EXPERIMENT 11
Diffraction and Interference

OBJECTIVES:

1) Observe Fraunhofer diffraction and interference from a single slit, double-slit and multiple-slit (a diffraction grating).
2) Calculate the slit width, which produces the single-slit diffraction pattern, and observe how the slit width affects the diffraction pattern.
3) Verify Babinet's Principle by observing the diffraction pattern from a thin wire.
4) Calculate the slit width and the slit spacing for double-slit from the interference pattern produced by light passing through the double slit.
5) Calculate the slit spacing of a diffraction grating and thereby determine the ruling density.

CAUTION!

The laser is a device that can produce an intense, narrow beam of light at one wavelength. NEVER look directly into the laser beam or its reflection from a mirror, etc.

INTRODUCTION

Diffraction occurs when a portion of a wave passes through a slit. Interference occurs when two or more coherent waves overlap. (Coherent means that the waves have a fixed phase relationship.) Constructive interference takes place at certain locations where two waves are in phase (for example, both waves have maximum). Destructive interference takes place where two waves are out of phase (for example, one wave has maximum, the other has minimum).

In the case of a single-slit, diffraction is the only effect present. In the case of two or more slits, two effects are present: a) diffraction from each individual slit; b) if the incident light is coherent and the diffraction patterns of each slit overlap, then interference takes place in the region of the overlap, (i.e., inside the diffraction envelope).

The simplest diffraction and interference patterns involve plane waves (collimated or parallel light beams). Diffraction patterns associated with plane waves are called Fraunhofer patterns, named after the German scientist who first explained the effect. In this experiment, we will use a laser as our light source. A laser produces collimated and coherent light beams at one wavelength.

Single-slit diffraction

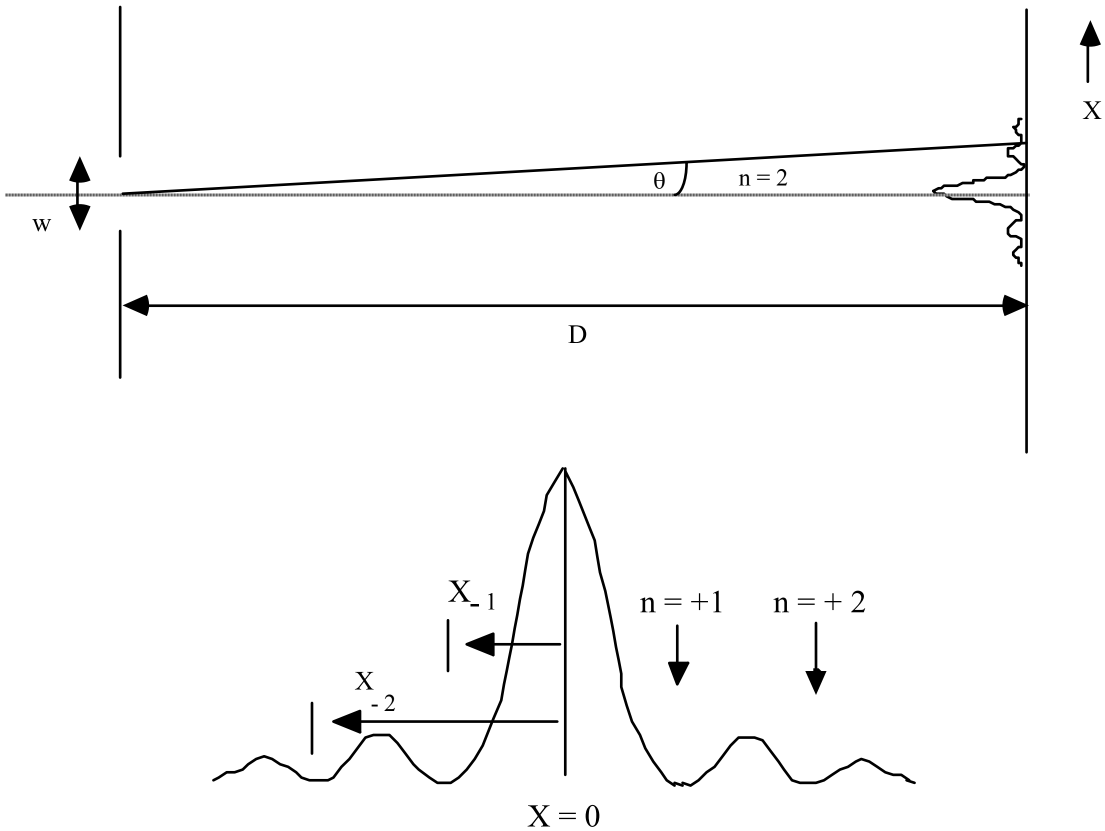


Figure 1: Intensity distribution in single-slit diffraction

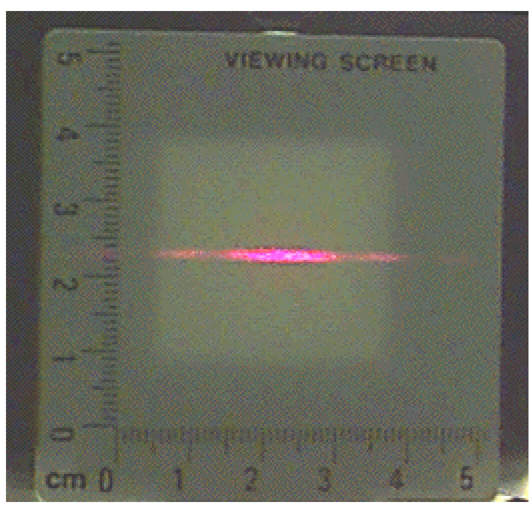


Figure 2: Diffraction pattern produced by laser light passing through a single slit

Figure 1 shows the pattern of light intensity as a function of position, with x = 0 at the position the beam would have taken without the slit. The top pane of the drawing shows the physical arrangement, with a sketch indicating the intensity at different points. The bottom part of the figure is a blowup of the intensity sketch. Figure 2 is a picture of a diffraction pattern. The first order minima (n = -1 and n = +1) are at 1.2 cm and 3.4 cm.

The intensity curve shows how much light appears as a function of position; the original beam of light is redistributed by diffraction. Maxima (peaks) of light intensity will be seen as bright spots if you look at the screen. Minima (valleys) of light intensity will be seen as dark spots. The principal maximum position, x=0, is the center of the central bright spot, which is where you might have thought all the light would go. To measure the location of a maximum, use the center of a bright spot. To measure the location of the minimum, use the center of a dark spot.

The pattern in Figures 1 and 2 is what happens for light of a single wavelength. Superimposing diffraction patterns from different wavelengths of light would blur the pattern, because maxima for some wavelengths would land on minima for other wavelengths. This is one reason we use a laser in this experiment.

For a single-slit, the condition for diffraction minimum to occur is:

$$w \sin \theta_n = n\lambda.$$

Since, $$\theta_n = \tan^{-1}\left(\frac{x_n}{D}\right)$$

∴ $$\sin \theta_n = \sin\left(\tan^{-1}\left(\frac{x_n}{D}\right)\right)$$

Therefore, $$w = \frac{n\lambda}{\sin\left(\tan^{-1}\left(\frac{x_n}{D}\right)\right)} \qquad (1)$$

Here,
- w = slit width
- n = order number of diffraction minimum
- λ = wavelength of light
- D = distance from screen to slit
- x_n = distance from the principal maximum to the nth diffraction minimum
- θ_n = angle from the center line to the nth diffraction minimum

Equation (1) is called the Diffraction Equation.

Note that x_n is a signed distance: it is positive for n positive. For small values of θ_n, $x_n \ll D$ and $\sin \theta_n \approx \tan \theta_n \approx \theta_n$; this is commonly called the small angle approximation. Using this approximation $\sin\left(\tan^{-1}\left(\frac{x_n}{D}\right)\right) \approx \frac{x_n}{D}$ and equation (1) becomes:

$$w = \frac{n\lambda D}{x_n} \qquad (2)$$

Double-slit diffraction and interference

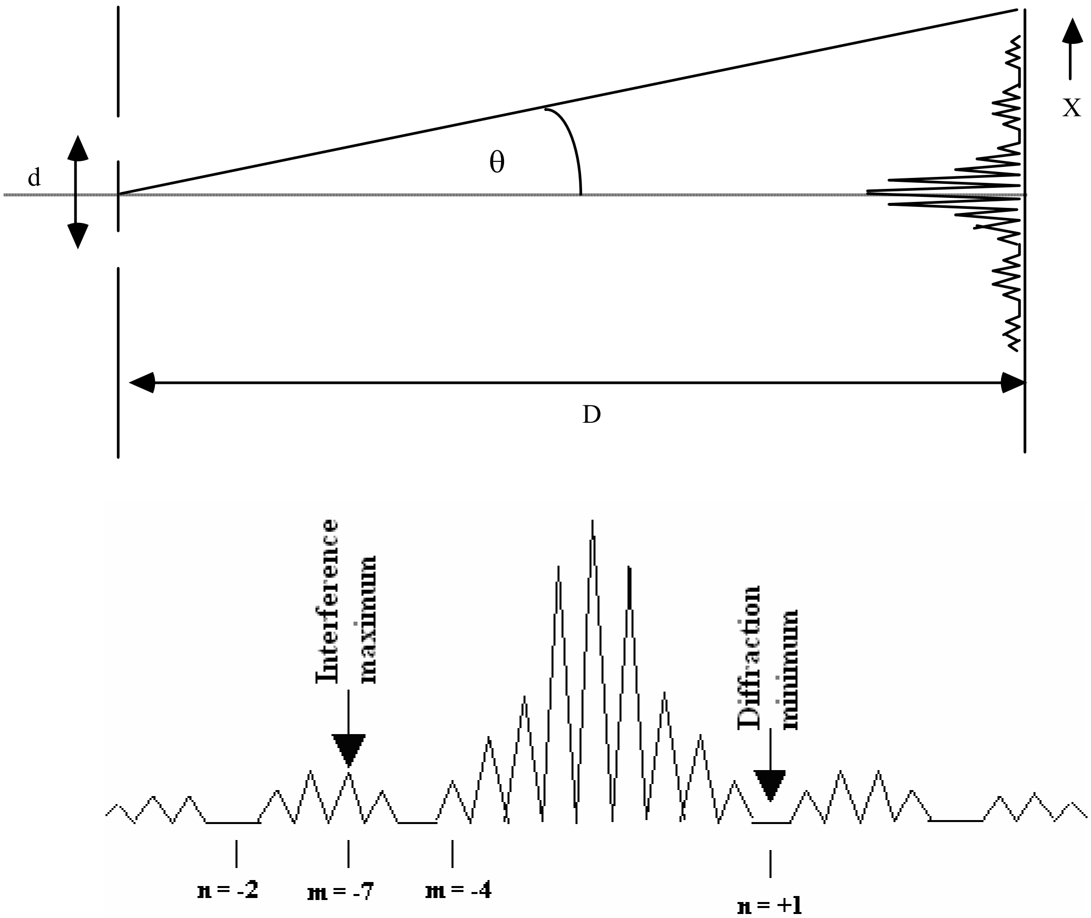


Figure 3: Geometry and intensity distribution in double-slit diffraction and interference. Arrows point to m = -7 interference maximum, and n = +1 diffraction minimum.
The m = -5 maximum is not seen here as it coincides with the n = -1 minimum.

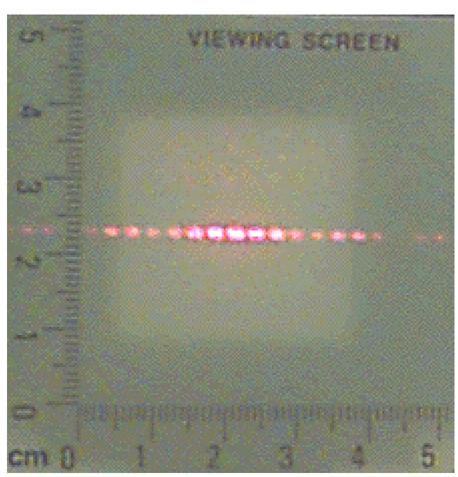


Figure 4: Diffraction and interference pattern produced by laser light passing through a double slit.

In Figure 4, the first order (n = +1) diffraction minimum is at 3.1 cm, between the third and fourth order (m = +3 and m = +4) inference maxima. The second order (n = +2) diffraction minimum is at 4.2 cm. Notice, the interference maxima m = +5, -5, +10 and -10 are not visible. These maxima coincide with the n = +1, -1, +2 and -2 minima, consequently they are not visible.

For two nearby slits (the double slit), the condition for an interference maximum to occur is:

$$d \sin \theta_m = m\lambda$$

Thus,

$$d = \sin\left[\tan^{-1}\left(\frac{m\lambda D}{x_m}\right)\right] \tag{3}$$

Here,
d = slit spacing
m = order number of interference maximum
λ = wavelength of light
D = distance from screen to slit
x_m =distance from the principal maximum to the mth interference maximum
θ_m = angle from the center line to the mth interference maximum

Equation (3) is called the Interference Equation. The algebra looks the same as for the Diffraction equation except that $\mathbf{x_m}$ refers to the location of **maxima,** not minima and **d** is the **spacing** between the two slits**,** not the width of the individual slits.

Using the small angle approximation, equation (3) simplifies to:

$$d = \frac{m\lambda D}{x_m} \tag{4}$$

Multiple-slit interference and diffraction (a diffraction grating)

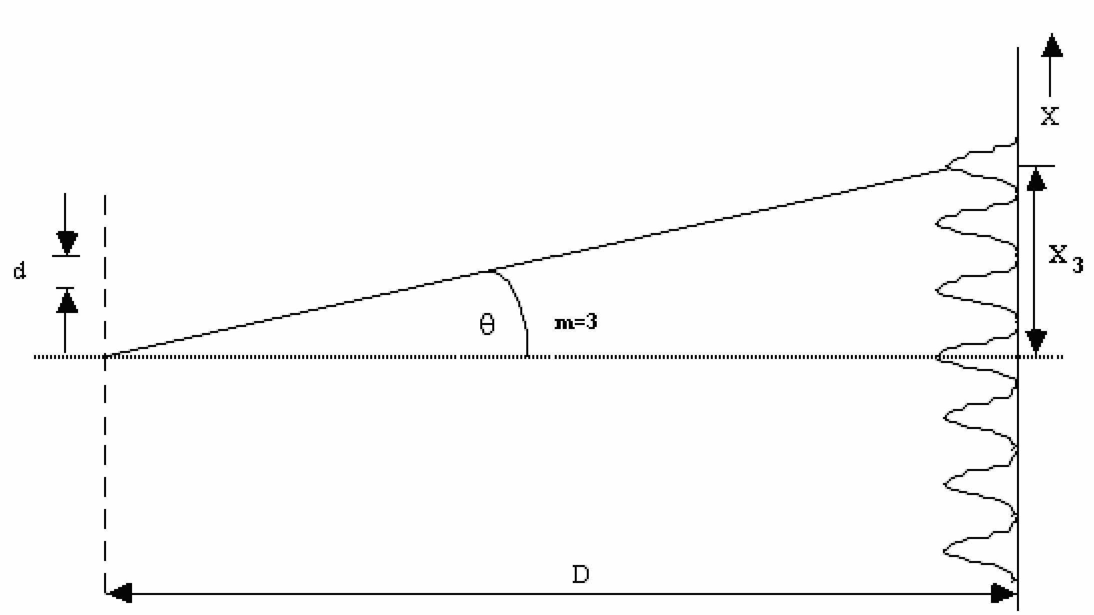


Figure 5: Intensity distribution of a diffraction grating

Although a multi-slit grating is commonly referred to as a diffraction grating, a more appropriate name for it is an interference grating. The phenomenon that is observed is interference and not as its name suggests diffraction. The condition here for interference maximum is the same as for double-slits, but the pattern may be very different because *d* (the slit spacing) for gratings is very small.

$$d \sin \theta_m = m\lambda$$

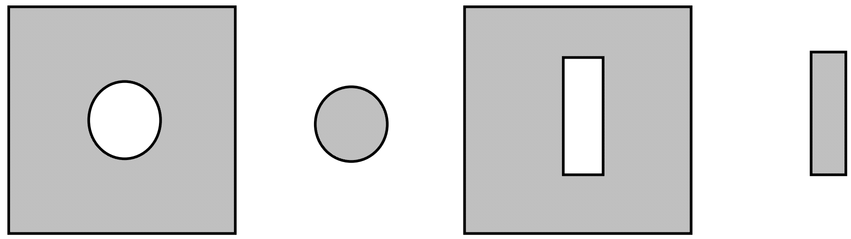
Thus

$$d = \frac{m\lambda}{\sin\left(\tan^{-1}\left(\frac{x_m}{D}\right)\right)} \qquad (5)$$

Note: the angles involved when using the diffraction grating are large; therefore you cannot use the small angle approximation here.

Babinet's Principle

Two screens are said to be complementary when the transparent regions on one exactly correspond to the opaque regions on the other and vice versa. Figure 6 shows two examples of complementary screens. Babinet's Principle implies that the Fraunhofer diffraction patterns from complementary screens are nearly identical. Thus, if a thin wire were placed in the laser beam, one would expect to obtain a diffraction pattern similar to that of a single-slit.



Examples of complementary screens
Figure 6

<u>IN A NUTSHELL</u>: For all these phenomena, the equation relating object size, S (such as, the wire diameter, slit width or grating spacing), and the location of maxima or minima is of the form

$$S = \frac{n\lambda}{\sin\left(\tan^{-1}\left(\frac{x_n}{D}\right)\right)}$$

If appropriate, using the small angle approximation this reduces to:

$$S \approx \frac{n\lambda D}{x_n}$$

This approximate equation can be rearranged to concentrate on $\frac{x_n}{n}$, the spacing between peaks or valleys. We thus estimate the fineness of the diffraction pattern to be

$$\frac{x_n}{n} \approx \frac{\lambda D}{S}$$

The peak spacing is **<u>smaller</u>** for **<u>larger</u>** object size S. Common objects are much larger than the wavelength of light, so any interference effects are also usually "too fine to notice". Further, in every day life, we use light with a variety of wavelengths, so that peaks of one wavelength tend to land on valleys of another wavelength. Thus diffraction effects are not only too small for our eyes to resolve, but are also smeared out because the wavelength is not a single well-defined value. Using small objects with a single well-defined wavelength helps us see these effects in the lab.

PROCEDURE

The wavelength (λ) of He-Ne laser used in this experiment is 632.8 nm = 6.328 x 10^{-4} mm.

The single slit

As you do this part of the experiment, you should answer questions 1 through 3.

a) Place the panel that holds four single-slits in the laser (between the laser and the screen). The pattern is most easily seen with the slit near the laser and the screen far away. Try different slits and observe their diffraction patterns on the screen.
b) Choose a slit that gives a diffraction pattern of reasonable size. Measure the distance from the slit to the screen (D) and record the labeled slit width in an Excel spreadsheet.
c) Attach a piece of paper across the screen where you see the diffraction pattern. Mark carefully the positions of the center of the principal maximum and the center of the diffraction minima of several orders on the paper. Remove the paper from the screen and attach it to your lab report (this is question 1).
d) Measure the distance of each minimum from the principal maximum (x_n) and record them in an Excel spreadsheet. Have Excel calculate the slit width (w) using equation (2). *Use negative x_n and n for the positions to the left of the principal maximum.*
e) Have Excel calculate the mean value of the slit width and the standard deviation of the mean value. (The Excel formula for standard deviation of the mean is: "=STDEV(cell1:cellN)/SQRT(N)" where, N refers to the total number of measurements). You can use the standard deviation of the mean value as the uncertainty in your slit width (δw).

Babinet's Principle

As you do this part of the experiment, you should answer questions 4 through 7.

a) Place a thin wire in the laser beam.
b) Measure the distance between the wire and the screen (D) and record the labeled wire diameter.
c) Tape a piece of paper to the screen. Mark the positions of the principal maximum and the diffraction minima.
d) Measure the distance of each diffraction minimum from the principal maximum and record them in an Excel spreadsheet. Have Excel calculate the diameter using the appropriate equation.
e) Have Excel calculate the mean value of the wire diameter and the standard deviation of the mean value.

The double slit

As you do this part of the experiment, you should answer questions 8 through 12.

a) Place the panel that holds four double-slits in the laser beam. Try different double-slits.
b) Choose a double-slit that gives a reasonable pattern. Measure the distance from the slit to the screen (D) and record the labeled slit width and slit spacing.

c) Tape a piece of paper across the screen where you see the pattern. Mark carefully the positions of the principal maximum, the interference maxima, and the diffraction minima on the paper. (You may want to distinguish the marks for each kind.) Remove the paper from the screen and attach it to your lab report (this is question 8).
d) Measure the distance of each interference maximum from the principal maximum (x_n) and record them in an Excel spreadsheet. Have Excel calculate the slit spacing using equation (4).
e) Have Excel calculate the mean value of the slit spacing and the standard deviation of the mean value.

(OPTIONAL) **Multiple-slit interference and diffraction: a diffraction grating**

Your lab instructor will tell you if you are to complete this section. As you do this part of the experiment, you should answer questions 13 and 14.

a) Place the grating in the laser beam (**close** to the screen, not far away).
b) Measure the distance from the *plane of the grating* to the screen (D) and record it in an Excel spreadsheet.
c) Record the labeled ruling density (grooves/mm) in your Excel spreadsheet.
d) Tape a piece of paper across the screen. Mark carefully the positions of the principal maximum and the interference maxima. Remove the paper from the screen and attach it to your lab report (this is question 13).
e) Measure the distance of each interference maximum from the principal maximum (x_n) and record them in your Excel spreadsheet. Have Excel use equation (5) to calculate the slit spacing (d) for each of the maxima and equation (6) to calculate the ruling density.

$$ruling\ density = \frac{1}{d} \tag{6}$$

f) Have Excel calculate the mean value of the slit spacing and the standard deviation of the mean value.

QUESTIONS

Single slit diffraction

1. Sketch the pattern you observed when the laser light passed through a single slit. Label some of the significant features.
2. How does the slit width affect the diffraction pattern?
3. Discuss the consistency of the mean value of the slit width with the labeled slit width.

Babinet's Principle

4. Is the position right behind the wire light or dark? (If you aren't surprised, you should be!)
5. What is the equation you will use to calculate the diameter of the wire? Clearly define each quantity.
6. Discuss the consistency of the mean value of the wire with the labeled wire diameter.
7. Is your hair thicker or thinner than the wire? How did you find out?

Double slit diffraction and interference

8. Sketch the pattern you observed when the laser light passed through a double slit. Label interference maxima and diffraction minima.
9. How is the double slit pattern different from the single slit pattern? Also, what causes the difference in the pattern?
10. How does the slit width affect the pattern?
11. How does the slit spacing affect the pattern?
12. Discuss the consistency of the mean value of the slit spacing with the labeled slit spacing.

(OPTIONAL)Diffraction grating

13. Sketch the pattern you observed when the laser light passed through a diffraction grating. Label the interference maxima.
14. Discuss the consistency of the mean value of the ruling density with the ruling density.