EXPERIMENT 11
The Spring
Hooke's Law and Oscillations

Objectives

- To investigate how a spring behaves when it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hooke's Law.
- Measure the spring constant, k in two independent ways

Apparatus

A spring, photogate system, and masses will be used.

Theory

Hooke's Law

An ideal spring is remarkable in the sense that it is a system where the generated force is **linearly dependent** on how far it is stretched. Hooke's law describes this behavior, and we would like to verify this in lab today. In order to extend a spring by an amount Δx from its previous position, one needs a **force** F which is determined by $F = k\Delta x$. Hooke's Law states that:

$$F_S = -k\Delta x \tag{1}$$

Here k is the **spring constant** which is a quality particular to each spring and Δx is the distance the spring is stretched or compressed. The force F_S is a restorative force and its direction is opposite to the direction of the spring's displacement Δx.

To verify Hooke's Law, we must show that the spring force F_S and the distance the spring is stretched Δx are proportional to each other (that just means linearly dependant on each other), and that the constant of proportionality is -k.

In our case the external force is provided by attaching a mass m to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$. See Figure 1. Consider the forces exerted on the attached mass. The force of gravity (mg) is pointing downward. The force exerted by the spring ($-k\Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point <u>where the two forces are equal but pointing in opposite directions</u>:

$$F_s - F_g = 0$$

or

$$mg = -k\Delta x \tag{2}$$

This point where the forces balance each other out is known as the **equilibrium point**. The spring + mass system can stay at the equilibrium point indefinitely as long as no additional external forces come to be exerted on it. The relationship in (2) allows us to determine the spring constant k when m, g, and Δx are known or can be measured. This is one way in which we will determine k today.

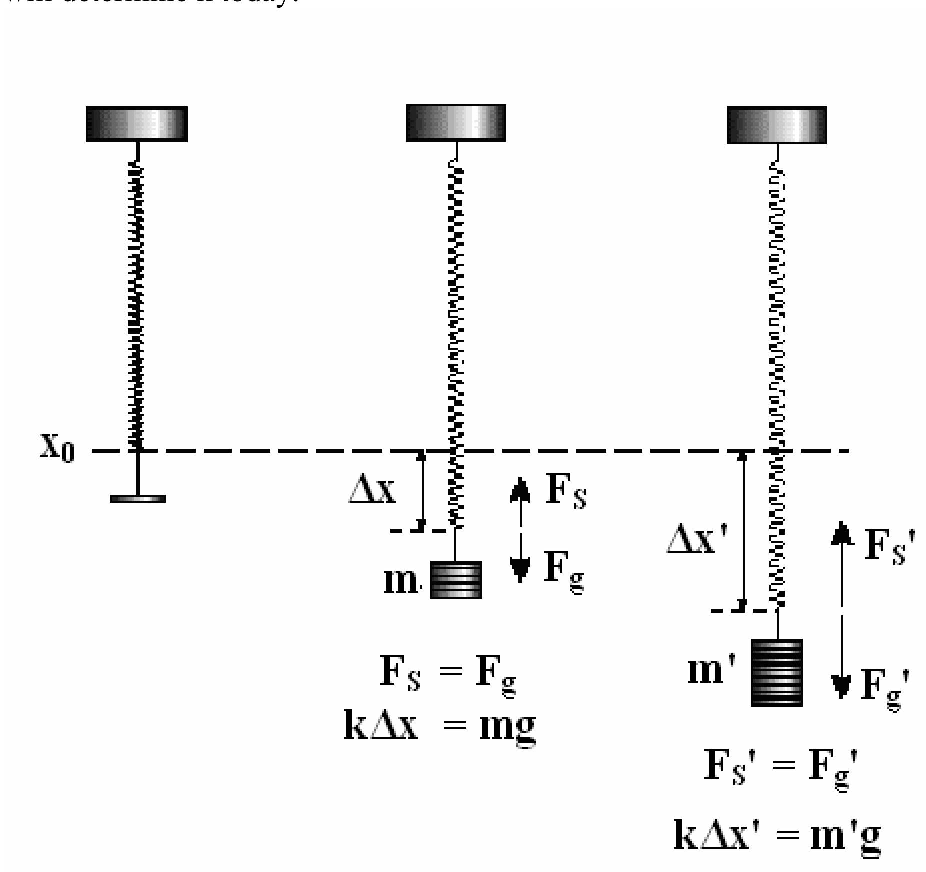

Figure 1: The Spring in Equilibrium

Oscillation

The position where the mass is at rest is called the equilibrium position ($x = x_0$). As we now know, the downward force due to gravity $F_g = mg$ and the force due to the spring pulling upward $F_S = -k\Delta x$ cancel each other. This is shown in the first part of Figure 2. However, if the string is stretched beyond its equilibrium point by pulling it down and then releasing it, the mass will accelerate upward ($a > 0$), because the force due to the spring is larger than gravity pulling down. The mass will then pass through the equilibrium point and continue to move upward. Once above the equilibrium position, the motion will slow because the net force acting on the mass is now downward (i.e. the downward force due to gravity is constant while the upwardly directed spring force is getting smaller0. The mass and spring will stop and then its downward acceleration will cause it to move back down again. The result of this is that the mass will oscillate around the equilibrium position. These steps and the forces (F), accelerations (a) and velocities (v) are illustrated in Figure 2 for the first complete cycle of an oscillation. The oscillation will proceed with a characteristic period, T, which is determined by the spring constant and the total attached mass. This period is the time it takes for the spring to complete one oscillation, or the time necessary to return to the point where the cycle starts repeating (the points where x, v, a, are the same). One complete cycle is shown in Figure 2 and the time of each position is indicated in terms of the period T.

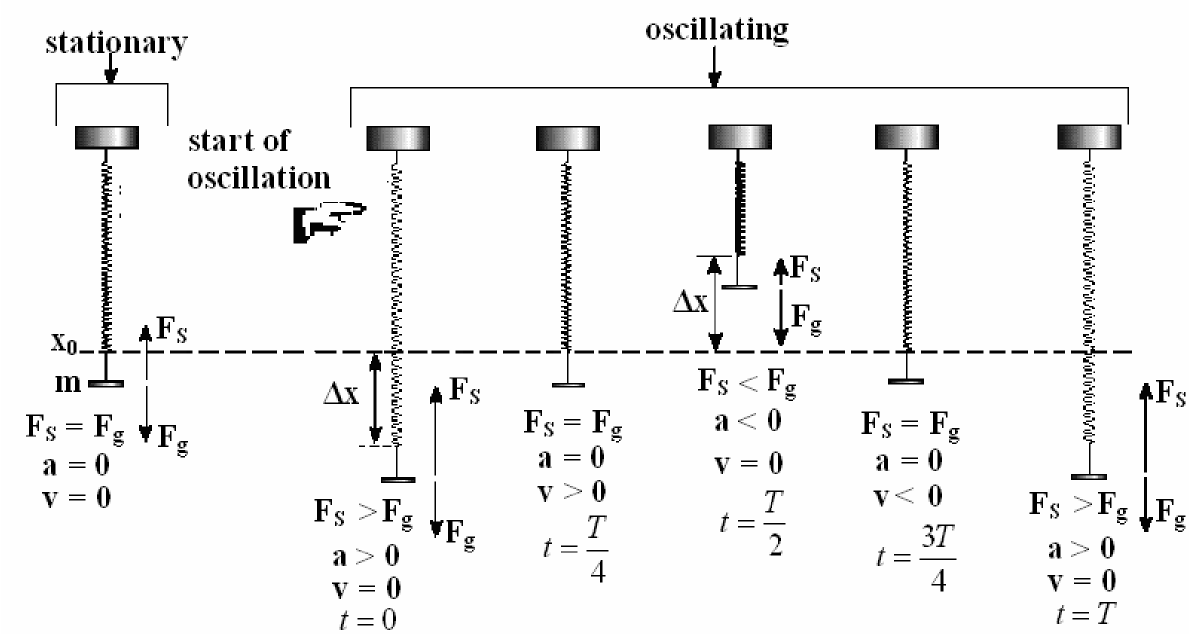

Figure 2: One Cycle of an Oscillation of the Spring

The period is given by

$$T = 2\pi\sqrt{\frac{m}{k}} \tag{3}$$

By measuring the period for given masses the spring constant can be determined. This is the second way we will determine k today. You will use this value of k to verify that the proportionality constant you determined for Hooke's Law in the first part is indeed the correct k for the spring.

Procedure

Part I: Hooke's Law

Determine the initial mass, m_0, by weighing the support table. Next attach the support table for the masses to the spring. With the zero end of a meter stick on the lab table, measure the position of the end of the spring after the support table has been attached. This position is the initial position x_0.

Start measuring by increasing the mass attached to the spring to 120 grams (this includes the mass of the support table). Then increase the mass by increments of 10 grams up to a total of 220 grams and measure the corresponding position of the spring for each mass. This results in a series of measurements m_i and x_i. To calculate the forces due to gravity and the spring calculate $\Delta x_i = x_i - x_0$ and $\Delta m_i = m_i - m_0$. The corresponding forces for gravity and the spring are $F_g = \Delta mg$ and $F_S = -k\Delta x$. Right now you do not know k, so you will only have your spreadsheet calculate F_g for you. But remember, at equilibrium positions the magnitude of F_g equals the magnitude of F_S! Therefore,

$$F_S = \Delta mg \tag{4}$$

The direction of F_S is upward (define this as positive). Graph F_S vs. Δx – include horizontal error bars on your data points. If you have a straight line you have already verified the first part of Hooke's Law, that force and distance the spring is stretched are linearly dependent. Have the computer fir your data with a best-fit line including the equation of the line and the uncertainty in the slope. Determine the spring constant k and its uncertainty from the slope. We will verify this value of k by determining k a second way that is independent of your ability to make accurate length measurements.

Part II: Period of Oscillation

Determine the period for attached masses varying from 120 to 220 g in steps of 20 g (the same masses as in Part I). When displacing the masses, DO NOT stretch the spring more than about 2 cm from its equilibrium position. Set the photogate to the "PEND" setting. Then use the photogate to measure the period of the oscillation by causing the masses to oscillate through the photogate. The "PEND" setting will start timing the first time the masses pass through it, continue timing through the second pass, and stop timing when it senses the masses a third time. Thus it measures the time for the masses, to complete a whole period of the oscillation.

If we square both sides of equation (3) we find:

$$T^2 = \frac{4\pi^2 m}{k} \tag{5}$$

Equation (5) has the same form as the equation of a straight line y = mx + b, with an intercept of zero (i.e. b = 0). Notice in this equation, T^2 corresponds to y and m corresponds to x.

The mass m in equation (5) is the total mass felt by the spring. The oscillating mass includes the entire mass of the support table. Furthermore, to get the total mass felt by the spring you should add 1/3 of the mass of the spring itself to the total mass attached. Your TA will give you the mass per unit length of your spring. Measure the length of the spring and use it to calculate the mass of the spring. Make a graph of T^2 vs. m. Fit the graph with a best fit line; find the slope of the line and its uncertainty.

Questions

1) Are your data consistent with Hooke's Law? Specifically, is the spring force linearly dependant on how much the spring is stretched and is it a restorative force? Why or why not?

2) Calculate the spring constant and its uncertainty using the information obtained from your graph of T^2 vs. m. Hint: The fractional uncertainty in the spring constant $\frac{\delta k}{k}$ is equal to the fractional uncertainty in the slope.

3) You obtained the spring constant in two independent ways. Discuss the consistency of your two measurements of the spring constant.

4) When a mass m is attached to a spring it exerts a force W=mg on the spring and the length of the spring is changed by Δx. If the single spring is replaced with a) two identical springs in series, what happens to Δx_{series} compared to the case of a single spring? b) If the single spring is replaced by two identical springs in parallel, what happens to $\Delta x_{parallel}$ compared to the case of a single spring? Assume all springs have the same spring constant and always compare to the single spring case. Answer each question by stating if Δx increases, decreases or remains unchanged. Also, what are Δx_{series} and $\Delta x_{parallel}$ in terms of Δx for the single spring case (Hint draw a force diagram of the system – the net force on the mass must be zero).