Physics 472 – Spring 2008

Homework #5, due Friday, February 15

(Point values are in parentheses.)

1. [5] Griffiths problem 6.1

2. [4] Griffiths problem 6.2. Griffiths wants you to use the Virial Theorem (problem 3.31) to do part (b). An alternative is to express the $\hat{x}$ operator in terms of $\hat{a}$ and $\hat{a}^{+}$, as we did in class.

3. [5] Griffiths problem 6.4.

4. [6] Two neutrons (s = ½) are confined inside a spherical cavity of radius a. Assume that we have solved the single-particle problem, and we know the correct radial wavefunctions $R_{nl}(r)$, which we label by the quantum number n and the orbital angular momentum l. Since the cavity is spherically symmetric, $\vec{L}$ is conserved, and we can write <u>single-particle</u> wavefunctions as

$$\Psi_{nlms}(r,\theta,\phi) = R_{nl}(r)Y_l^m(\theta,\phi)\chi(s)$$

(If you want to see what the $R_{nl}(r)$ look like, see Griffiths Section 4.1.3, Example 4.1.) In Dirac notation, the single particle eigenstates can be written as:

$$|n,l,m_l,s,m_s\rangle \equiv |n,l\rangle \otimes |l,m_l\rangle \otimes |s,m_s\rangle$$

(Writing the s is redundant, because we know s = ½, but I'm being precise.) If we ignore the weak interaction between the neutrons, then the two-particle states look something like this:

$$|\Psi\rangle = |n_1,l_1,m_{l1},s_1,m_{s1}\rangle^{(1)} \otimes |n_2,l_2,m_{l2},s_2,m_{s2}\rangle^{(2)} \qquad (1)$$

a) The problem is, the state written above is not an eigenstate of the exchange operator. Assuming that $n_1 \neq n_2$ or $l_1 \neq l_2$, write down a properly antisymmetrized version of (1).

b) Eigenstates of exchange are more easily written down as tensor products of a two-particle spatial state with a two-particle spin state:

$$|\Psi\rangle = |n_1,n_2,l_1,m_{l1},l_2,m_{l2}\rangle \otimes |s_1,m_{s1},s_2,m_{s2}\rangle \qquad (2)$$

But even this isn't the most convenient basis set. We'd be better off using eigenstates of the total spin $\vec{S} = \vec{S}_1 + \vec{S}_2$. So we'll use the following basis states:

$$|\Psi\rangle = |n_1,n_2,l_1,m_{l1},l_2,m_{l2}\rangle \otimes |s_1,s_2,s,m_s\rangle \qquad (3)$$

Again, the state written above is not an eigenstate of the exchange operator. Assuming that $n_1 \neq n_2$ or $l_1 \neq l_2$ or $m_{l1} \neq m_{l2}$, write down all four properly antisymmetrized versions of (3).

c) If $n_1 = n_2$, $l_1 = l_2$, and $m_{l1} = m_{l2}$, then the states you wrote down in part (b) are not right (three of them are zero). Write down the only properly antisymmetrized version of (3) for this special case.

d) When $n_1 = n_2$ and $l_1 = l_2$, it may be more convenient to change bases once again, and use eigenstates of the total orbital angular momentum $\vec{L} = \vec{L}_1 + \vec{L}_2$. In that case the states are:

$$|\Psi\rangle = |n_1, n_2, l_1, l_2, l, m_l\rangle \otimes |s_1, s_2, s, m_s\rangle \tag{4}$$

For the case $n_1 = n_2 = 3$, $l_1 = l_2 = 1$, write down one properly antisymmetrized state for each possible value of *l*.

e) Now consider two neutrons in free space. From classical mechanics, you know that we can specify their motion using the center-of-mass coordinate $\vec{R} = \frac{1}{2}(\vec{r}_1 + \vec{r}_2)$ and the relative coordinate $\vec{r} = \vec{r}_1 - \vec{r}_2$. (To remind yourself how this works, look at Griffiths problem 5.1). For this problem, we'll assume that the center-of-mass momentum is zero, so we'll ignore $\vec{R}$ and $\vec{P}$. We could thus write the wavefunction for the <u>two-neutron</u> system in the following form:

$$\Psi_{nlms_1s_2}(r,\theta,\phi) = \Phi_{nl}(r) Y_l^m(\theta,\phi)\chi(s_1,s_2).$$

where again the radial wavefunction depends both on the quantum number *n* and the orbital angular momentum *l*. Instead, let's use eigenstates of total spin S^2 and S_z, where $\vec{S} = \vec{S}_1 + \vec{S}_2$. In Dirac notation, a complete basis set of two-neutron quantum states is thus:

$$|n, l, m_l, s, m_s\rangle \equiv |n, l\rangle \otimes |l, m_l\rangle \otimes |s, m_s\rangle$$

When the spatial state is written in terms of the relative coordinate $\vec{r}$, spatial exchange corresponds to the parity transformation, $\vec{r} \to -\vec{r}$. In terms of spherical polar coordinates, that corresponds to $r \to r,\ \theta \to \pi - \theta,\ \phi \to \phi + \pi$. If you look at the spherical harmonics in Table 4.3, you'll see that they have even parity if *l* is even, and odd parity if *l* is odd. In operator notation, that means $\hat{P}|l, m_l\rangle = (-1)^l |l, m_l\rangle$, where $\hat{P}$ is the parity operator.

For each allowed value of *s*, what are the allowed values of *l* (up to *l*=4)? Write down all allowed combinations of *s*, *l*, and *j*, using the archaic spectroscopic notation from atomic physics, ${}^{2S+1}L_J$.