on how the period depends on each of the three parameters. If any of the graphs should be a straight line, have the computer fit it with a best-fit line and give the equation of the line.

2) Use the slope of the graph of τ vs. square root of L to calculate g and its uncertainty.
   $$\delta g = g\frac{\delta(slope)}{slope}$$

3) Is your value of g consistent with 981 cm/sec^2?

4) For one of your measurements calculate the angular acceleration (α) at the starting angle θ_o, at $\theta = 0$, and at the far end of the swing.

CHECKLIST

1) the spreadsheet with your data
2) three graphs with best-fit line and equation of best-fit line where appropriate
3) comments on each of the three graphs on how the period depends on each of the three parameters m, θ_0, and L
4) answers to the questions