To Frame the World—25 Sept

- Kepler found orbit of Mars relative to earth’s orbit.
- Goal was to measure the absolute distance (in miles or km) of the solar system
- Cassini & Richer 1672

Hipparchus measures the moon’s distance~200BC

- At the Hellespont, the solar eclipse of 189BC was total. (Sparta defeated Athens there in 405 BC.)
- In Alexandria, the moon covered \( \frac{3}{4} \) of the sun.
- Draw a picture to show the relationship between the sun, the moon, the two locations, and the difference between a total & \( \frac{1}{4} \) eclipse. (The diameter of the sun is \( \frac{1}{2} \) degree.)
What triangle did Hipparchus use?

• Parts of triangle
  • Angle is due to parallax: moon in foreground shifts with respect to sun in the background.
  • One leg of triangle is the baseline.
  • Other leg is distance to moon
• Method of parallax.
  • Angle is the “parallactic shift.”

Small angle approximation

• Measure angles in radians
• Arc = radius × angle
  • For entire circle,
    Length of arc = circumference
  • Therefore angle = 2π for the .
  • 2π radians = 360°
• For small angles, arc is appi equal to the chord.
  Chord = radius × angle
• Application
  • Baseline = Distance × angle
Method of parallax

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  - Angle is the “parallactic shift.”
  - Distance = Baseline/ angle

Difficulties

- Small angles are hard to measure
  - Naked eye $1/30^\circ = 1/1700 \text{ rad} = 6 \times 10^{-4} \text{ rad} = 600 \mu \text{rad}$
  - Modern telescope used under ideal conditions: $5 \mu \text{ rad}$
  - Modern telescope with correction for atmospheric turbulence: $0.5 \mu \text{ rad}$
  - Moon using Hellespont & Alexandria
    Angle = baseline/distance = $1000 \text{ km} / 400,000 \text{ km}$
    $= 1/400 \text{ rad} = 2500 \mu \text{ rad (1/7}^\circ)$
  - Mars
    Angle = baseline/distance = $1000 \text{ km} / 80,000,000 \text{ km}$
    $= 13 \mu \text{ rad using Hellespont & Alexandria}$
- Need a reference nearby in the sky
  - Measuring with a reference on the ground is impossible.
Cassini & Richer 1672

- Angle = baseline/distance
- What baseline should C&R use to measure distance to Mars?
  - Cayenne-Paris baseline is 7000 km.
    - Angle = baseline/distance = 7000 km / 60,000,000 km = 120 μrad
    - Shift is 25 times width of a star seen with modern telescope.
• We are pretending to be Jean Richer and Giovanni Cassini in 1672. We are measuring the distance to Mars by making observations from Paris and Cayenne.

• Facing the screen, hold a pencil at arms length. Without moving the pencil, look at it with your left and then your right eye. The pencil tip shifts with respect to something on the screen.

1. What corresponds to Mars?  
   A. Left eye or right eye  
   B. Tip of pencil  
   C. Something in the screen  
   D. The shift of the pencil tip with respect to the screen.

2. What corresponds to Paris?
3. What corresponds to the star?
4. What is proportional to the parallax angle?

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5. If Mars were closer, the shift is
   A. larger  
   B. same.  
   C. smaller.

6. If observing stations were farther apart, the shift is ___ larger
To Frame the World

• We pretend to be Jean (Giovanni) Cassini, Director of the Paris Observatory.

• We propose a grand plan to “Frame the World” to the Louis XIV. The expedition to Cayenne will determine the distance to the outermost planet in the solar system!

Proposal to Louis XIV

• I draw the orbit of Mars & Earth at the best opposition.
  – Mars is 0.38AU from Earth at the closest.
  – Jean (in Cayenne) & I (in Paris) will measure the distance between Earth & Mars by triangulation.
  – We will then know the length of an AU.

• Kepler’s 3rd Law
  – Known to Kepler, period of Saturn is 29.5 years
  – \( P^2 = R^3 \Rightarrow R = 9.54\text{AU} \)
  – Because we will have measured the length an AU, we will know the size of the solar system.