You may use three sheets of notes. You may not use books or additional notes.

| Name |  |
| :--- | ---: |
| PID |  |
| Signature |  |
| 1 | $/ 15$ |
| 2 | $/ 13$ |
| 3 | $/ 40$ |
| Total |  |



Fig. 3.-Upper, major axis heliocentric velocities on plane of sky, as a function of distance from the nucleus. Lower, minor axis velocities as a function of distance from the nucleus; note change in scale from upper plot. The steep velocity gradient in nuclear region along minor axis is prominent.

From Rubin, Vera, Thonnard, Norbert, and Ford, W. Kent, jr., 1977, Astrophysical Journal 217, L1.

## 1. Short answers

a. (1 pt.) The sun will become a giant 5 billion years from now. Will Deneb, a star of spectral class A visible in the summer, be visible then? (2 pts.) Explain your reasoning.

Because Deneb is a much more massive star, its lifetime is much shorter that the sun's, and it will have become a supernova and hence have disappeared.
b. The sun will become a planetary nebula. ( 3 pts.) How has the composition of the material in the outer parts changed from when it was a protostar? (3 pts.) What prevents material in the star in the center of the planetary nebula from fusing?

The outer parts will have the products of fusion, primarily He. The pressure of the white dwarf in the center is degeneracy pressure. This pressure exists without having to heat and reach the temperature needed for fusion of heavier elements.
c. (3 pts.) Your parents experienced an exciting period in astronomy. (For this problem, assume your parents were born in 1960 and are still living.) Name three discoveries in cosmology made in their lifetime.
Many possibilities: Radiation from the Big Bang. Evidence of the epoch of the first stars. Dark matter in galaxies. Production of helium in the big bang.
d. ( 3 pts .) What is the evidence for the black hole in the center of our galaxy?

The orbit of a star nearly at the center of our galaxy shows that it orbits a 3 million solar mass object with a very small orbital radius.
2. The picture shows a sample of the universe 0.001 s after the big bang, when the expansion parameter was $6 \times 10^{-12}$. (Recall that the expansion parameter is distance/(present distance). The box was a cube $2 \times 10^{-11} \mathrm{~m}$ on a side. There are 8 neutrons, 8 protons, and lots of light in the box.
a. (5 pts.) The box expands with the universe. Draw its contents just before helium formed (at 3 minutes, when the expansion parameter is $2.3 \times 10^{-9}$ ). The number must be precise to $10 \%$; for example, drawing 15 protons is OK if the actual number of protons is 16 . ( 3 pts .) What is the temperature of the radiation in the box?
Just before helium formed, the ratio of neutrons to protons was $1 / 7$, just as it is today. Since the box expands with the universe, it will have the same number of nuclei inside. Therefore there are 2 neutrons and 14 protons.
The temperature scales inversely as the expansion parameter: $\mathrm{T}=\mathrm{T}_{\text {today }} / \mathrm{a}=2.7 / 2.3 \times 10^{-9}$ $=1.2 \times 10^{9} \mathrm{~K}$. A common mistake was to use different times for the temperature and expansion parameter.
b. The box expands with the universe. (3 pts.) Draw its contents at the present time. (2 pts.) How big is the box now? Assume the box is not from some special place such as in a star, or in a galaxy.
At the present time, the neutrons are locked up in helium. The box has 1 helium and 12 protons (hydrogen).
The size of the box is $\mathrm{L}=\mathrm{L}_{\text {then }} / \mathrm{a}_{\text {then }}=2 \times 10^{-11} \mathrm{~m} / 6 \times 10^{-12}=3.3 \mathrm{~m}$. A common mistake was to use different times for the size of the box and the expansion parameter.
3. Consider the measurements of the speed of gas in the galaxy NGC 3672 from a paper by Rubin, Thonnard, \& Ford on the front of this test.
a. (3 pts.) If this galaxy were moved to twice the distance, how and by how much would the speeds in lower plot change?
The expansion of the universe, not the rotation of the galaxy, is the reason for motion in the lower plot. Hubble's Law says $\mathrm{v}=\mathrm{H}$ D. The speed in the lower plot doubles from 1850 to $3700 \mathrm{~km} / \mathrm{s}$.
b. (3 pts.) If this galaxy had twice as much mass, how and by how much would the speeds in the upper plot change?
With twice the mass, the rotation speed is $\sqrt{2}$ of its original value. Use $v^{2}=G M / R$. If $M$ doubles, $v^{2}$ doubles, and $v$ is a factor of $\sqrt{ } 2$ of its original value. The galaxy as a whole moves at the same speed, since that depends on Hubble's Law and not the mass. The speed of the center is unchanged, since its rotation speed is zero. The speed of the left edge changes from $2050 \mathrm{~km} / \mathrm{s}$ to $(2050-1850) \sqrt{ } 2+1850$ $=200 * 1.4+1850=280+1850=2130 \mathrm{~km} / \mathrm{s}$. The speed of the right edge changes from $1650 \mathrm{~km} / \mathrm{s}$ to $(1650-1850) \sqrt{ } 2+1850=-200 * 1.4+1850=-280+1850=1570 \mathrm{~km} / \mathrm{s}$.
c. (3 pts.) Explain the evidence for dark matter in this galaxy.

The amount of mass is inferred from the measurement of the rotation speed according to Kepler's $3^{\text {rd }}$ Law $v^{2}=G M / R$. The visible mass (stars and gas) in the outer parts of the galaxy is small, but the rotation speed indicates a large amount of mass the outer parts of the galaxy.
d. (3 pts.) Sketch the speed beyond 20 kpc (to the right) if there is no dark matter beyond 20 kpc .
Since there is no mass beyond $\mathrm{R}=20 \mathrm{kpc}$, the mass enclosed within R will be constant for $R>20 \mathrm{kpc}$. Therefore the rotational speed $v(R)=\sqrt{ }[G M$ (within 20 kpc$) / R]$ drops to zero. However, the part of the velocity that is due to the expansion of the universe does not change with position. See sketch on front sheet.

