

# EXPERIMENT 12

## THE GRATING SPECTROMETER AND ATOMIC SPECTRA

### OBJECTIVES

- Learn the theory of the grating spectrometer
- Observe the spectrum of mercury and hydrogen
- Measure the grating constant of a diffraction grating
- Measure the Rydberg Constant

### INTRODUCTION:

In the previous experiment diffraction and interference were discussed and at the end a diffraction grating was introduced. In this lab you will apply what you learned to measure the wavelengths of light in the visible bright line spectrum of some elements. You will use a diffraction grating mounted on a spectrometer to measure the angular position of the interference maxima. Knowing the grating spacing and the angular position of an interference maxima, the wavelength of the light can be calculated. In the first part of the lab, you will use the known wavelengths of the mercury spectrum and the angles measured using the grating spectrometer to determine the grating spacing of the diffraction grating. In the second part of the lab you will use the calculated grating spacing and the spectrometer to determine the wavelengths of light in the visible hydrogen spectrum.

White light is made up of all the colors of the rainbow - red, orange, yellow, green, blue, and violet. Different colors correspond to different wavelengths. Human eyes are sensitive to light with wavelengths in the range 390 nm (violet) to 750 nm (red) (1 nm = one nanometer =  $10^{-9}$  m).

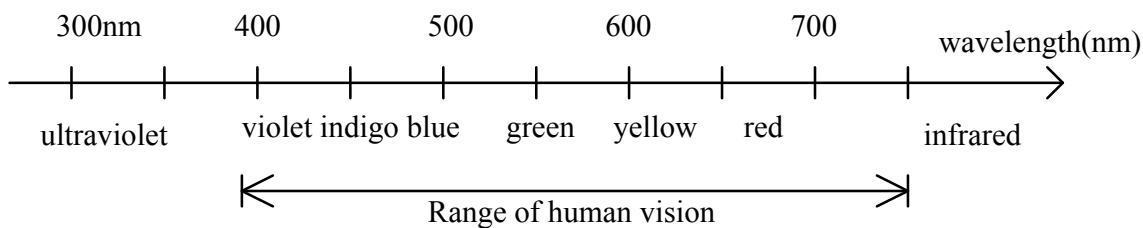


Figure 1

Recall from the previous lab:

$$m\lambda = d \sin \theta_m \quad (1)$$

Here,  $d$  = distance between slits  
 $m$  = order number of interference maximum  
 $\lambda$  = wavelength of light  
 $\theta_m$  = angle from the center line to the  $m$ th interference maximum

Unlike the previous lab, the grating spectrometer allows you to directly measure the angle  $\theta_m$  in degrees and minutes of arc (60 arc minutes = 1 degree). If a plot of  $m\lambda$  vs.  $\sin \theta_m$  is made, it has the form of a straight line with  $m\lambda$  playing the role of y and  $\sin \theta_m$  playing the role of x. The equation of the line should have an intercept of zero and a slope of  $d$ , the spacing between adjacent slits in the diffraction grating.

When the wavelengths of light emitted by excited atomic gases were first studied with precision spectrometers, scientists observed that most of the light was emitted at a few discrete wavelengths. In this experiment, you will observe the discrete emission lines of hydrogen and mercury gases. These discrete lines originate from the deexcitation of atoms. That the lines are discrete was initially a great surprise because it implied that the valid energies of electrons in atoms were also discrete. If any energies were possible, a continuous spectrum would be expected not a discrete line spectrum. This discovery led to the development of quantum mechanics, which is one of the foundations of modern physics.

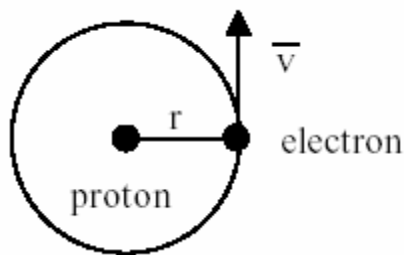


Figure 2

To put this discovery in perspective, consider what one might have expected if Newton's laws governed the energies of atomic electrons. Atomic hydrogen consists of a proton and an electron, which is bound to the proton by the attractive Coulomb force:

$$F_c = \frac{ke^2}{r^2} \quad (2)$$

where  $k=8.99 \times 10^9 \frac{Nm^2}{C^2}$ ;  $e=1.6 \times 10^{-19} C$  is the magnitude of the electron and proton charges; and  $r$  is the distance between the electron and proton. If the electron is in a circular orbit about the proton, as shown above, Newton's second law states  $F_c = ma$ , where  $a$  is the centripetal acceleration of the electron  $\frac{v^2}{r}$  (here,  $v$  is the speed of the electron). Combining Newton's second law with equation (2):

$$\frac{mv^2}{r} = \frac{ke^2}{r^2} \quad (3)$$

(The proton is so much heavier than the electron that one can assume that it remains stationary while the electron orbits about it.) The potential energy  $U(r)$  of the electron is given by:

$$U(r) = -\frac{ke^2}{r} \quad (4)$$

The total energy of the electron is the sum of the kinetic (T) and potential (U) energies  $E=T+U$ . Rearranging equation (3) yields:

$$\frac{mv^2}{2} = \frac{ke^2}{2r^2} \quad (5)$$

The left hand side of equation (5) is the kinetic energy, therefore the total energy is:

$$E = \frac{ke^2}{2r} - \frac{ke^2}{r}$$

or

$$E = -\frac{ke^2}{2r} \quad (6)$$

It is negative because the electron is bound in the attractive Coulomb potential of the proton's electric field.

This derivation, which follows from Newton's laws, implies that the energy of the electron is negative and has an arbitrary magnitude that depends only on the radius  $r$  of the electron's orbit. This turns out to be **incorrect** for small systems like an atom. In fact, the electron in a hydrogen atom cannot be in any arbitrary orbit, but only those orbits with energies given by the Bohr formula:

$$E = -\frac{E_0}{n^2} \quad (7)$$

where,  $E_0 = 2.2 \cdot 10^{-18} \text{ J} = \mathbf{13.6 \text{ eV}}$  (eV is an electron volt and  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$ ) and  $n = 1, 2, 3, 4, \dots$ . The negative sign in the equation for the  $E_n$  means that the system has less energy than a free electron and proton or that the electron is bound to the proton, and it takes 13.6 eV of energy to remove the electron from the atom.

### Bohr's Model of the Hydrogen Atom

Bohr proposed to modify this planetary model of the atom by making the following assumptions:

1. Electrons can exist only in certain special orbits about the central nucleus. The orbits are called stationary orbits or stationary states. The possible orbits are those with angular momenta that are multiples of a number called Planck's constant (denoted by  $h$ ).
2. The dynamic equilibrium of the system in the stationary states is governed by Newtonian mechanics.
3. Transitions between different stationary states are accompanied by the emission or absorption of radiation the frequency of which is given by the Planck formula,

$$hf = \Delta E \quad (8)$$

where  $f$  is the frequency and  $\Delta E$  is the difference in the energy between the two stationary states.

The first assumption was made to eliminate the problem of the electrons radiating due to the acceleration in their circular orbits. This assumption was made *ad hoc* with no justification other than the resulting model reproduced the results of experiments. The second assumption stated that the features of the model could be calculated with classical mechanics. The third assumption was the most controversial because little was known about how these transitions might occur.

According to the Bohr model, whenever a hydrogen atom changes from one stationary state to another, energy is emitted or absorbed. The energy takes the form of electromagnetic radiation and is given by:

$$hf = |E_f - E_i| \quad (9)$$

where  $E_f$  and  $E_i$  are the final and initial energies respectively. If  $E_i > E_f$ , then energy is radiated; whereas, if  $E_f > E_i$ , then energy is absorbed. The energy level diagram of the hydrogen atom is shown in Figure 3.

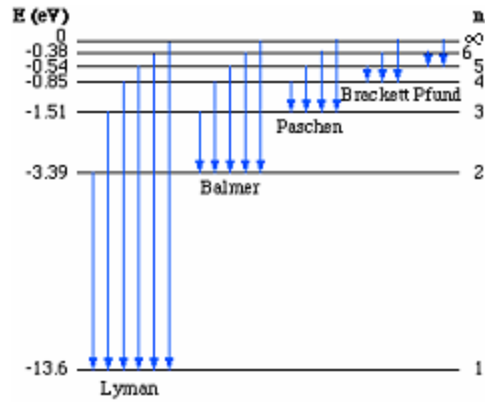


Figure 3

In the diagram above the first six levels are shown as well as the zero energy level ( $n = \infty$ ). Various transitions are shown as vertical arrows. If the transition takes place from any  $n$  to  $n = 1$ , it is referred to as a Lyman line. Transitions to  $n = 2$  are called Balmer lines and so on. Four of the Balmer lines are the visible range.

Combining equations (7) and (9) yields:

$$hf = E_0 \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \quad (10)$$

The frequency of a photon is related to its wavelength and the speed of light ( $c$ ) by  $f = \frac{c}{\lambda}$ .

Substituting this relationship into equation (10) and solving for  $\frac{1}{\lambda}$ :

$$\frac{1}{\lambda} = \frac{E_0}{hc} \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

$\frac{E_0}{hc} = R$ , the Rydberg constant ( $R = 1.097 \times 10^7 \text{ m}^{-1} = 109.7 \text{ nm}^{-1}$ ). In this experiment we will study the Balmer series ( $n_f = 2$ ) for this series:

$$\frac{1}{\lambda} = R \left( \frac{1}{4} - \frac{1}{n_i^2} \right)$$

Solving this for  $R$  yields:

$$R = \frac{1}{\lambda \left( 0.25 - \frac{1}{n_i^2} \right)} \quad (11)$$

PROCEDURE

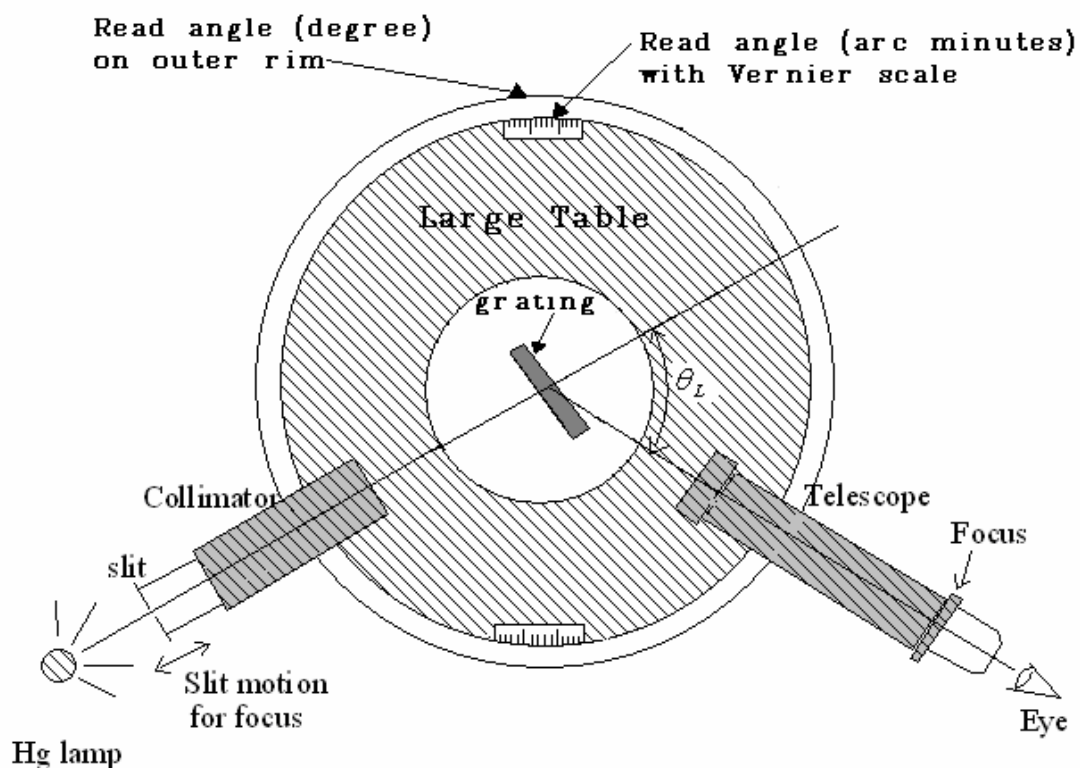


Figure 4

1. Become familiar with the spectrometer
  - a) Identify each component: the large table, the grating table, the outer rim, the collimator, and the telescope (see Figure 4 above).
  - b) Note the clamping screws and the fine adjustment screws for the telescope and the large table. Note the clamping screw for the grating table.
  - c) Note how to adjust the telescope focus and the eyepiece.
  - d) Note how to adjust the slit focusing in the collimator tube. Note how the slit width can be adjusted and how the slit orientation can be rotated.

2. Practice reading the angle from a precise protractor scale on the outer rim of the table. Use the Vernier scale with the little magnifying glass to read the angle to the nearest arc minute. (1 arcmin = 1' = 1/60 degree.) **The following is an example:**

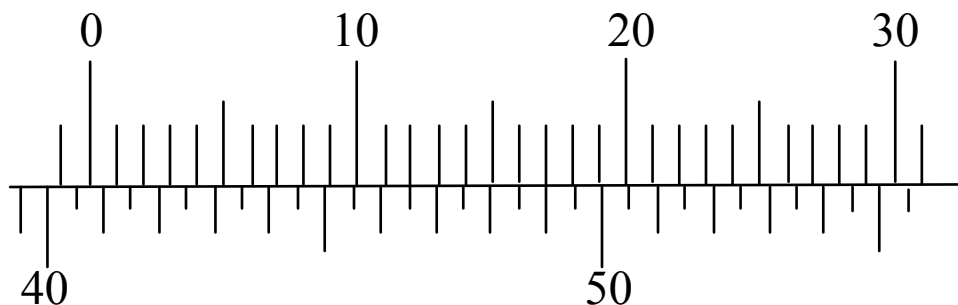


Figure 5

In this example, the zero line of the Vernier scale (the upper scale) is between  $40.5^\circ$  and  $41^\circ$ , so the angle is somewhere between  $40^\circ 30'$  and  $41^\circ$ . The Vernier scale tells exactly where in between. Look along the Vernier for the line that exactly lines up with the line below it. In this case, it's the 17' line. So the angle is  $40^\circ 47'$ , which we get by adding 17' to  $40^\circ 30'$ . In order to use this measurement, we must convert it to decimal degrees:  $40 + (47/60)$  degrees =  $40.78^\circ$ .

3. In order to correctly measure angles with the spectrometer, we must first align it. To do so, use the following steps:
- Crosshair focus:** Look through the eyepiece of the telescope. You should see crosshairs which you will use to accurately locate the spectral lines. Rotate the eyepiece to focus the crosshairs.
  - Telescope focus:** There are two knobs under the large table. If you are facing the spectrometer, on the side opposite the collimator, loosen the right hand knob (BLUE) under the large table – this allows you to rotate the telescope. Turn the telescope so that it is not pointing at the collimator, but instead is aimed at something as far away from you in the room as possible. Loosen the thumb screw on top of the telescope and slide the eyepiece in/out until the distant object comes clearly into focus. Tighten the thumbscrew. **After this adjustment, you should not adjust the focus of the telescope again.**
  - Telescope alignment:** Place a mercury vapor lamp in front of the slit at the end of the collimator. Look through the collimator, you should see the slit. If you do not, the slit is most likely closed – use the (BLACK) screw at the far end of the collimator to open/adjust the width of the slit. Now, move your light source around until the slit as seen through the collimator appears the brightest. Rotate the telescope until it is pointing directly at the collimator. By looking through the telescope, you should be able to line up the slit with the crosshair. By locking down the telescope (tightening the knob from part b) and using the fine adjustment you should be able to do this very accurately. The fine adjust knob is at the bottom of the telescope's support on its right hand side. If the slit does not have very crisp edges when looking through the

telescope, loosen the thumbscrew on top of the collimator and slide it in/out to focus it – it’s best to start with the far end pushed all the way in. Also, if the slit is not vertical, adjust it at this time and then tighten the thumbscrew on top of the collimator. Once you have a nice thin, well-focused slit with your crosshairs centered on it **and your telescope locked down**, you are ready to align your angular scales.

- d) **Angle adjustment:** The outer rim of the large table is marked in  $0.5^\circ$  increments. The inner half of the large table has two Vernier scales divided into 30 one minute increments – we’ll only use the scale on the right hand side of the telescope. Loosen the left hand screw (GREEN) under the large table – this unlocks the outer rim of the large table. Rotate the outer rim and align the zero degree mark on the outer rim with the zero minute mark on the inner Vernier scale. Use a magnifying glass to assist in this alignment. Lock the outer rim so it will no longer rotate.
- e) **Grating placement:** Place the grating on the grating table. The face of the grating should be perpendicular to a line between the collimator and the telescope. Now, find mercury’s bright line spectrum without the telescope. To do this, you should look toward the grating from the telescope side to the left and to the right of the telescope. Once you have found the spectral lines, unlock the telescope by loosening the BLUE thumbscrew and rotate the telescope to point at the spectral lines. Observe the spectral lines through the telescope. If your spectral lines are difficult to see, too wide or not in focus, you may have to adjust the focus on the collimator or the slit width.

4. Notice on the outer rim the angles increase counterclockwise from  $0^\circ$  to  $360^\circ$ . You should also notice that the spectrum is symmetric, that is you should see the same spectrum to the left of the telescope that you see to the right. This corresponds to  $m = +1$  and  $m = -1$ , respectively from equation (1). For our calculations, we will use the average value of the angular location of the two lines on the right and left. The reading on the right must be converted using  $\theta_R = |360 - \theta'_R|$ , where  $\theta'_R$  is the decimal equivalent of the angle measured by using the spectrometer (it should be between  $270^\circ$  and  $360^\circ$ ). The value of  $\theta$  to use in equation (1) is:

$$\theta = \frac{\theta_R + \theta_L}{2} \quad (12)$$

5. Measure the angle for each of the first order spectral lines of the Mercury lamp. The wavelengths and colors of the spectral lines are given in the Excel spreadsheet. When measuring very closely spaced lines, like the double yellow lines, make the slit very narrow and check the focus; when measuring dim lines, make the slit wider.
6. Have Excel convert each of your angles from degree/minute format to degree (decimal). Calculate the average value of the angular location for each of the lines using equation (12) and calculate the sine of the angle. Note, to calculate  $\sin \theta$  in Excel use the



following Excel equation: “=sin(RADIANS(CELL))” where CELL is the Excel spreadsheet cell containing the angle (in degrees).

7. Import the wavelength and  $\sin\theta$  data into Kaleidagraph and make a plot of  $\lambda$  vs.  $\sin\theta$ . Fit your plot with a best-fit line and have Kaleidagraph display the equation of the line and the uncertainties in the slope and intercept.
8. Allow the mercury lamp to cool, and then replace it with a hydrogen lamp. Measure  $\theta$  for the first and second order maxima for each of the four lines in the visible hydrogen spectrum.
9. Use the grating spacing you found in step 7 and equation (1) to calculate the wavelength for each of these maxima.
10. Use equation (11) to calculate the Rydberg constant for each of the wavelengths you calculated in step 9. Calculate the mean value of the Rydberg constant and the standard deviation of mean value of the Rydberg constant.

### QUESTIONS

1. What is the slope of your graph and its uncertainty?
2. What is the calculated spacing between adjacent slits on the diffraction grating and its uncertainty?
3. What is your measured value of the grating constant and its uncertainty? The grating constant and its uncertainty are given by:  $\frac{1}{d}$  and  $\frac{\delta d}{d^2}$ , respectively.
4. Does the grating constant agree with the manufacturer's stated value? What could cause your experimentally measured value to differ from the manufacturer's stated value (do not say human error)?
5. What is your measured value of the Rydberg constant and its uncertainty?
6. Does your measured value agree with the accepted value of  $1.097 \times 10^7 \text{ m}^{-1}$ ?

### Checklist

- Spreadsheet both numerical and formula view
- Sample calculations
- Graph with comments
- Answers to questions