EXPERIMENT 3
Analysis of a freely falling body
Dependence of speed and position on time

Objectives

- to verify how the distance of a freely-falling body varies with time
- to investigate whether the velocity of a freely-falling body increases linearly with time
- to calculate a value for g, the acceleration due to gravity

Apparatus

A Behr Free-Fall Apparatus and Spark Timing System will be used in this experiment. A schematic representation of the apparatus is shown below in Figure 1 and a digital photograph of the apparatus is shown in Figure 2 on the following page.

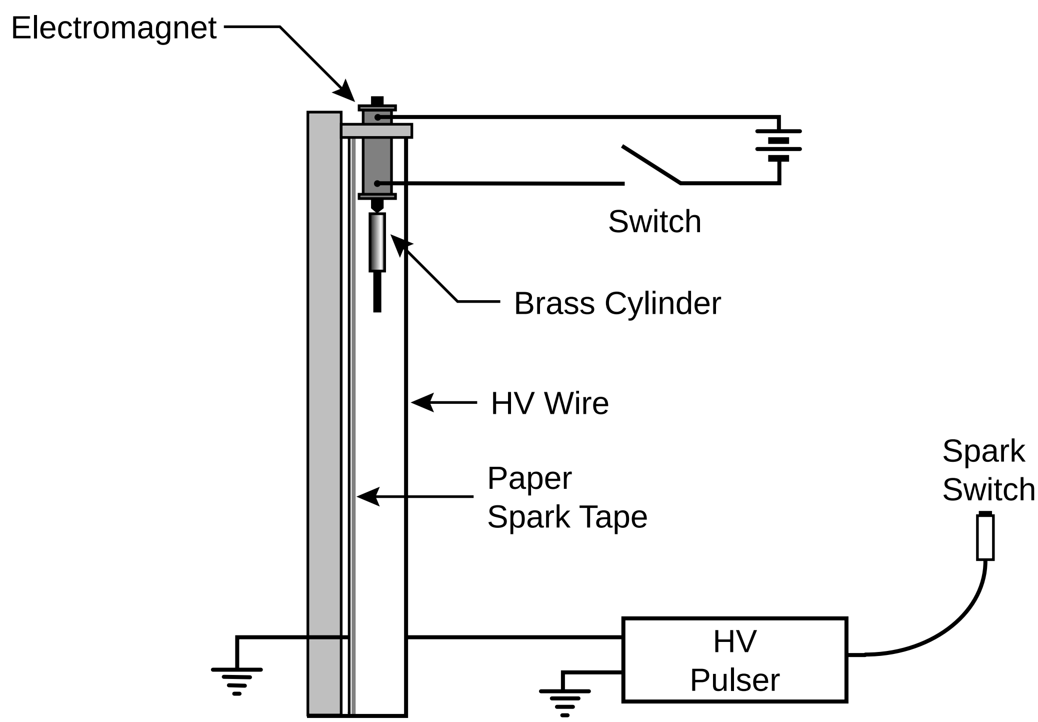

Figure 1: Schematic of the Behr Free-Fall Apparatus

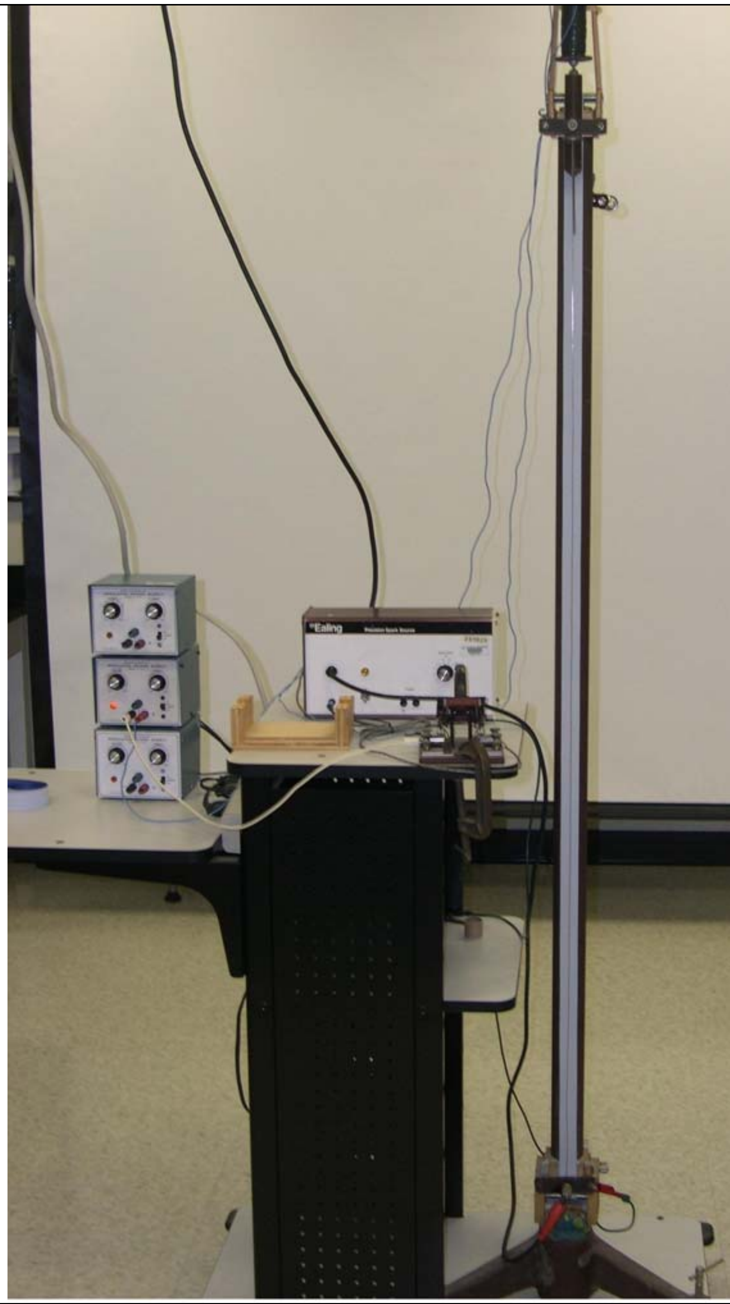

Figure 2: The Behr Free-Fall Apparatus

Theory

In this experiment a brass cylinder is dropped and a record of its free fall is made. Before the measurement, the cylinder is suspended at the top of the stand with the help of an electromagnet. When the electromagnet is turned off, the cylinder is released and starts to fall. Simultaneously, the spark timer starts to send evenly-spaced, high-voltage pulses through two wires which are stretched along the cylinder's path.

At the time of each pulse a spark goes through the wires and the cylinder, leaving a mark on the special paper tape that lies between the cylinder and one of the wires. The time interval between two adjacent sparks, τ, is 1/60 of a second.

Measuring the distances between any two marks, Δy, and knowing the times between the corresponding sparks, Δt, it is possible to calculate the average velocity during this interval using the formula

$$v = \frac{\Delta y}{\Delta t} \tag{1}$$

If Δt is small enough, we can assume that the velocity at any instant within this interval is approximately equal to this average velocity. In the case where acceleration is constant, the instantaneous velocity at the middle of the time interval Δt is exactly equally to the average velocity of the object during the time interval Δt.

In general, for the motion of a body with a constant acceleration a, the velocity v is given by the equation

$$v = at + v_0 \tag{2}$$

where v_o is the velocity of the cylinder at t = 0. Since in our case the brass cylinder is falling freely,

$$a = -g, \tag{3}$$

where $g=9.81\ m/s^2$ is the magnitude of the acceleration due to gravity. The negative sign in front of g is to indicate that the direction of the acceleration is in the negative direction (i.e. downward). Therefore it follows from (2) that for a freely-falling body

$$v = v_0 - gt \tag{4}$$

Thus g can be determined from a plot of v vs. t since the slope of any velocity versus time graph is just the acceleration. The obtained value of g can then be compared with the known value of the acceleration due to gravity. The position of the cylinder, y, as a function of time is given by the standard equation for an object that is undergoing constant acceleration. If at time = 0 the object has height y_o and velocity in the vertical direction v_o, then this equation looks like

$$y = y_0 + v_0 t - \frac{1}{2} g t^2 \tag{5}$$

Procedure

Using the free fall apparatus, drop the brass cylinder and record on spark tape the location of the falling cylinder at a series of equally spaced time intervals, $\tau = 1/60$ s.

The quantities needed to analyze the motion are the position (y), velocity (v) and time (t) of the points on your spark tape. The choices of t = 0 and y = 0 are arbitrary and do not necessarily refer to your first or last data points. However, as your object falls the position *y* must decrease (becoming negative if necessary).

After you have performed the experiment, tape the paper strip to your lab table. Label the points on your tape, starting with the **third** point from BOTTOM OF THE OBJECT'S FALL as #25 and label them in DESCENDING numerical order. The reason for using the third from the last point is you want to be sure you do not include the points obtained when the brass cylinder is comes to rest at the bottom or when its begins to enter the putty at the bottom of the apparatus. NOTE: Point #1 will not necessarily correspond to your very first point, but the very first points are somewhat ambiguous anyway. *It does not matter if you do not use some of the first points*. Next, put a ruler on your tape such that height y=0.0 cm corresponds to point #25 and that height INCREASES as you move towards point #1. Measure the locations of each marker with this ruler and write the y position on the tape next to each marked point. Make sure you are labeling each point with its TOTAL distance from 0.0cm, NOT just with its distance from the previous point.

After completing this, transfer the y positions into the spreadsheet. Using the spreadsheet, calculate the velocity at all points during the fall except for the first and last points. Be sure you do at least one sample calculation to include in your lab report. As discussed in the theory section, the instantaneous velocity at time t_i are found by finding the average velocity during the time interval of t_{i-1} to t_{i+1} for each of your points, $v = \frac{\Delta y}{\Delta t}$. In this equation Δy is the difference between the position of the spark FOLLOWING and the position of the spark PRECEEDING the spark for which you are trying to calculate a velocity. Similarly, Δt is the time interval between the FOLLOWING spark and the PRECEEDING spark. For example, the instantaneous velocity at $y = y_2$ is $v_2 = \frac{y_3 - y_1}{t_3 - t_1}$.

Transfer the columns of "Time," "y," and "v" from your spreadsheet to Kaleidagraph for graphing.

Prepare the following graphs:

I -- a graph showing Vy vs. t fitted with error bars and fitted with a best-fit line. Use the procedure from Experiment 1. Make sure the equation of the best-fit line is on your graph. From the slope of the line determine the gravitational acceleration, g, in cm/s^2 and its uncertainty.

II -- a graph of y vs. time

After your perfected graphs are printed out, write a 2 to 3 sentence "Graph Analysis" at the bottom of each graph. This should describe what is happening on the graph ("as the quantity along the horizontal axis does such and such, the quantity along the vertical does this"), and how what is happening on the graph relates to what actually occurs with those quantities physically.

Questions

1. What is the y-intercept determined from your Graph I (or from the equation of its best-fit line)? What does it mean?

2. From Graph I, or the equation of its best-fit line, find the time at which v=0 cm/s.

3. What is your value of the gravitational acceleration in cm/s^2 determined from the slope from graph I? Is this value compatible with the accepted value of the gravitational acceleration? If not suggest a possible source of error (do not suggest human error or a mistake).

4. What general equation describes Graph II?

5. When the initial velocity is zero, what would you plot to make graph II linear: y^2 vs. t, y^2 vs. t^2, or y vs. t^2? As always, justify your response.

Checklist

Your lab report should include the following eight items:

1) your spreadsheet
2) sample calculations
3) plot of the height vs. time Graph II
4) plot of velocity vs. time with slope (in cm/s^2) Graph I
5) interpretation of the two plots
6) answers to questions
7) one member of each group should turn in your spark tape record of the free-fall
8) if your lab instructor tells you that you will need to use this data for next the next lab, make sure you save your spreadsheet

Formulae, Definitions, and Errors for the Free Fall Experiment

DEFINITIONS

In this experiment you measure the position of a falling mass, m, at fixed time intervals. The fixed time interval is determined by a high-voltage spark source. Read off the time between sparks (τ) from the setting on the spark source.

You will measure the positions at each spark as y_1, y_2, y_3, y_4, etc. in centimeters [cm]. These positions will be referred to as y_i.

To measure the speed at point a particular point "i" first calculate

$$\Delta y_i = y_{i+1} - y_{i-1} .$$

For example for the sixth point

$$\Delta y_6 = y_7 - y_5 .$$

On your data sheet this is labeled as Δy(i). You are now ready to calculate the speed at point i by dividing the distance Δy_i by the time elapsed between the two points, or 2τ. So the speed V_{y_i} is given by

$$V_{y_i} = \frac{\Delta y_i}{\Delta t} = \frac{\Delta y_i}{2\tau} \text{ [cm/s].}$$

ERRORS

For each measured y_i you assign an error based on how accurately you can measure that point. This error is called δy. This error determines all other errors in this lab. For this lab and for the following formulae it is assumed that the error in τ and m are zero.

The error in Δy at each point i is the same and is given by $\delta(\Delta y) = 2\,\delta y$

The error in the speed at each point i is

$$\delta(V_y) = V_y \frac{\delta(\Delta y)}{\Delta y} = V_y \frac{2\delta y}{\Delta y} = \frac{\Delta y}{2\tau}\frac{2\delta y}{\Delta y} = \frac{\delta y}{\tau}$$

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