Physics 472 – Spring 2008

Homework #6, due Friday, February 29

1. [6] Griffiths problem 6.7. For part (b), please don't blindly use equation 6.27, or you won't know what you are doing. Instead, write the 2 x 2 matrix representation of H' in the basis $|n\rangle$ and $|-n\rangle$, where $\langle x|n\rangle \equiv \Psi_n(x) = \frac{1}{\sqrt{L}} e^{2\pi i n x / L}$. I.e. evaluate the four elements in the matrix:

$$\begin{pmatrix} \langle n|H'|n\rangle & \langle n|H'|-n\rangle \\ \langle -n|H'|n\rangle & \langle -n|H'|-n\rangle \end{pmatrix}$$

   (Griffiths calls this matrix **W**.) The idea in degenerate P.T. is to find the eigenvalues and eigenvectors of this matrix. After you evaluate these four numbers, I suggest you call the off-diagonal term δ_n while you are finding the eigenvalues and eigenvectors. After you find the eigenvectors in part (c), don't bother to evaluate the matrix elements again, as Griffiths requests. It's a waste of time. You may find the following formula useful, which we derived in PHY471:

$$\int_{-\infty}^{\infty} e^{-\alpha x^2 + \beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{(\beta^2 / 4\alpha)}.$$

   Hints: $\delta_n \propto e^{-(2\pi n a / L)^2}$; the answer to part (d) is Parity.

2. [6] Griffiths problem 6.9. To simplify your notation, label the three eigenvectors of H^0 as $|1\rangle$, $|2\rangle$, and $|3\rangle$. When you get to part (c), just write down the 3 x 3 matrix form of H' in that basis. You can then read all the matrix elements directly from the matrix, without performing any matrix-vector multiplications. When you get to part (d), don't be faked out when you discover that H' is already diagonal in the 2D subspace of degenerate states.

   Add a part (e) to the problem: Calculate the second-order shifts to states $|1\rangle$ and $|2\rangle$. You use the same formula [6.15] for second-order P.T., but now the sum is only over the states outside of the degenerate subspace, i.e. m=3 only. (That is because you have already exactly diagonalized H' within the 2D subspace.) With this last bit of calculation, you should now have agreement with the expansion of the exact results to order ε^2 for all three energies.

3. [5] Griffiths problem 6.21.

4. [3] Consider a hydrogen molecule made from standard hydrogen atoms (no deuterium or tritium allowed). Let's forget about the electrons. Let $\vec{L}$ be the orbital angular momentum of the two protons about their center of mass, and $\vec{S} = \vec{S}_1 + \vec{S}_2$ be their total spin. For each allowed value of s, what are the allowed values of l? The Hamiltonian for molecular rotations is $H = \frac{L^2}{2I}$, where I is the rotational inertia of the molecule. If the distance between the protons is $d = 74$ pm, calculate the energy difference in Joules between the rotational ground state and first excited state. The Wikipedia article entitled "Spin Isomers of Hydrogen" says that "the ratio between the ortho and para forms is about 3:1 at standard temperature and pressure, but the para form dominates at low temperatures." Try to guess why that is.