EXPERIMENT 10
Thin Lenses

Objectives

1) Measure the power and focal length of a converging lens.
2) Measure the power and focal length of a diverging lens.

Apparatus

A two meter optical bench, a meter stick, a small ruler, a screen, a lighted object a converging lens (f = +25.0 cm) and a diverging lens (f = -30.0 cm) will be used in this experiment.

Introduction

A converging lens will cause light rays passing through it to be bent toward each other and the principal axis of the lens. If parallel light rays are incident on a converging lens, the light rays will converge at the focal point, a distance f from the lens (see Figure 1). The distance f is called the focal length of the lens. For converging lenses, f is positive. In general, converging lenses are thicker in the middle than they are at the outer edge of the lens.

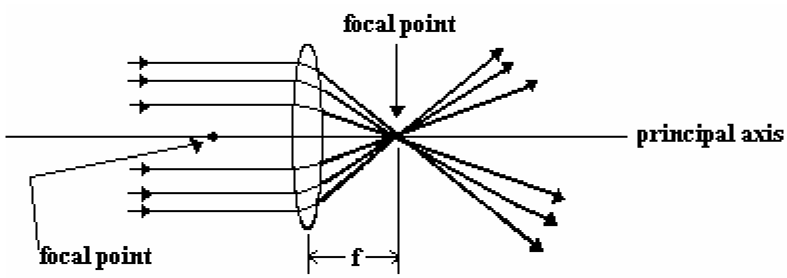


Figure 1: Parallel light rays passing through a converging lens

A diverging lens will cause light rays passing through it to bend away for each other and the principal axis of the lens. If parallel light rays are incident on a diverging lens, the light rays will appear to diverge from the focal point, a distance f from the lens (see Figure 2). For diverging lenses, f is negative. In general, diverging lenses are thicker at the outer edge than they are in the middle.

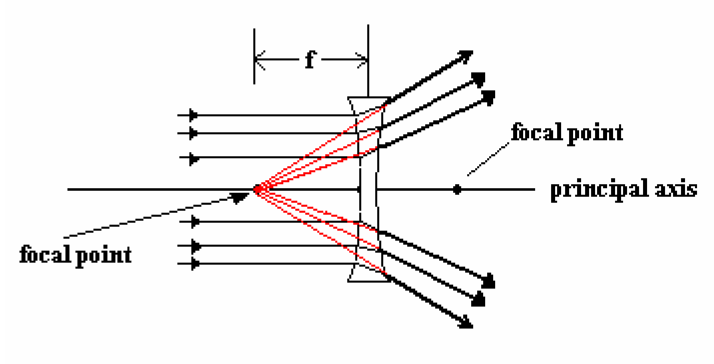


Figure 2: Parallel light rays passing through a diverging lens

Ray diagrams can be a useful tool for analyzing the behavior of light rays passing through thin lenses. Here are some rules for drawing light rays in ray diagrams.

1. A light ray approaching the lens parallel to the principle axis will pass through the focal point on the opposite side of the lens for a converging lens or appear to diverge from the focal point on the same side of the lens for a diverging lens (see Figures 1 and 2 above).
2. A light ray passing through the center of the lens will not change direction as it passes through the lens.
3. A light ray passing through the focal point will be parallel to the principal axis when it leaves the lens.

Consider the ray diagrams shown in Figures 3 and 4 below. For clarity, only rays 1 and 2 are shown. The image is formed where the rays intersect.

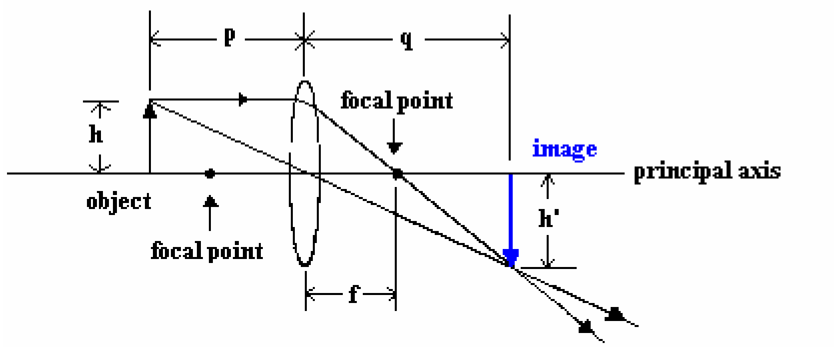


Figure 3: Real image formed by a converging lens

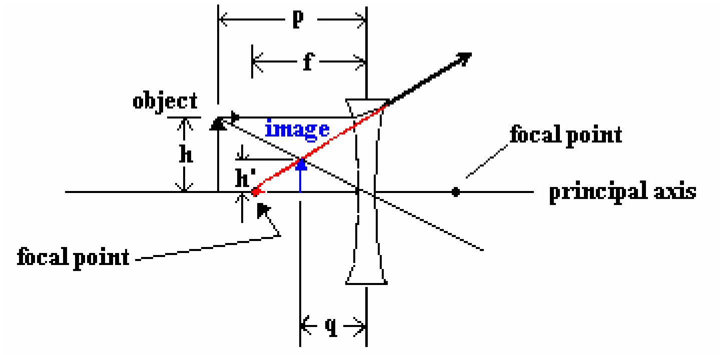


Figure 4: Virtual image formed by a diverging lens

In Figures 3 and 4, h is the height of the object, h’ is the height of the image, p is the object distance (measured from the lens) and q is the image distance (measured from the lens). There are two types of images real and virtual. Real images occur when the light rays actually converge on a point and for an image. These types of images can be displayed on a screen. A virtual image is formed when the light rays appear to diverge from a point. Because they rays do not actually diverge from this point (that is, pass through the point), they cannot be displayed on a screen. Converging lenses can form both real and virtual images. A diverging lens by itself can only form a virtual image. In the last part of this lab, you will use an image formed by a converging lens as the object for a diverging lens. The resulting image from this two lens system can be real – more about this later.

Sign conventions for lenses

The front side of the lens is defined as the side of the lens from which the light is traveling (in the diagrams above it is the left hand side of the lens). The back side of the lens is defined as the side to which the light is traveling (in the diagrams above it is the right hand side of the lens).

- The object distance is positive when the object is on the front side of the lens. The object distance is negative when it is on the back side of the lens.
- The image distance is positive when the image is on the back side of the lens. The image distance is negative when it is on the front side of the lens.
- Image height is negative when the image is inverted (compared to the object). The image height is positive when the image is upright.

For systems with a single optical element, a real image will always have a negative image height (inverted) and a positive image distance. In contrast, a virtual image will always have a positive image height (upright) and a negative image distance.

Magnification

The intuitive definition of magnification is how much bigger (or smaller) an image is than the original object.

$$M = \frac{h'}{h} \tag{1}$$

Notice, for a real image formed by a single lens the magnification is negative; the negative sign just means the image is inverted. Referring back to Figure 3, we can use similar triangles to find an alternate expression for magnification in terms of the image and object distances.

$$\frac{h}{p} = \frac{-h'}{q}$$

Rearranging this expression we find:

$$-\frac{q}{p} = \frac{h'}{h}$$

The right hand side is the same as equation (1), therefore:

$$M = -\frac{q}{p} \tag{2}$$

Focal length

The focal length of a thin lens is related to the object and image distances by the following expression:

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} \tag{3}$$

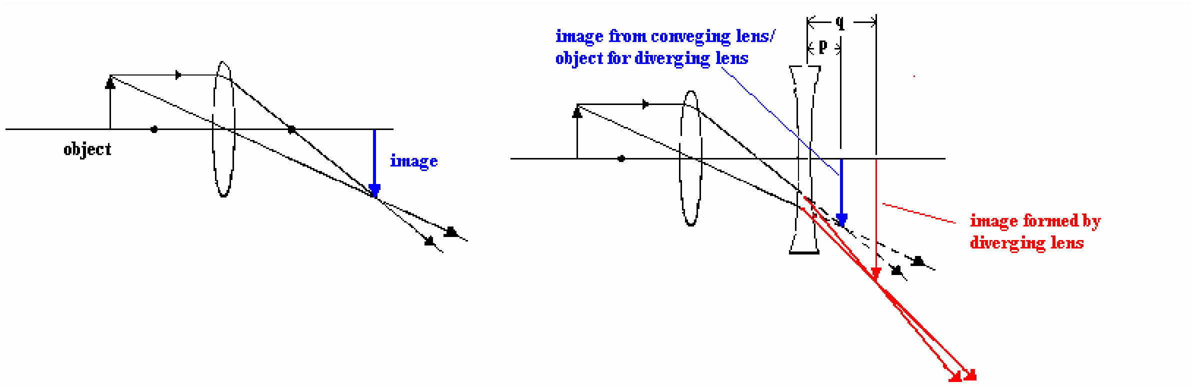
If we rearrange equation (3) we obtain:

$$\frac{1}{q} = -\frac{1}{p} + \frac{1}{f} \tag{4}$$

Notice equation (4) has the same form as the equation of a straight line y = mx + b. With $\frac{1}{q}$ playing the role of y, $\frac{1}{p}$ playing the role of x and $\frac{1}{f}$ is the intercept. In this experiment, you will measure a series of object and corresponding image distances to calculate the focal length of a converging lens. You will do this by constructing a graph of $\frac{1}{q}$ vs $\frac{1}{p}$. If the focal length f is in meters, the quantity $\frac{1}{f}$ is the power of the lens, D in diopters.

Measuring the focal length of a diverging lens

To measure the focal length of a diverging lens, we will use a converging lens to create a real image. This real image will be the object for the diverging lens. We will place the diverging lens such that the image formed by the converging lens is on the diverging lens's back side. The result is a negative object distance from the diverging lens's perspective. See Figure 5 below. Using the object distance, image distance and using equation (3) the focal length of the diverging lens can be calculated.



Figures 5 A converging lens forms a real image then a diverging lens is added to form a new real image.

Procedure

Focal length of a converging lens

1. Measure the height of the object and record it in the Excel Spreadsheet.
2. Place a 25.0 cm focal length lens at a distance of p = 60.0 cm away from the object and adjust the screen until a clear image is formed. Assign a reasonable uncertainty to your object distance p and record it in your Excel spreadsheet. You should notice as you move the screen back and forth that there is a range over which the image appears clear. You should place the screen in the center of this range and use half of the range as your uncertainty in the image distance q (is this distance large compared to your ability to read the meter stick?). Measure the image distance q and record it and its uncertainty in your spreadsheet. Measure the height of your image and record it and a reasonable estimate of its uncertainty in your Excel spreadsheet.
3. Repeat step 2 for object distances p = 55 cm, 50 cm, 45 cm, and 40 cm.
4. Have Excel calculate the magnification (M_1) for each of your measurements using equation (1). Also, calculate the uncertainty using: $\delta M_1 = M_1\left(\frac{\delta h}{h} + \frac{\delta h'}{h'}\right)$
5. Have Excel calculate the magnification (M_2) for each of your measurements using equation (2). Also, calculate the uncertainty using: $\delta M_2 = M_2\left(\frac{\delta p}{p} + \frac{\delta q}{q}\right)$

6. Have Excel calculate $\frac{1}{p}$, $\frac{1}{q}$ and $\delta\left(\frac{1}{q}\right) = \frac{1}{q}\left(\frac{\delta q}{q}\right) = \frac{\delta q}{q^2}$. Import these data columns into Kaleidagraph and construct a graph of $\frac{1}{q}\, vs\, \frac{1}{p}$. Include vertical error bars and have Kaleidagraph fit it with a best-fit line with relevant uncertainties.

Focal length of a diverging lens

1. Return the converging lens to a distance of 50.0 cm from the object and adjust the screen until a clear image is formed.
2. Place a diverging lens having a focal length of -30.0 cm at a distance of 10.0 cm from the screen (i.e. the diverging lens should be between the screen and the converging lens. By placing it in this location, your OBJECT distance for the diverging lens is -10.0 cm. Assign a reasonable uncertainty to this distance (remember you should include the uncertainty in finding the image formed by the converging lens).
3. Adjust the screen until a clear image is formed. Measure the image distance q; that is, distance between the diverging lens and this image. Use equation (3) to calculate the focal length of the diverging lens. Use $\delta f = |f|\left(\frac{\delta p}{|p|} + \frac{\delta q}{q} + \frac{\delta p + \delta q}{|p+q|}\right)$ to calculate its uncertainty (remember, uncertainties are ***ALWAYS*** positive). These calculations are question # 5.

Questions

1. Are your measurements for the magnification consistent? Explain.
2. What is the slope of your graph? Is it consistent with what you expect from equation (4)? Explain.
3. From your graph, what is the focal length of the converging lens? Does it agree with the established value of f = +25.0 cm. Explain.
4. Based on your measurement, what is the power of the converging lens?
5. From your measurements, calculate the focal length of the diverging lens and its uncertainty.
6. Is your calculated focal length in question 5 consistent with the established value of f = -30.0 cm? Explain.
7. Based on your measurement, what is the power of the diverging lens?