EXPERIMENT 5A
RC Circuits

Objectives

1) Observe and qualitatively describe the charging and discharging (decay) of the voltage on a capacitor.

2) Graphically determine the time constant for the decay, $\tau = RC$.

Apparatus

Signal Generator
Resistors and Capacitors
Two-color Light Emitting Diode (LED)
Digital Multimeter
Circuit Breadboard and Cable

Introduction

The Capacitor

A capacitor is a device that stores electrical charge. The simplest kind is a "parallel plate" capacitor that consists of two metal plates separated by an insulating material such as dry air, plastic or ceramic. Such a device is shown schematically below.

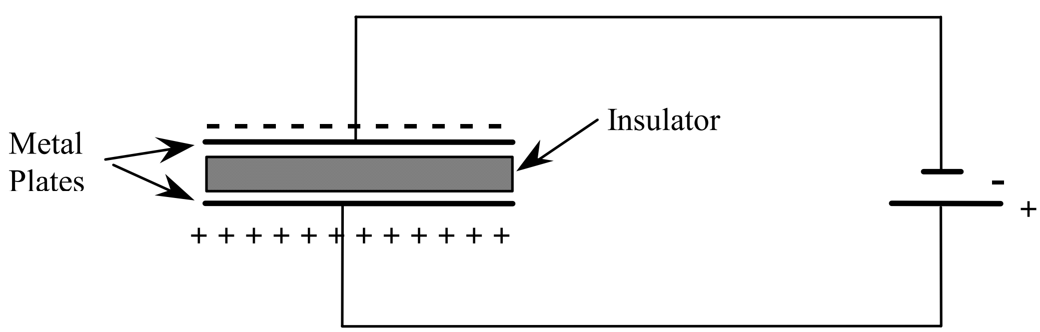


Figure 1. A simple capacitor circuit.

It is straightforward to see how it could store electrical energy. If we connect the two plates to each other with a battery in the circuit, as shown in the figure above, the battery will drive charge around the circuit as an electric current. But when the charges reach the plates they can't go any further because of the insulating gap; they collect on the plates, one plate becoming positively charged and the other negatively charged. The voltage across the plates due to the electric charges is opposite in sign to the voltage of the battery. As the charge on the plates builds up, this back-voltage increases, opposing the action of the battery. As a consequence, the current flowing in the circuit decays, falling to zero when the back-voltage is exactly equal and opposite to the battery voltage.

If we quickly remove the wires without touching the plates, the charge remains on the plates. Because the two plates have different charge, there is a net electric field between the two plates. Hence, there is a voltage difference between the plates. If, sometime later, we connect the plates again, this time with a light bulb in place of the battery, the plates will discharge: the electrons on the negatively charged plate will move around the circuit to the positive plate until all the charges are equalized. During this short discharge period a current flows and the bulb will light. The capacitor stored electrical energy from its original charge up by the battery until it could discharge through the light bulb. The speed with which the discharge (and conversely the charging process) can take place is limited by the resistance of the circuit connecting the plates and by the capacitance of the capacitor (a measure of its ability to hold charge). In this lab you will test the theory that describes this behavior by measuring some discharge rates with an oscilloscope and comparing them to predictions of the theory.

RC circuit

An RC circuit is simply a circuit with a resistor and a capacitor in series connected to a voltage source (battery).

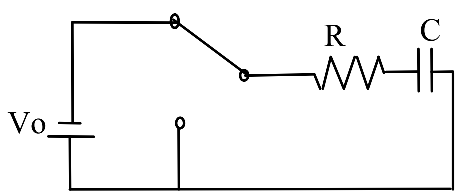


Figure 2a Charging

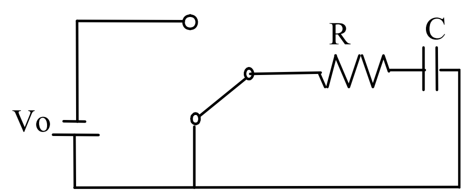


Figure 2b Discharging

As with circuits made up only of resistors, electrical current can flow in this RC circuit, with one modification. A battery connected in series with a resistor will produce a constant current. The same battery in series with a capacitor will produce a time varying current, which decays gradually to zero. If the battery is removed and the circuit reconnected without the battery, a current will flow (for a short time) in the <u>opposite</u> <u>direction</u> as the capacitor "discharges". A measure of how long these transient currents last in a given circuit is given by the time constant τ.

The time it takes for these transient currents to decay depends on the resistance and capacitance. The resistor resists the flow of current; it thus slows down the decay. The capacitance measures capacity to hold charge: like a bucket of water, a larger capacity container takes longer to empty than a smaller capacity container. Thus, the time constant of the circuit gets larger for larger R and C. In detail:

$$\tau \text{ (seconds)} = R(\text{Ohms}) \times C(\text{Farads})$$

The current does not fall to zero at time τ; τ is the time it takes for the voltage of the discharging capacitor to drop to 37% its original value. It takes 5 to 6 τ 's for the current to decay to 0 amps. Just as it takes time for the charged capacitor to discharge, it takes time to charge the capacitor. Due to the unavoidable presence of resistance in the circuit, the charge on the capacitor and its stored energy only approaches a final (steady-state) value after a period of several times the time constant of the circuit elements employed.

What's going on in this RC Circuit

1) Initially, the switch is open, and no current is flowing.
2) The switch is closed as in Figure 2. (a). The capacitor will charge up, its voltage will increase. During this time, a current will flow, producing a voltage across the resistor according to Ohm's Law, $V_R = IR$. As the capacitor is being charged up, the current will be decreasing, with a certain time constant τ, due the stored charge producing a voltage across the capacitor.

   In terms of τ = RC, the voltage across the resistor and the voltage across the capacitor when the capacitor is charging look like:

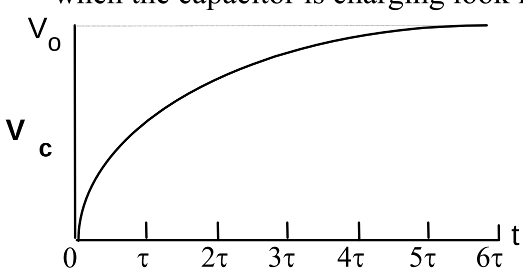


Figure 3a: Voltage across the capacitor V_c as a function of time. Time constant τ = RC

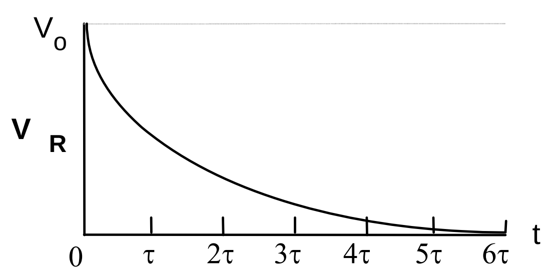


Figure 3b: Voltage across the resistor V_R as a function of time. Time constant τ = RC

V_C and V_R while the capacitor is charging can be expressed as:

$$V_c(t) = V_0\left(1 - e^{\frac{-t}{RC}}\right) \quad \text{and} \quad V_R(t) = V_0\, e^{\frac{-t}{RC}}$$

Where, e is an irrational number and is the base of the natural logarithm. The value of e is approximately 2.718.

When t = τ = RC,

$$V_C = V_0(1 - e^{-1}) \quad \text{and} \quad V_R = V_0(e^{-1})$$

$$V_C = 0.63V_0 \qquad V_R = .37V_0$$

So, after t = RC seconds, the capacitor has been charged to 63% of its final value and the voltage across the resistor has dropped to 37% of its peak (initial) value. After a very long time, the voltage across the capacitor will be V_0 and the voltage across the resistor will be zero.

3) If we flip the switch as shown in Figure 2(b), we will discharge the capacitor.

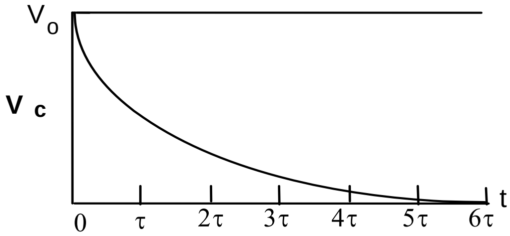
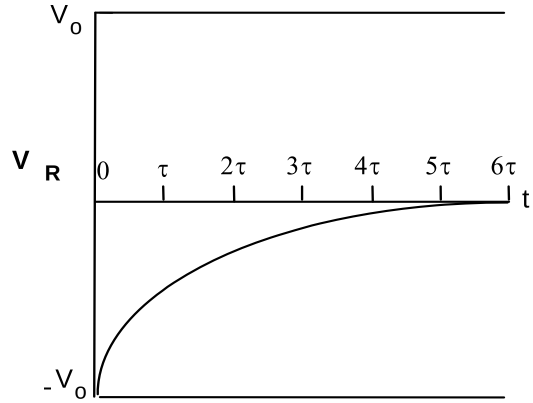


Figure 4a Voltage across the capacitor V_c for the discharging capacitor. τ = RC

Figure 4b Voltage across the resistor V_R for the discharging capacitor. τ = RC



The voltage of the resistor is exponentially increasing from -Vo to zero. It is critical to remember that the total voltage between the capacitor and the resistor must add up to the applied voltage. If the circuit is disconnected from power supply, then the sum of the voltage must be zero.

The voltage across a capaitor and a resistor in a discharging RC circuit is given by:

$$V_C = V_0 e^{\frac{-t}{RC}} \quad \text{and} \quad V_R = -V_0 e^{\frac{-t}{RC}}$$

Calculating the natural logarithm of the voltage across the capacitor (V_C) yields:

$$\ln(V_C) = \ln(V_0 e^{\frac{-t}{RC}}) = \ln(V_0) + \ln\left(e^{\frac{-t}{RC}}\right) = \ln(V_0) - \frac{t}{RC}$$

or

$$\ln(V_C) = -\frac{t}{RC} + V_0$$

Notice, the result has the same form as the equation of a straight line y=mx+b. In our case, the slope is $\frac{-1}{RC}$ and the intercept is V_0.

4) If we now repeat this process and alternate the switch position every 6τ seconds, the voltages will look like:

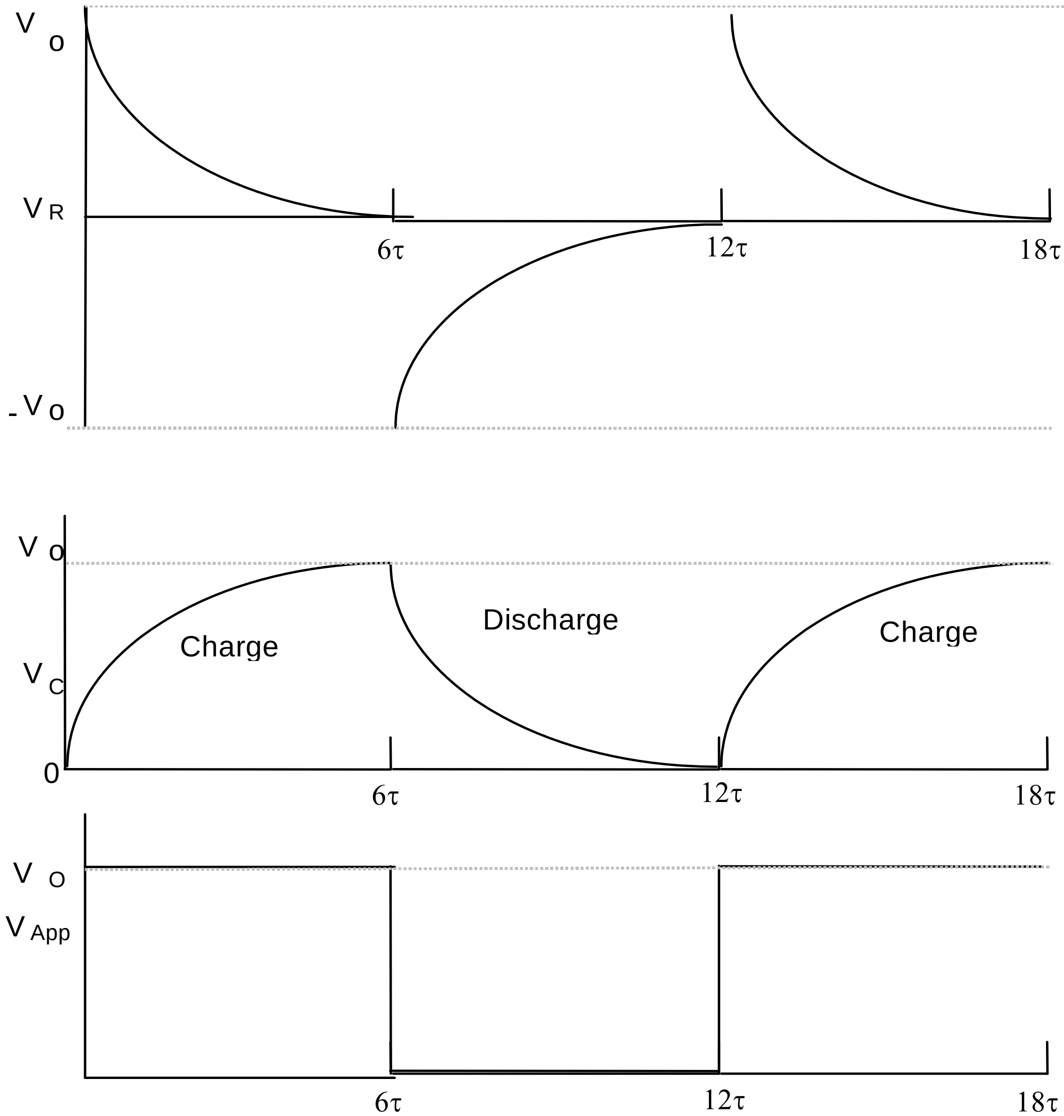


Figure 5. Repeated cycles between Figure 2a and Figure 2 b

5) In this particular experiment, we are hooking up a wave generator to a RC circuit, which allows us to reverse the applied voltage. In effect, it will allow us to drive the circuit with V_0 and $-V_0$ as input voltage. The voltage as a function of time for both the resistor and the capacitor are shown below.

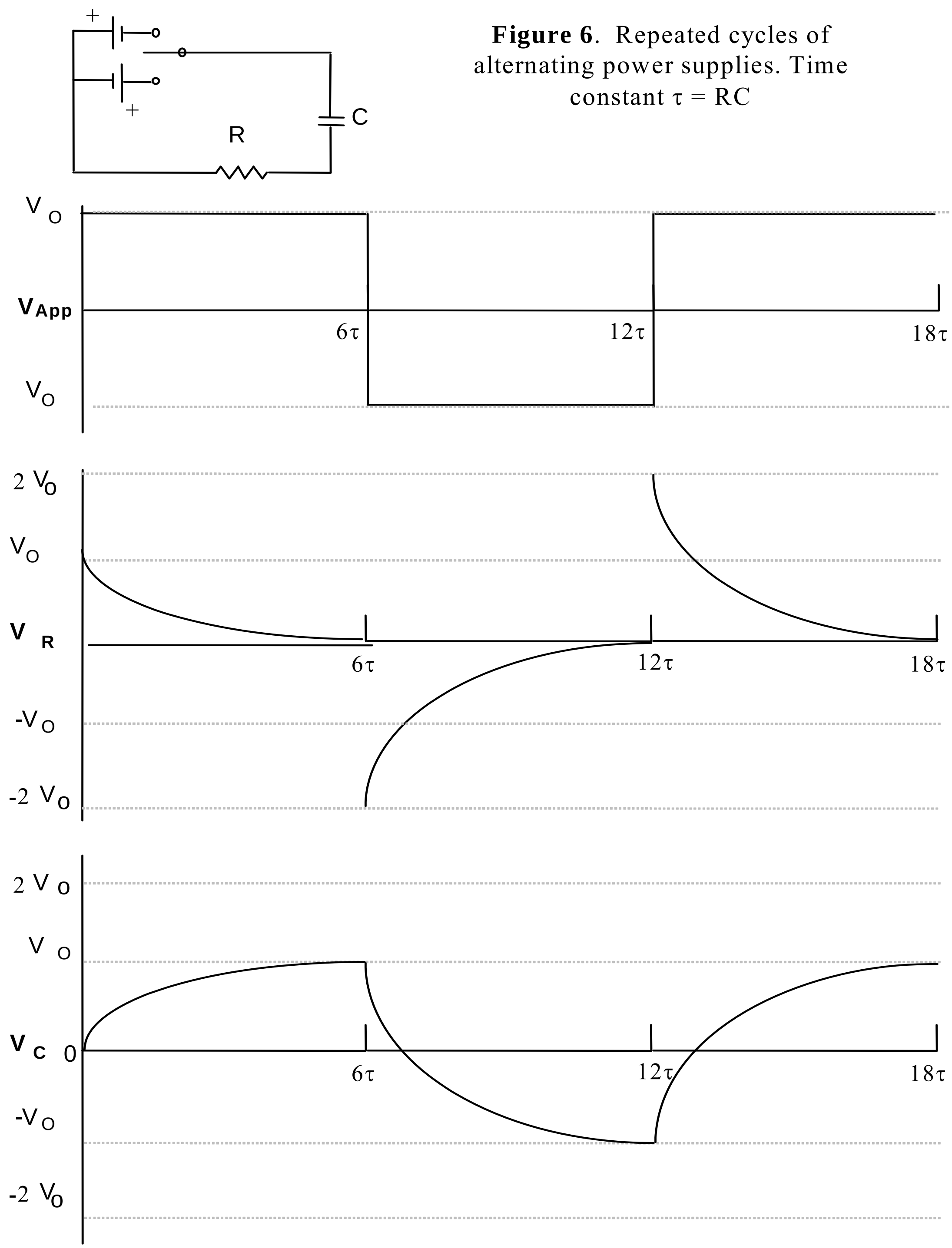


Figure 6. Repeated cycles of alternating power supplies. Time constant τ = RC

Procedure

PART I

In this part of the experiment you will make measurements using a digital multimeter and an electronic timer. You will charge the capacitor and measure the voltage across the resistor as the capacitor discharges. Set the multimeter to read DC voltage.

1. Build the circuit shown in Figure 7a. Use the DC power supply, a 100 kΩ resistor, and a 1,000 μF capacitor. Check the color code of the resistor to verify it is 100 kΩ; also check the tolerance of the resistor. Your instructor will tell you the tolerance of the capacitor.

2. Use the power supply to charge the capacitor to about 12 V to 13 V. Then disconnect the power supply from the circuit and notice that the voltage across the capacitor slowly decreases – what could possibly cause this effect (see question 2)? Disconnect the multimeter from the circuit.

3. Build the circuit shown in figure 7b. After building the circuit shown in 7b, the capacitor should still be charged and you should still notice that the voltage is slowly decreasing. For the branch of the circuit containing the switch, use a single wire with a banana plug on each end. For now, only connect one end of this wire. Connecting the second end serves to close the switch. When this is done, the capacitor will start to discharge through the resistor.

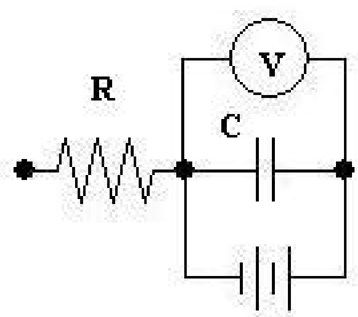


figure 7a

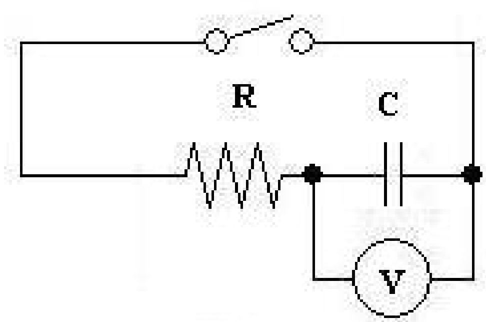


figure 7b

4. Close the switch and when the voltage across the capacitor has dropped to about 10 to 11 volts start the timer and record the time and voltage in Table 1. Continue recording the voltage across the resistor once every 10 seconds until your timer has reached 300 seconds.

5. Have *Excel* calculate ln(V_C) and import your data into *Kaleidagraph.* Make a plot of the capacitor voltage versus time.

Make a second plot of ln(V_C) versus time. Have *Kaleidagraph* fit you graph with a best-fit line. Use the curve fit parameters to determine the time constant of the circuit and its uncertainty.

PART II

In this part of the experiment you will use a square wave generator and a two color light emitting diode (LED) to observe an RC circuit. The color which the LED displays depends on the direction of the current flowing through it. The brightness of the LED depends on how much current is flowing through it. The LED glows brighter for higher currents.

1. Use a 100 Ω resistor, a 1,000 μF capacitor, an LED and a signal generator to build the circuit shown in figure 8.

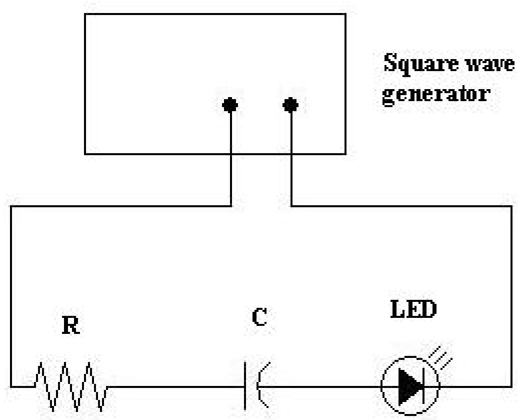


figure 8

2. On the signal generator, change the waveform from a sine wave to a square wave. Set the amplitude of the signal generator to maximum and set the frequency to 0.500 Hz. For question 3, record your observations.

Uncertainties:
From the multiplication rule for uncertainties,

$$\delta(RC) = RC\left(\frac{\delta R}{R} + \frac{\delta C}{C}\right)$$

The uncertainty in the time constant τ obtained from the graph of $\ln V_c$ versus time is given by:

$$\delta\tau = \tau\left(\frac{\delta slope}{slope}\right)$$

Questions:
Part I

1. Discuss the consistency between your measurement of τ from the graph and RC calculated from circuit values.
2. After charging the capacitor and disconnecting the power supply, you observed that the voltage measured by the voltmeter across the capacitor slowly decreased. What are possible explanations for this observation?

Part II

3. What are your observations of the circuit?
4. Qualitatively, how does your observation of the apparent brightness of the LED compare with your findings from Part I?