

To Frame the World—25 Sept

- Kepler found orbit of Mars relative to earth's orbit.
- Goal was to measure the absolute distance (in miles or km) of the solar system
- Cassini & Richer 1672



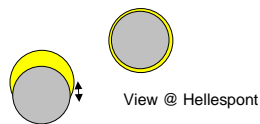
Giovanni Domenico Cassini, (1625 - 1712)
engraving by N. Dupuis
www.sil.si.edu/digitalcollections/hst/scientific-identity/fullsize/SIL14-C1-18a.jpg

Hipparchus measures the moon's distance~200BC

- At the Hellespont, the solar eclipse of 189BC was total. (Sparta defeated Athens there in 405 BC.)
- In Alexandria, the moon covered $\frac{1}{4}$ of the sun.
- Draw a picture to show the relationship between the sun, the moon, the two locations, and the difference between a total & $\frac{1}{4}$ eclipse. (The diameter of the sun is $\frac{1}{2}$ degree.)



<http://www.livius.org>



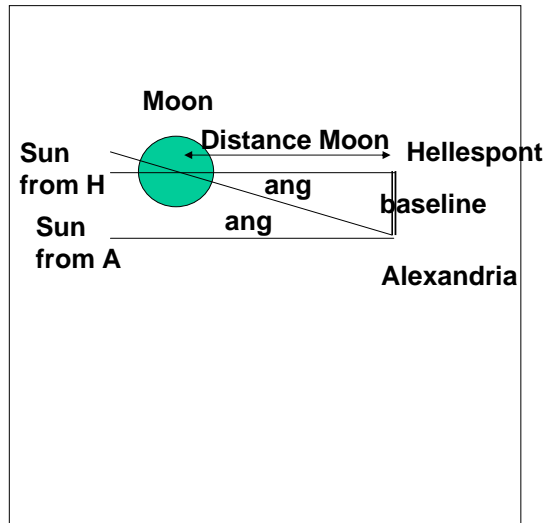
View in Alexandria.
Moon is offset by $\frac{1}{4}$
diameter of sun



http://mkatz.web.wesleyan.edu/medea_lecture/hellespont.gif

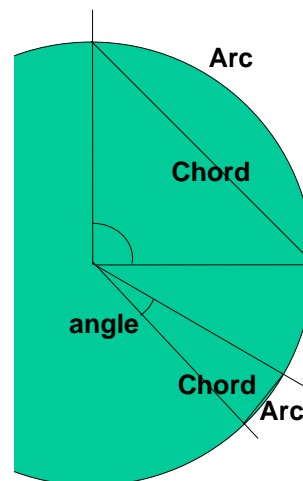
What triangle did Hipparchus use?

- Parts of triangle
 - Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
 - One leg of triangle is the baseline.
 - Other leg is distance to moon
- Method of parallax.
 - Angle is the “parallactic shift.”



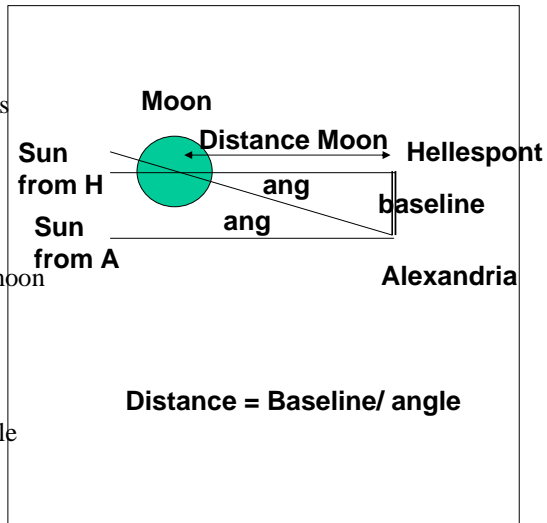
Small angle approximation

- Measure angles in radians
- Arc = radius \times angle
 - For entire circle,
 - Length of arc = circumferen
 - Therefore angle = 2π for the
 - 2π radians = 360°
- For small angles, arc is appr equal to the chord.
 - Chord = radius \times angle**
- Application
 - **Baseline = Distance \times angle**



Method of parallax

- Parts of triangle
 - Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
 - One leg of triangle is the baseline.
 - Other leg is distance to moon
- Method of parallax.
 - Angle is the “parallactic shift.”
 - Distance = Baseline/ angle



Difficulties

- Small angles are hard to measure
 - Naked eye $1/30^\circ = 1/1700 \text{ rad} = 6 \times 10^{-4} \text{ rad} = 600 \mu\text{rad}$
 - Modern telescope used under ideal conditions: $5 \mu\text{rad}$
 - Modern telescope with correction for atmospheric turbulence: $0.5 \mu\text{rad}$
 - Moon using Hellespont & Alexandria
 $\text{Angle} = \text{baseline}/\text{distance} = 1000\text{km} / 400,000\text{km}$
 $= 1/400 \text{ rad} = 2500 \mu\text{rad} (1/7^\circ)$
 - Mars
 $\text{Angle} = \text{baseline}/\text{distance} = 1000\text{km} / 80,000,000\text{km}$
 $= 13 \mu\text{rad}$ using Hellespont & Alexandria
- Need a reference nearby in the sky
 - Measuring with a reference on the ground is impossible.

Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?



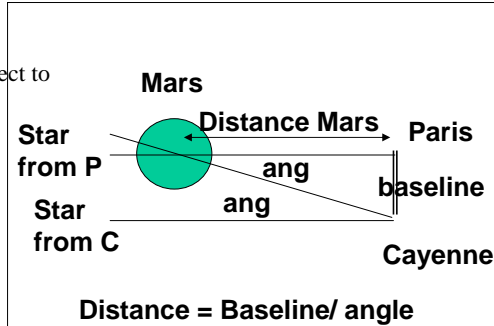
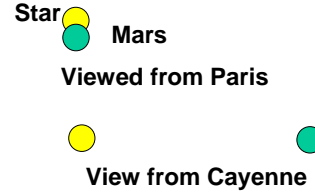
Cassini & Richer 1672

- Angle=baseline/distance
- What baseline should C&R use to measure distance to Mars?
- Cayenne-Paris baseline is 7000km.
 - Angle=baseline/distance=7000 km/60,000,000km
= 120 μ rad
 - Shift is 25 times width of a star seen with modern telescope.



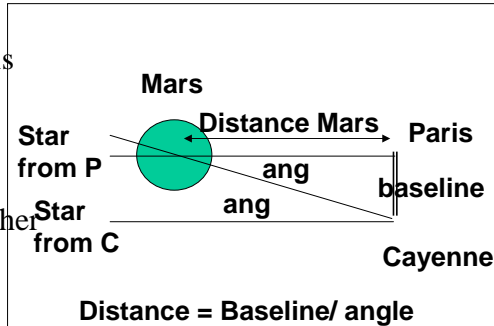
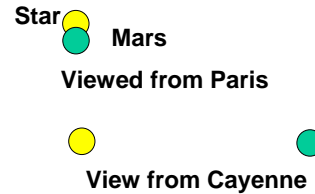
- We are pretending to be Jean Richer and Giovanni Cassini in 1672. We are measuring the distance to Mars by making observations from Paris and Cayenne.
 - Facing the screen, hold a pencil at arms length. Without moving the pencil, look at it with your left and then your right eye. The pencil tip shifts with respect to something on the screen.
- What corresponds to Mars?
 - Left eye or right eye
 - Tip of pencil
 - Something in the screen
 - The shift of the pencil tip with respect to the screen.
 - What corresponds to Paris?
 - What corresponds to the star?
 - What is proportional to the parallax angle?

Cassini & Richer



- What corresponds to Mars?
 - Left eye or right eye
 - Tip of pencil
 - Something in the screen
 - The shift of the pencil tip with respect to the screen.
- What corresponds to Paris? A
- What corresponds to the star? C
- What is proportional to the parallax angle? D
- If Mars were closer, the shift is
 - larger.
 - same.
 - smaller.
- If observing stations were farther apart, the shift is larger

Cassini & Richer



To Frame the World

- We pretend to be Jean (Giovanni) Cassini, Director of the Paris Observatory.
- We propose a grand plan to “Frame the World” to the Louis XIV. The expedition to Cayenne will determine the distance to the outermost planet in the solar system!



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Proposal to Louis XIV

- I draw the orbit of Mars & Earth at the best opposition.
 - Mars is 0.38AU from Earth at the closest.
 - Jean (in Cayenne) & I (in Paris) will measure the distance between Earth & Mars by triangulation.
 - We will then know the length of an AU.
- Kepler’s 3rd Law
 - Known to Kepler, period of Saturn is 29.5 years
 - $P^2 = R^3 \Rightarrow R = 9.54\text{AU}$
 - Because we will have measured the length an AU, we will know the size of the solar system.

