# EXPERIMENT 9A
## Rotational Motion 1
## The Relationship Between Linear and Angular Quantities

## Objectives

- to measure the radius of a cylinder using the relationship between linear and angular displacement
- to measure the radius of a cylinder using the relationship between linear and angular acceleration.
- To investigate the relationship between linear and angular velocity

## Apparatus

The experimental apparatus is shown in Figure 1 below.  A light string (negligible mass) is wound around a central pulley of radius R; it passes over a side pulley and is attached to a mass M.  Compressed air is supplied to the apparatus making the rotating central pulley and the disk to which it is attached (shown in green in figure 1) rotate with very little frictional losses.  The side pulley also is supplied with compressed air to minimize frictional losses.  The rotating disk (shown in green in Figure 1) has alternating black and white bars on its circumference which are detected by a photo-diode detector as they sweep past it.  The computer then uses a program written in the LabVIEW programming language to create a data table containing the angular velocity of the rotating disk and the corresponding times. You can then open the data table, copy it and paste its contents into Kaleidagraph.   Recall the slope of the angular velocity versus time plot is the angular acceleration (typically denoted by the Greek letter α) of the rotating disk.  A desktop timer will be used to measure the time the mass M takes to accelerate to the floor.  A meter stick will be used to measure the linear displacement of the mass M.
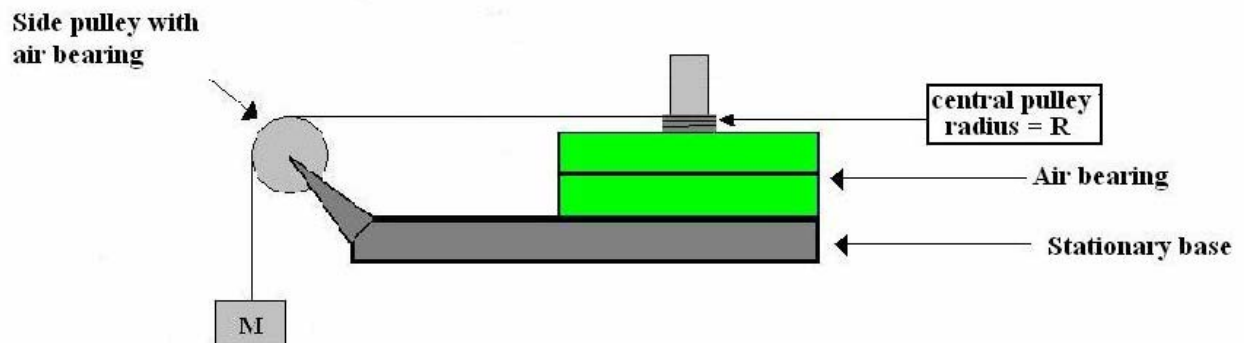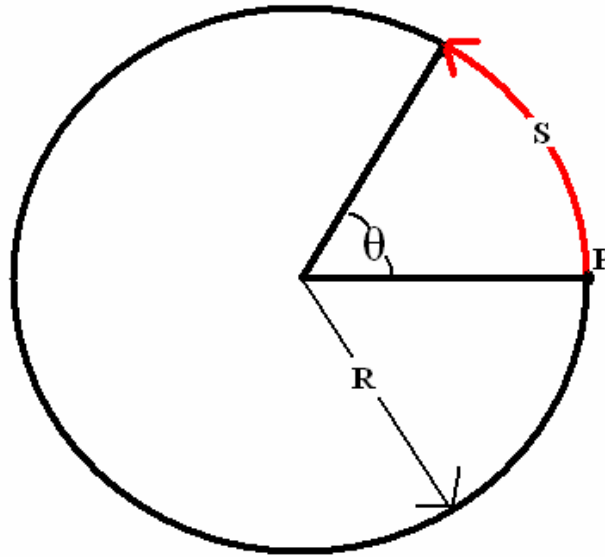


Figure 1

**Theory**



Figure 2

Consider a point P on the surface of a cylinder having a radius of R shown in Figure 2 above. If the cylinder is rotated about an axis perpendicular to the plane of the page and through the center of this circular cross-section, the arc length (S) swept out by P is given by:

$$S = R\theta \tag{1}$$

S and R have units of length and the angle $\theta$ is measured in radians. When the arc length S is equal to the radius R, the angle $\theta$ is 1 radian. For a complete 360° circle, the angle, in radians is $2\pi$ radians. If a thin string is wrapped around a cylinder N complete times, then the length of the string wrapped around the cylinder is:

$$\Delta y = N(2\pi)R = 2\pi NR \tag{2}$$

The linear velocity (v) of a point on the cylinder's surface is related to the angular velocity of the cylinder (typically denoted by the Greek letter omega: $\omega$) by:

$$v = R\omega \tag{3}$$

Similarly, the linear tangential acceleration of a point on the cylinder's surface (a) is related to the angular acceleration of the cylinder by:

$$a = R\alpha \tag{4}$$

If the hanging mass M in Figure 1 is released from rest and allowed to fall a distance h to the ground, it will have an acceleration equal in magnitude to the tangential linear acceleration of

the string that is wound around the central pulley of the apparatus. This acceleration is given by:

$$a = \frac{2h}{t^2} \qquad (5)$$

t is the time the mass took to fall from rest (v=0) to the ground.

## Procedure

**The apparatus is delicate. Do not rotate the rotating table or side pulley unless air is flowing through the bearings.**

**Displacement**

1. Turn on the air supply to the rotational apparatus and make sure the string passes over the side pulley as shown in Figure 1 and that the string is taught with the hanging mass M just barely touching the floor. The 0 cm end of the meter stick should be paced on the floor ($y_i = 0.0$cm).

2. Turn the rotational apparatus at least N=8 complete revolutions (DO NOT USE FRACTIONAL REVOLUTIONS!). Record the total number of revolutions and the final position of the hanging mass in the Displacement Data Table.

3. Use equation (2) and Excel to calculate the radius of the central pulley R.

4. Repeat steps 1 through 3 three additional times. Have Excel calculate the mean value of R and the standard deviation (s) and the standard deviation of the mean value ($s_m$). The Excel formula for the standard deviation (s) is "=STDEV(CELL1:CELL4)" and the Excel formula for the standard deviation of the mean ($s_m$) is "=s/SQRT(4). **You will need the mean value of the radius and the standard deviation of this mean value for the next experiment.**

**Acceleration**

As with the displacement measurements you will do 4 trials in this part. In each of the four trials you will simultaneously measure the angular acceleration of the rotating apparatus and the linear acceleration of the falling mass (and therefore the string). Refer to the LabVIEW Supplement after the Questions section for additional help using the LabVIEW program to measure the angular acceleration. Each trial MUST start with the hanging mass at the same height (h) above the floor.

1. Open the LabVIEW program located in the Rotation1 folder,

2. With the air supply on, attach the hanging mass (M) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Measure and record the position of the hanging mass above the floor – each trial MUST start with the mass at this same elevation.

3. Simultaneously release the hanging mass and start the desk top timer; then immediately click on the START button in LabVIEW. The START button is the white arrow at the far left hand edge of the toolbar at the top of the screen. When the mass hits the floor stop the desk top timer.

4. LabVIEW will display an angular velocity versus time plot on the monitor. In addition, it will construct a data table with the values of the angular velocity and time from its plot. LabVIEW will ask you to assign your data file a name. Transfer your data file into Kaleidagraph. To do this left click on the data file and drag it over the Kaleidagraph icon on the computer's desktop. Make a plot of angular velocity versus time. Make sure you only graph data that correspond to when the hanging mass was moving downward (i.e. before it hit the floor). Fit your data with a best fit line and have Kaleidagraph display the equation for this best fit line. The slope of this line is the angular acceleration of the rotating apparatus. Transfer this value into the Acceleration Data Table in Excel. **In this experiment you will create four angular velocity versus time graphs -- you only need to write comments on ONE of these four graphs.**

5. Transfer the time (t) from the desk top timer into the Acceleration Data Table and use equation (5) to calculate the linear acceleration (a) of the falling mass. Again, the magnitude of this acceleration is equal to the magnitude of the tangential acceleration of a point on the outside surface of the central pulley.

6. Have Excel calculate the radius of the central pulley using equation (4). In addition have Excel calculate the mean value and the standard deviation of the measured time, t; and the mean value, standard deviation and standard deviation of the mean value of the radius R in the Acceleration Data Table. **You will need the standard deviation for your time measurements for the next experiment.**

## Questions

1. Discuss the consistency of your measured values of the radius of the central pulley.

2. The angular velocity of an object undergoing constant angular acceleration and starting from rest is given by $\omega = \alpha * t$. Use your data (average values), this relationship and equation (3) to calculate the magnitude of the linear velocity of a piece of string on the central pulley's surface at the instant before the hanging mass hit the ground. In addition, calculate the uncertainty in this velocity. The uncertainty is given by: $\delta v = v \left[ \dfrac{\delta R}{R} + \dfrac{\delta \alpha}{\alpha} + \dfrac{\delta t}{t} \right]$  In the above equation, the uncertainties δR, δα and δt are the standard deviations of the mean values of R, α and t respectively.

3. The linear velocity of an object undergoing constant linear acceleration and starting from rest is given by v = a*t.  Use your data (average values) to calculate the magnitude of the velocity of the hanging mass at the instant before it hit the ground. In addition, calculate the uncertainty in this velocity using $\delta v = v\left(\dfrac{\delta a}{a} + \dfrac{\delta t}{t}\right)$.

4. Discuss the consistency in the magnitudes of the velocities calculated in questions 2 and 3.

## Checklist

Your lab report should include the following three items:

1) Your Excel spreadsheet

2) Formula view of your spreadsheet

3) Sample calculations (you do not need to do calculations for the mean values, standard deviation or standard deviation of the mean values)

4) All four of your angular velocity versus time graphs with comments only on one of the four graphs.

5) Answers to questions

## LabView Supplement

1. Open the Labview program – your TA will tell you where to find it.  When you initially open the program, you will see a red line moving across a black chart that Labview is acquiring data (your screen should look like Figure 1S below).   When you are asked to save a file, click *cancel* and then click *stop* in response to the error message.
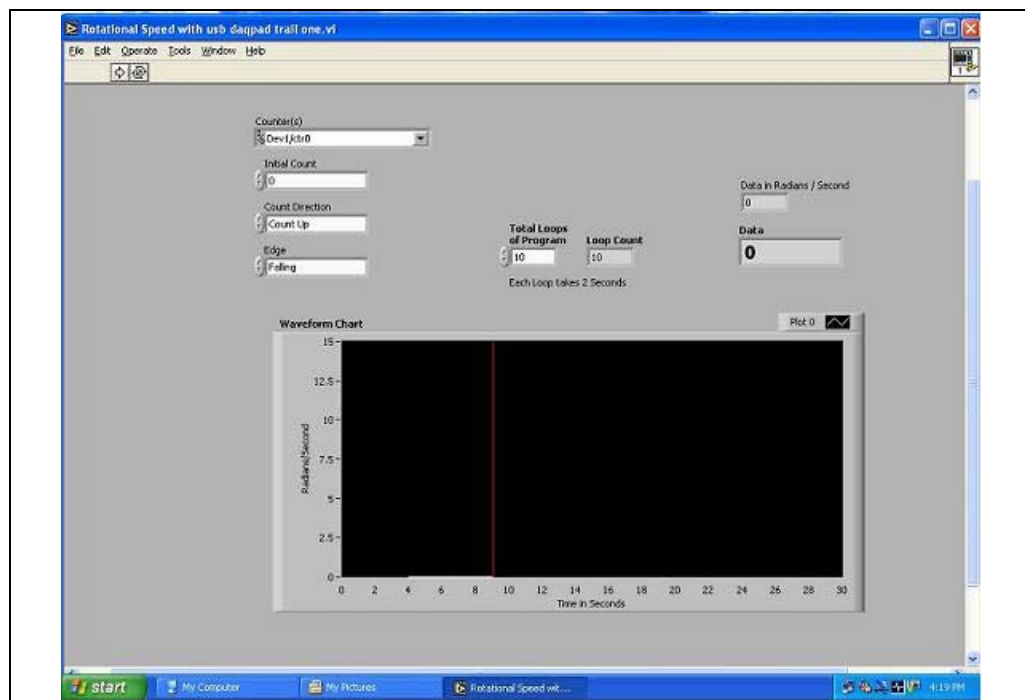


Figure 1S

2. Right click your mouse over the black portion of the chart and select *clear chart* (see Figure 2S below) this will clear the previous data from the chart.
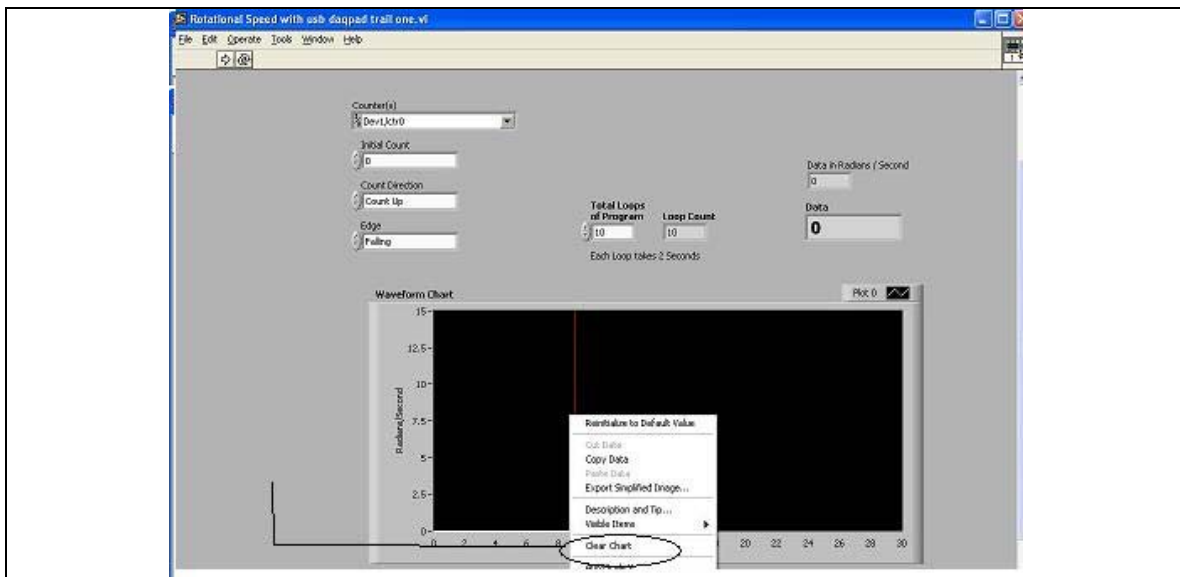


Figure 2S

3. When you are ready to take data on your first run, click the white "**run"** on the toolbar near the top of the screen. This is circled in Figure 3S below. When you see the red line appear on the screen, the computer is acquiring data and you can release the hanging mass. Your screen should look like Figure 4S when the computer is nearly finished acquiring data.
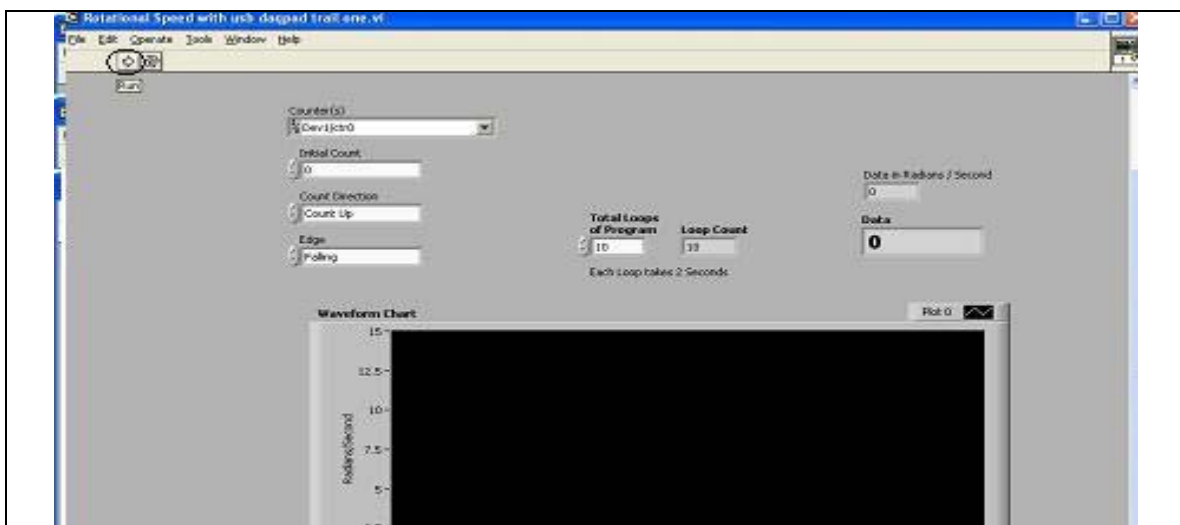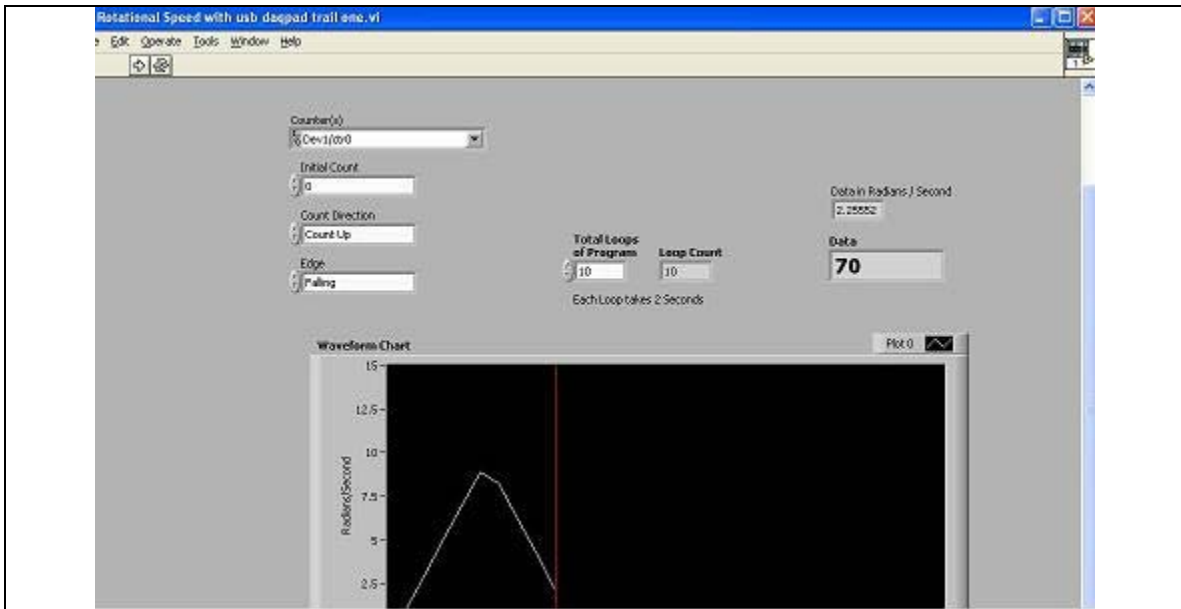


Figure 3S

Figure 4S

4. When the computer is finished acquiring data, you will be asked to choose a file to write to. Enter u: in the filename box hit enter and then give your file a name to be saved as on the U drive. You can now follow step 2 to clear this data from the chart and acquire data from a new run using Labview.

Importing your data to Kaleidagraph

1. Open Kaleidagraph and select **open** from the file menu.
2. Change the file type to **all files(\*.\*)**
3. Select your saved file from the U drive and click open.
4. Make sure the **read titles** check box is not checked, then click **OK** (see Figure 5S below).
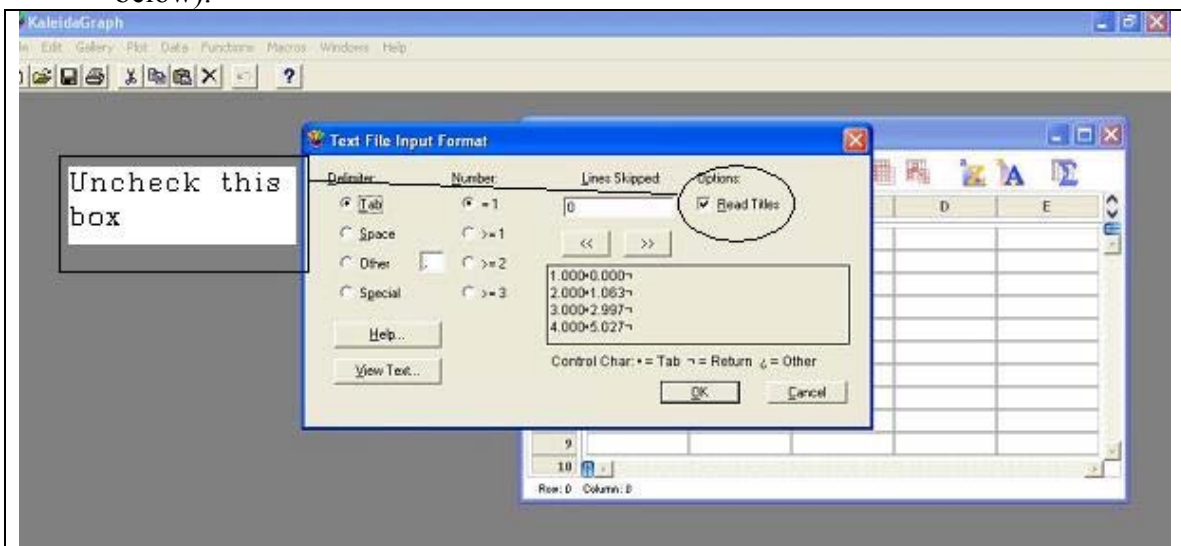

Figure 5S

5. Generate a plot – this will help you notice which points correspond to when the mass was moving downward (see the circled points in Figure 6S). Then discard this plot.
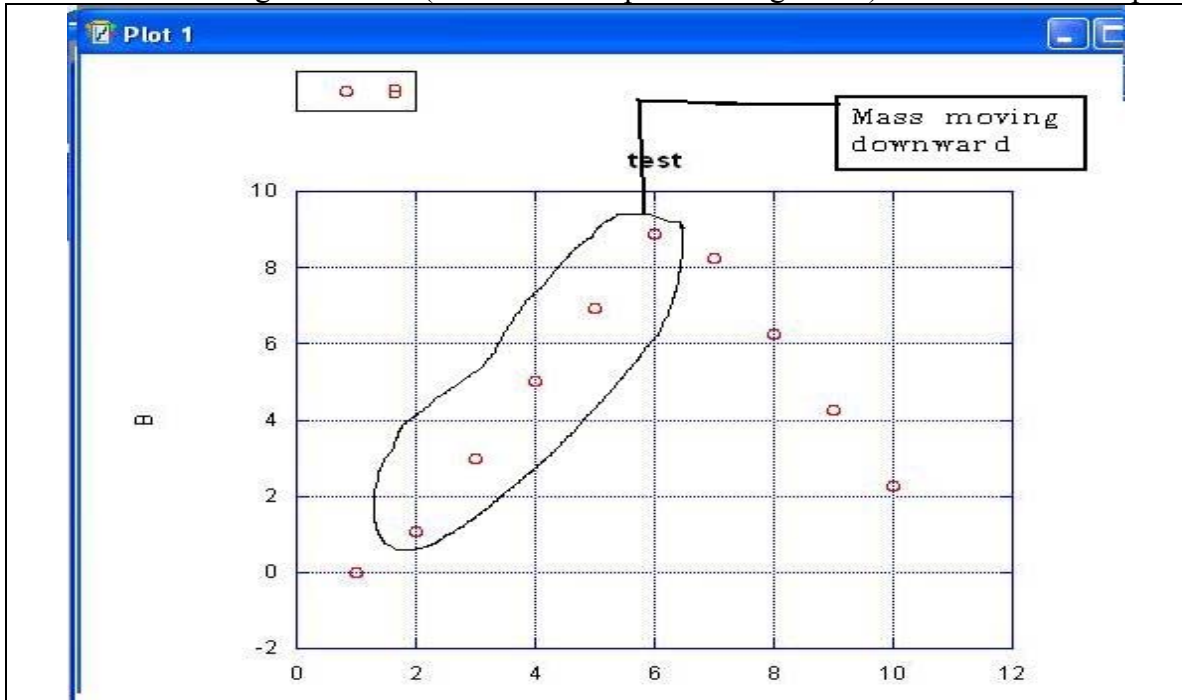


Figure 6S

6. Highlight the data points in your Kaleidagraph data file which correspond to when the mass was moving downward (see Figure 7S below), and then generate your new plot to be included with your lab report. (Don't forget to properly label your graph!)



Figure 7S