Physics 472 – Spring 2008

Homework #4, due Friday, February 8

(Point values are in parentheses.)

1. [2] Griffiths problem 5.4. This problem is much easier if you re-write Equation (5.10) in Dirac notation, but of course you may use wavefunctions if you insist.

2. [2] Consider an electron in the n=3, l=2, m_l=1 state of a hydrogen atom. If we add a term in the Hamiltonian of the form $\overline{V} = \lambda\vec{L}\cdot\vec{S}$, the energy will shift according to whether the electron is in the state with j=5/2 or j=3/2. Calculate the energy shifts for those two cases.

3. [4] a)Griffiths problem 5.5. Notice that Griffiths doesn't mention spin in this problem, so you must satisfy the correct exchange symmetry requirements with the spatial wavefunctions alone.

   [4] b) Now let's put spin into the previous problem. Repeat part (b) of Griffiths 5.5 for the fermion and boson cases only. For the fermion case, assume that the two identical fermions each have spin ½. Answer part (b) for both the spin triplet (s=1) case and for the spin singlet (s=0) case. For the boson case, assume that the two identical bosons each have spin 1. Answer part (b) for all three values of the total spin: s=2, s=1, and s=0. Use Table 4.8 to figure out the exchange symmetry of the spin state for each of those three cases.

4. [8] On a previous problem set you found the matrices that represent the operators for the three component of spin, for a spin-1 particle. They are:

   $$\hat{S}_x = \begin{pmatrix} 0 & \hbar/\sqrt{2} & 0 \\ \hbar/\sqrt{2} & 0 & \hbar/\sqrt{2} \\ 0 & \hbar/\sqrt{2} & 0 \end{pmatrix},\ \hat{S}_y = \begin{pmatrix} 0 & -i\hbar/\sqrt{2} & 0 \\ i\hbar/\sqrt{2} & 0 & -i\hbar/\sqrt{2} \\ 0 & i\hbar/\sqrt{2} & 0 \end{pmatrix},\ \hat{S}_z = \begin{pmatrix} \hbar & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\hbar \end{pmatrix}$$

   An SG apparatus is used to prepare a beam of s=1 atoms in the initial state: $|\chi\rangle = \begin{pmatrix} -3i/4 \\ \sqrt{6}/4 \\ i/4 \end{pmatrix}$.

   a) If you send the beam into a second SG apparatus with its spin axis oriented along the z-direction, calculate the probabilities associated with each of the three output ports of the apparatus, i.e. calculate the probabilities that measurement of S_z will produce $\hbar$, 0, and $-\hbar$. (You should be able to do this just by looking at $|\chi\rangle$, without any fancy calculation.)

   b) Now change the spin orientation of your second SG apparatus to the x-axis, and calculate the probabilities associated with each of the three output ports. This time you will need to do a real calculation. To find the eigenstates of S_x, you can use your results from problem 3 from the final exam from last semester if you are sure that you did it correctly.

c) Do the same thing with the second SG apparatus oriented along the y-axis. This will require some real work.

d) Now calculate the three expectation values, $\langle S_x \rangle$, $\langle S_y \rangle$, and $\langle S_z \rangle$, using the probabilities you calculated in parts (a) – (c). Check your answers using direct matrix multiplication.

e) Expectation values of spin operators behave like classical vectors in the following sense. A spin eigenstate oriented along any direction in space labeled by θ and ϕ will have expectation values along the x, y, and z axes given by the standard projection formulas in spherical coordinates. For example, the state $|\chi\rangle$ given above has the maximum value $\langle S_{\theta,\phi} \rangle = +\hbar$ along the unknown direction labeled by θ, ϕ, and will therefore have the following expectation values along the x, y, and z axes:

$$\langle S_x \rangle = \hbar \sin\theta \cos\phi \qquad \langle S_y \rangle = \hbar \sin\theta \sin\phi \qquad \langle S_z \rangle = \hbar \cos\theta \,.$$

Using these relationships, find the direction in space in which the spin state $|\chi\rangle$ is pointing. In other words, find the values of θ and ϕ for which your results from part (d) satisfy these three equations.