AST 207 Homework 10 (25 pts.) 9 December 2009

1. A black hole is in the center of our galaxy, and its mass is about 3 million times the mass of the sun.

    a. (5 pts.) What is the evidence for the black hole in the center of our galaxy?

       Astronomers measured the orbits of the stars. The mass pulling on the stars is very large. The stars orbits are elliptical, which means the large mass is confined to a size of order Pluto's orbit. If the mass were a million stars, they would collide.

    b. (4 pts.) How big is the Schwarzschild radius of that black hole? (Read about black holes in section 18.3 of the textbook.) (1 pt.) Find something in the solar system that is comparable in size to the Schwarzschild radius of the black hole in the center of our galaxy.

       The Schwarzschild radius of one solar mass is 3km, and it is proportional to mass. The Schwarzschild radius of 3 million solar mass is $3\times10^6\times3$km = 10^7km. The radius of Mercury's orbit is 6 times bigger. The radius of the sun is 13 times smaller.

2. The temperature of the radiation was 3000K during recombination, and it is 2.728K today. The distance to the Coma cluster of galaxies is 100 Mpc.

    a. (5 pts.) At the time of recombination, what was the distance between what was to become the Coma cluster and what was to become you?

       Key idea: Temperature of the radiation changes inversely with the expansion parameter. The expansion parameter at recombination is 2.728K/3000K=0.001. The distance to proto Coma was 100Mpc $\times$ 0.001 = 0.1Mpc.

    b. (5 pts.) At the time of recombination, what elements were present?

       At the time of recombination, stars had not formed yet. The only elements were hydrogen and those made at 3min, which is ^{4}He, and trace amounts of ^{2}H, ^{3}He, and ^{7}Li.

3. The temperature of the cosmic background radiation is not perfectly isotropic.

    a. (5 pts.) Because we are moving with respect to the Big Bang, the temperature is hottest in a certain direction in the sky and coolest in the opposite direction. The temperature in the hottest direction is hotter by 0.004 K than the average. Find our speed with respect to the Big Bang. (You must show your calculation.) Hint: Wien's Law connects the temperature and the wavelength of the peak of the radiation, and Doppler's effect relates wavelength and speed.

       Let λ_h and λ_a be the wavelength of the peak of the radiation in the hottest direction and in the average. Wein's Law:

       $\lambda_h T_h = \lambda_a T_a$.

       $\lambda_a / \lambda_h = T_h / T_a = (2.728+0.004)/2.728 = 1.0015$.

       Doppler:

       $\lambda_a / \lambda_h = 1+v/c$.

       $v = c (\lambda_a / \lambda_h - 1) = 0.0015\times300{,}000\text{km/s} = 440\text{km/s}$