1. Hot-plate model of a star. Imagine that you have made "stars" out of hot plates and you are plotting them on a Hertzsprung-Russell diagram
a. (3 pts.) How can you make two hot plates with the same spectral class and differing absolute magnitude?

The luminosity (and absolute magnitude) depends on the size and temperature of the star. Make the star bigger.
b. (3 pts.) If you moved the hot plate to a greater distance, how would its place on the H -R diagram change?

Since the vertical axis of the H-R diagram is related to luminosity (not flux), its position does not change.
c. (3 pts.) If you turned the setting on the hot plate from "high" to "medium," how would its place in the H -R diagram change?
Since the temperature is cooler, the star moves to the right. Because $\mathrm{L}=\mathrm{R}^{2} \mathrm{~T}^{4}$, the luminosity is less, and the star moves down also.
2. Life on Deneb. Here you will find out what it means to live near a giant like Deneb. Recall that the luminosity of a star $L=$ const $T^{4} R^{2}$, where $T$ is its temperature, $R$ is its radius, and const is a constant.

| Star | App mag | Abs mag | Spectral type | Distance (pc) |
| :--- | :--- | :--- | :--- | :--- |
| Sun | -26.74 | 4.83 | G2 | $1 / 200,000$ |
| Deneb | 1.25 | -7.3 | A2 | 500 |

a. ( 5 pts.) In class we found that a star 10 times fainter has a magnitude +2.5 greater. This relationship between the flux $f_{A}$ and $f_{B}$ of two stars $A$ and $B$ and their magnitudes can be expressed mathematically as $m_{A}-m_{B}=-2.5 \log \left(f_{A} / f_{B}\right)$. How much brighter is Deneb than the sun if both are placed at the same distance?

Use absolute magnitude, since both stars would then be at the same distance and $\mathrm{L}_{\mathrm{D}} / \mathrm{L}_{\mathrm{S}}=\mathrm{f}_{\mathrm{D}} / \mathrm{f}_{\mathrm{s}}$.

$$
\mathrm{m}_{\mathrm{D}}-\mathrm{m}_{\mathrm{S}}=-7.3-4.83=-12.1 .
$$

Deneb is $12.1 / 2.5=4.84$ steps of 2.5 mag brighter than the sun. Therefore

$$
\mathrm{L}_{\mathrm{D}} / \mathrm{L}_{\mathrm{S}}=10^{4.84}=70,000
$$

b. ( 5 pts .) The temperature of the sun is 5700 K , and the temperature of Deneb is 9800 K . How much larger is Deneb than the sun?
Since $L=R^{2} T^{4}, R_{D} / R_{S}=\left(L_{D} / L_{S} T_{S}^{4} / T_{D}\right)^{1 / 2}=90$.
c. ( 2 pts.) The sun subtends $1 / 2$ degree in the sky. If Deneb replaced the sun, what angle would our replacement subtend?

Since Deneb is 90 times larger, Deneb would subtend $45^{\circ}$ in the sky.
3. M15. Figure 1 is the Hertzsprung-Russell diagram of the star cluster M15.
a. (2 pts.) What is the absolute magnitude of the hottest main-sequence stars? 3.5
b. (3 pts.) Why are there no hotter main-sequence stars?

Hotter stars have exhausted their hydrogen and ceased to be main-sequence stars.
c. Stars with a color $\mathrm{B}-\mathrm{V}=0.6$ span a range of 5 magnitudes. ( 2 pts.) What property of


Figure 1 Hertzsprung-Russell diagram of the star cluster M15. B-V is a measure of color. The vertical scale on the left is apparent magnitude, and the scale on the right is absolute magnitude.
the stars accounts for this observation? ( 3 pts.) What is the range of this property?
The luminosity $\mathrm{L}=\mathrm{R}^{2} \mathrm{~T}^{4}$. Since the horizontal axis of an HR diagram is related to temperature, stars with the color $\mathrm{B}-\mathrm{V}=0.6$ have the same temperature. The range of luminosity must therefore be due to the sizes of the stars. Since a range of 5 mag is two steps of 2.5 mag , the luminosity differs by two factors of 10 , or 100 . Therefore the range in radius is a factor of 10 .
d. (3 pts.) The apparent magnitude of a star is 5 magnitudes fainter than the absolute magnitude. Find its distance. (Recall that the absolute magnitude is the apparent magnitude if the object is moved to a distance of 10 pc .)
The apparent mag is fainter than the abs mag, the distance of the star must be greater than 10 pc . The flux is 100 times fainter than it would be at a distance of 10 pc . (There are two steps of 2.5 mag.) Since flux depends on distance ${ }^{2}$, the distance is $10 * 10 \mathrm{pc}=100 \mathrm{pc}$.
e. (2 pts.) Suppose the apparent magnitude of M15 is exactly 15 magnitudes fainter than the absolute magnitude. Find its distance.
The flux is $10^{6}$ times fainter than it would be at a distance of 10 pc . (There are 6 steps of 2.5 mag.) Since flux depends on distance ${ }^{2}$, the distance is $10^{3} * 10 \mathrm{pc}=10^{4} \mathrm{pc}$.

