

EXPERIMENT 9B

Rotational Motion 2

Moment of inertia

Objectives

- to familiarize yourself with the concept of moment of inertia, I , which plays the same role in the description of the rotation of a rigid body as mass plays in the description of linear motion
- to investigate how changing the moment of inertia of a body affects its rotational motion

APPARATUS

See Figure 3a.

THEORY

If we apply a single unbalanced force, F , to an object, the object will undergo a linear acceleration, a , which is determined by the unbalanced force acting on the object and the mass of the object. The mass is a measure of an object's inertia, or its resistance to being accelerated. Newton's Second Law expresses this relationship:

$$F = ma$$

If we consider rotational motion, we find that a single unbalanced torque

$$\tau = (\text{Force})(\text{lever arm})^\#$$

produces an angular acceleration, α , which depends not only on the mass of the object but on how that mass is distributed. The equation which is analogous to $F = ma$ for an object that is rotationally accelerating is

$$\tau = I \alpha. \tag{1}$$

where the Greek letter tau (τ) represents the torque in Newton-meters, α is the angular acceleration in radians/sec² and I is the **moment of inertia** in kg*m². The moment of inertia is a measure of the way the mass is distributed on the object and determines its resistance to angular acceleration.

[#] In this lab the lever arm will be the radius at which the force is applied (the radius of the axle). This is due to the fact that the forces will be applied tangentially, i.e., perpendicular to the radius. The general form of this relationship is $\tau = \text{force} * \text{lever arm} * \sin \theta$, where θ is the angle between the force and the lever arm. However, in this experiment θ is 90° and $\sin(90^\circ) = 1$.

Every rigid object has a definite moment of inertia about any particular axis of rotation. Here are a couple of examples of the expression for I for two special objects:

One point mass m on a weightless rod of radius r ($I = mr^2$):

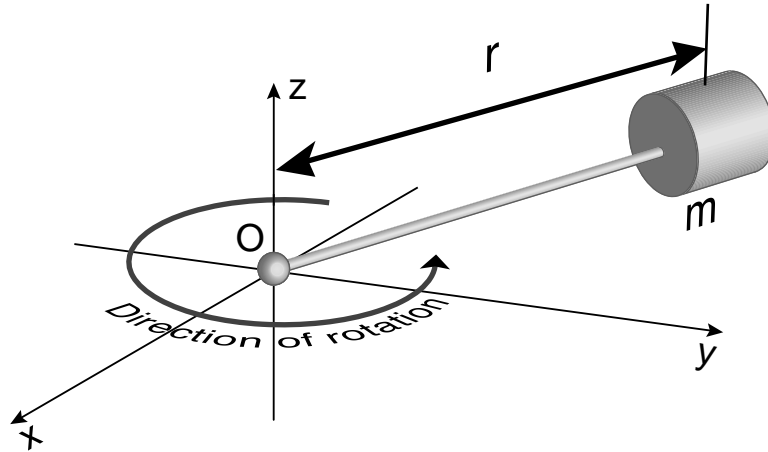


Figure 1

Two point masses on a weightless rod ($I = m_1r_1^2 + m_2r_2^2$):

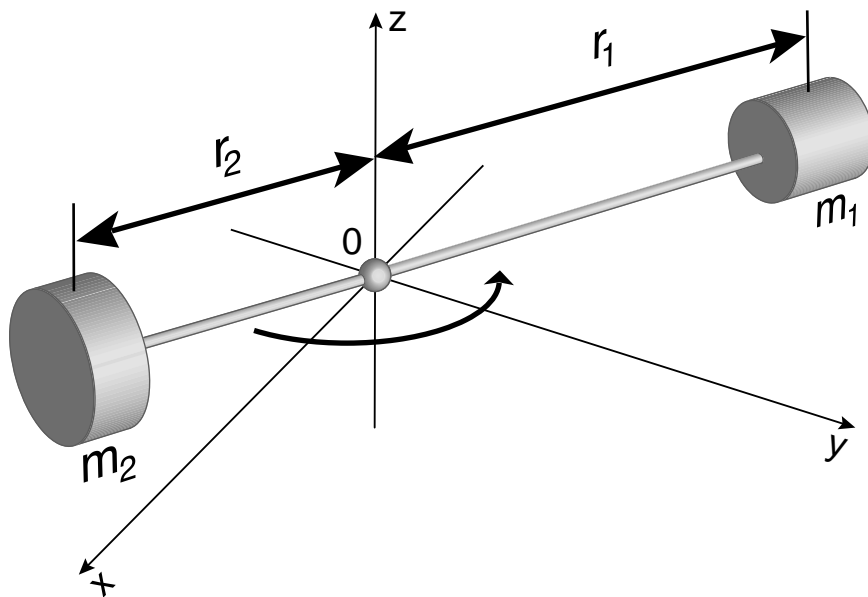


Figure 2

To illustrate we will calculate the moment of inertia for a mass of 2 kg at the end of a massless rod that is 2 m in length (object #1 above):

$$I = mr^2 = (2 \text{ kg}) (2 \text{ m})^2 = 8 \text{ kg}\cdot\text{m}^2$$

If a force of 5 N were applied to the mass perpendicular to the rod (to make the lever arm equal to r) the torque is given by:

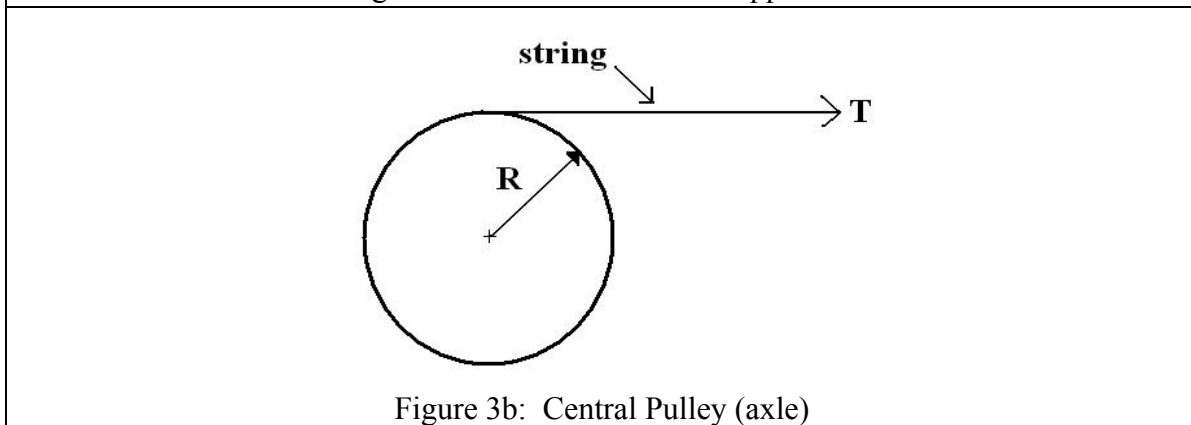
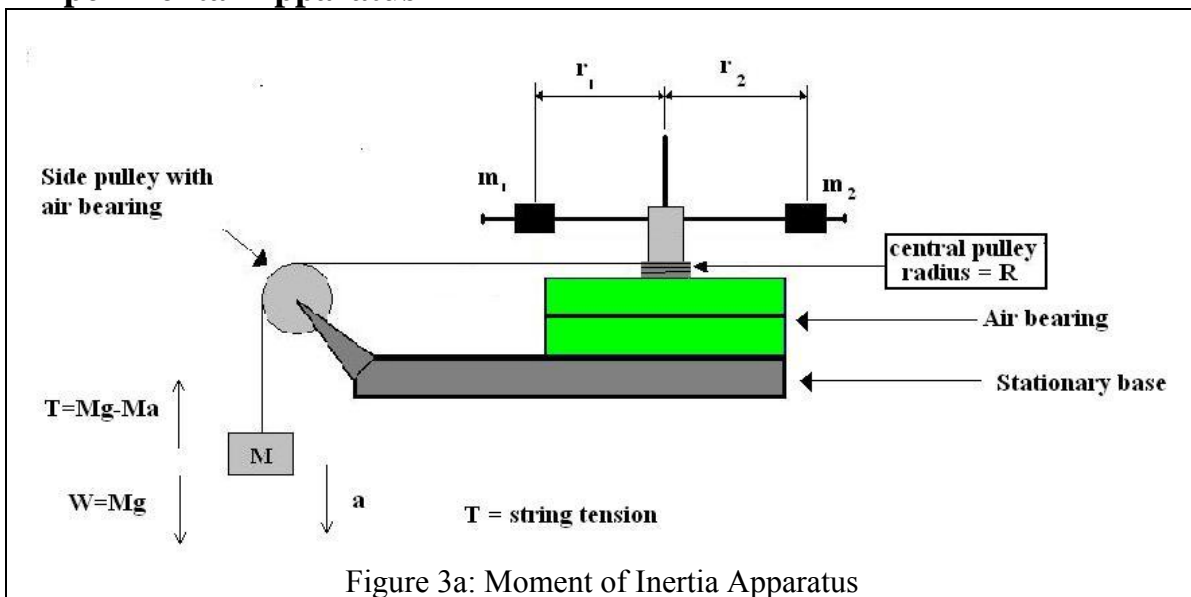
$$\tau = Fr = (5 \text{ N}) (2 \text{ m}) = 10 \text{ N}\cdot\text{m}$$

By equation (1) we can now calculate the angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{10 \text{ N}\cdot\text{m}}{8 \text{ kg}\cdot\text{m}^2} = 1.25 \frac{\text{rad}}{\text{sec}^2}$$

NOTE: The moment of inertia of a complicated object is found by adding up the moments of each individual piece (object #2 above is the sum of two object #1 components).

Experimental Apparatus



In our case, the rigid body consists of two cylinders, which are placed on a metallic rod at varying radii from the axis of rotation. The cylinders and rod are supported by a rotating platform attached to a central pulley and nearly frictionless air bearings. A side view of the apparatus is shown in Figure 3a and a top view of the central pulley is shown in Figure 3b.

In this experiment, we will change the moment of inertia of the rotating body by changing how the mass is distributed on the rotating body. We will place the two cylindrical masses at four different radii such that $r = r_1 = r_2$ in each of the four cases. We will then use our measurements to calculate the moment of inertia (I) for each of the four radial positions of the cylindrical masses (r). The sum of the two cylindrical masses (m_1+m_2) can then be found from a graph of I versus r^2 .

To set up your rigid body, wrap the string around the central pulley (axle) and run it over the side pulley to a known weight as shown in Figure 3a.

Consider the following steps:

If we release the weight from rest, the tension in the string will exert a torque on the rigid body causing it to rotate with a constant angular acceleration α . The angular acceleration of the rigid body is related to the linear acceleration of the falling mass by:

$$\alpha = \frac{\text{Linear acceleration}}{\text{radius of axle}} = \frac{a}{R}$$

or

$$a = R\alpha \quad (2)$$

From Figure 3a and Newton's Second Law, the tension in the string is:

$$T = Mg - Ma \quad (3)$$

The tension in the string causes a net torque on the rigid body. Since torque = (Lever arm) (Force), the net torque on the rigid body is given by:

$$\tau = R \times T \quad (4)$$

The moment of inertia of the rigid body is then found from equation 1 ($\tau = I\alpha$).

PROCEDURE

The apparatus is delicate. Do not rotate the rotating table or side pulley unless air is flowing through the bearings.

1. Measure and record the masses of the hanging mass (M) and the two cylinders (m_1 and m_2).
2. Place the cylinders on the horizontal rod such that the axes of the cylinders are along the horizontal rod (as shown in Figure 4 below). Make sure the thumbscrew on each cylinder is tightened. The center of mass of each cylinder must be the same distance (r) from the axis of rotation (i.e. $r_1 = r_2$ in Figure 3a). Estimate the uncertainty in r called δr . This should include both the uncertainty in reading your ruler and the uncertainty in locating the cylinder's center of mass.

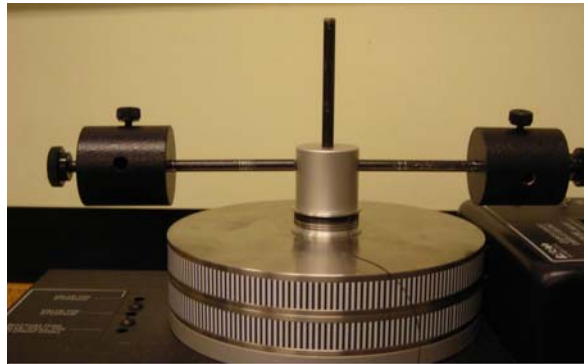


Figure 4

3. With the air supply on, attach the hanging mass (M) to one end of a string and wind the other end around the central pulley. The string should also pass over the side pulley such that the hanging mass is just below the side pulley. Hold the hanging mass stationary and measure its elevation (y) using the floor as your reference level. Record this elevation in your spreadsheet and assign an appropriate uncertainty to this measurement. Then release the hanging mass and simultaneously start the desktop timer. When the mass hits the floor, stop the timer. For the uncertainty in this time (δt), use the standard deviation of a measurement (denoted by s) from your time measurements in the previous experiment. We are assuming your ability to stop and start the timer has not changed appreciably in the past week or so.
4. In the previous experiment we saw:

$$a = \frac{2y}{t^2}$$

Calculate the linear acceleration of the falling mass (M) and use

$$\delta a = a \left(2 \frac{\delta t}{t} + \frac{\delta y}{y} \right) \text{ to calculate its uncertainty.}$$

5. Use equation (3) to calculate the tension in the string (T) and use $\delta T = \left(\frac{\delta M}{M} + \frac{\delta a}{a} \right)$ to calculate its uncertainty.
6. Use equation (2) and the value you obtained for the radius of the central pulley R in the displacement part of the previous experiment to calculate the angular acceleration of the rotating apparatus. In addition, use $\delta \alpha = \alpha \left(\frac{\delta a}{a} + \frac{\delta R}{R} \right)$ to calculate its uncertainty.
7. Use equation (4) to calculate the unbalanced torque on the rotating apparatus and use $\delta \tau = \tau \left(\frac{\delta T}{T} + \frac{\delta R}{R} \right)$ to calculate the uncertainty in this torque. (Note: in this equation the Greek letter τ (tau) is the torque and T is the tension in the string.)
8. Use equation (1) to calculate the moment of inertia of the rotating apparatus;

The uncertainty in moment of inertia is given by:

$$\delta I = I \left[\frac{\delta \tau}{\tau} + \frac{\delta \alpha}{\alpha} \right]$$

Calculate in r^2 and its uncertainty, $\delta(r^2)$.

$$\delta(r^2) = 2r\delta r$$

9. Repeat steps 2 through 8 for two additional (non-zero) values of r.
10. We would like to place the two cylinders at $r = 0$. To do this, we will use the vertical bar on the support (see Figure 5). When you place the cylinders on the vertical bar, make sure they are oriented the same way as in your previous trials, i.e. with the axes of the two cylinders perpendicular to the vertical bar. As before, make sure to tighten the thumbscrews on the cylinder. Follow the procedure in steps 3 through 8 to calculate the moment of inertia of the body with the two cylinders at $r = 0$. Include this data in your data table.



Figure 5

Transfer your data into Kaleidagraph and make a plot of I vs. r^2 . Your data points should have both horizontal and vertical error bars. Also, fit your data with a best fit line, display its equation with the uncertainties in the slope and intercept.

When the two cylinders are placed *on the axis of rotation*, the measured moment of inertia I_0 is the moment of inertia of the rotating apparatus alone plus the moment of inertia of each of the two cylinders about an axis through their own centers of mass.

$$I = I_0 \quad (5)$$

If the two masses are now each placed a distance r from the axis of rotation then equation 5 becomes:

$$I = (m_1 + m_2)r^2 + I_0 \quad (6)$$

If you compare equation 6 to the form of an equation for a straight line:

$$y = mx + b$$

You can see that a plot of I vs. r^2 should be a straight line. The slope of this line is the sum of the masses ($m_1 + m_2$) and the intercept is I_0 .

QUESTIONS

- 1) The moment of inertia of a body depends not only on its mass, but also on how the mass is distributed. Do your data support this? Why or why not?
- 2) In your plot of I vs. r^2 , why did you use r^2 and not r in the plot? What are the units of the slope of I vs. r^2 ?
- 3) Discuss the consistency of the slope of your I vs. r^2 with the value you measured for m_1+m_2 .