Rotation Curves

Hwk 2 due Sept 23
Hwk 3 not yet assigned, but due Sept 30
Midterm 1 Wed, Oct 2

- Spherical mass shell, uniform density

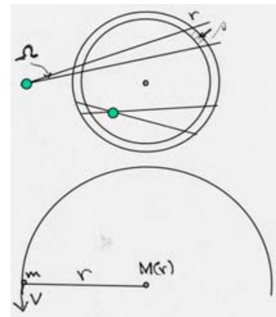
  Inside shell: no effect

  Outside shell:

  acts as if all mass at center

$$\frac{dF_{gravity}}{d\Omega} \propto \frac{\rho \times d(vol)}{r^2}$$

$$d(vol) = d\Omega \; r^2 dr$$



- $F_{centripital} = F_{gravity}$

$$\frac{mv^2}{r} = \frac{GM(r)m}{r^2}$$

$$v = \left(\frac{GM(r)}{r}\right)^{1/2}$$

- Inside spherical mass distribution

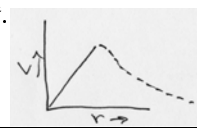
$$M(r) = \int_0^r \rho(r') 4\pi r'^2 dr' = \bar{\rho}\frac{4}{3}\pi r^3$$

$$v \propto r$$

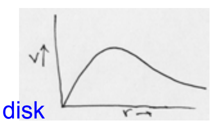
- Outside spherical mass distribution

$$M(r) = const.$$

$$v \propto r^{-1/2}$$



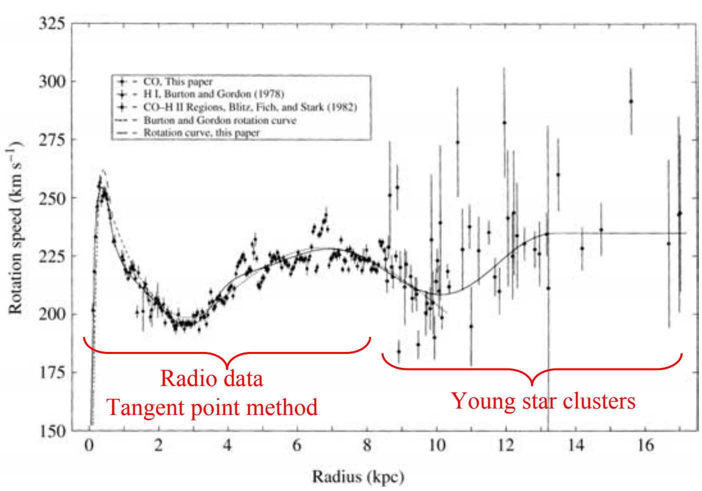
- Spher. Distr.
  \+ exponential disk



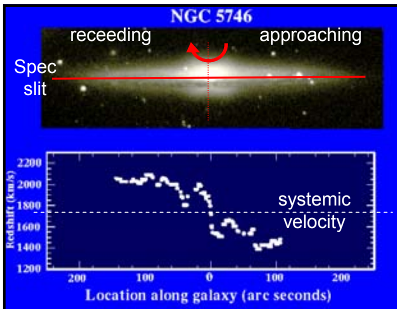
$$V = \left(\frac{G}{r}\right)^{1/2} \left\{ M(r) + 4 m_d s^3 \left[I_0(s)K_0(s) - I_1(s)K_1(s) \right] \right\}^{1/2}$$

$s = \frac{1}{2} r/h$ I, K = Bessel functions

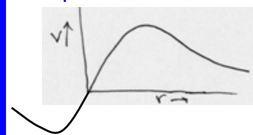
The Milky Way's Rotation Curve



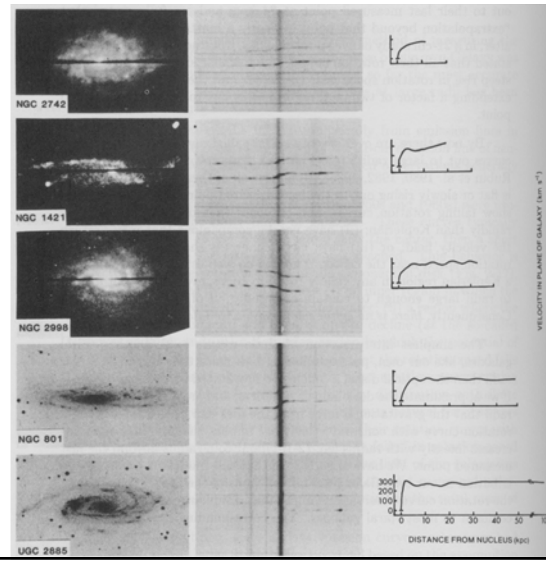
Rotation curves in other galaxies



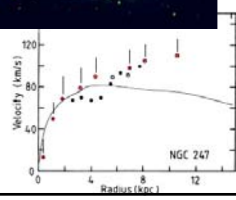
Expected:



Observed:



- Vera Rubin & Kent Ford (late 1970's)
- Image tube spectrograph



Density as shown by flat rotation curves

[CO pg. 917]

Back to $F_{centripetal} = F_{gravitational}$

$$\frac{mv^2}{r} = \frac{GM(r)\,m}{r^2}$$

$$M(r) = \frac{v^2 r}{G}$$

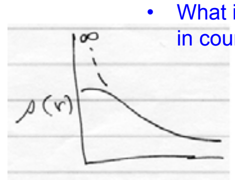
$$\frac{dM(r)}{dr} = \frac{v^2}{G}$$

but also $\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$

$$\Rightarrow \rho(r) = \frac{v^2}{4\pi G r^2}$$

$v \sim$ constant $\Rightarrow \rho(r) \propto \frac{1}{r^2}$

Use $\rho(r) = \frac{C_0}{(a^2 + r^2)}$

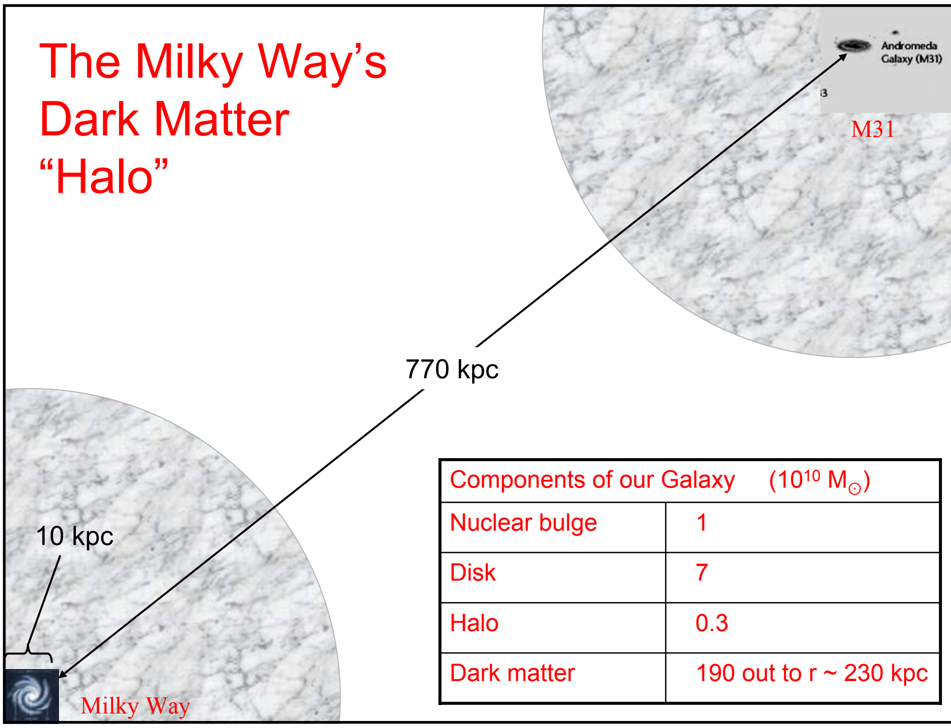


- dM(r)/dr ~ constant
  - Unbounded mass distribution??
- NFW profile

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

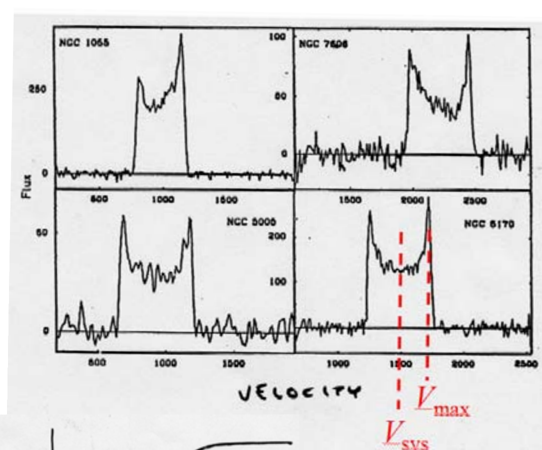
  - Predicted for Cold dark matter (CDM)
  - Actual derived dark matter profiles often slightly different than this
  - What is CDM? Coming later in course.

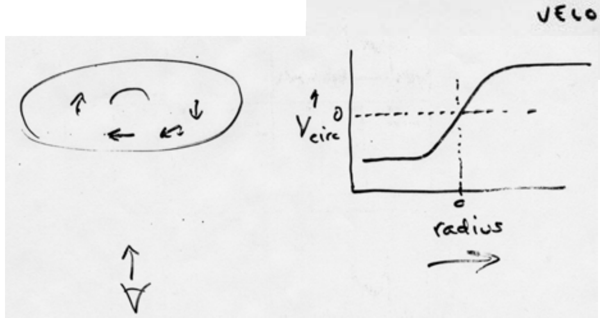
The Milky Way's Dark Matter "Halo"



Components of our Galaxy	(10^{10} $M_\odot$)
Nuclear bulge	1
Disk	7
Halo	0.3
Dark matter	190 out to r ~ 230 kpc

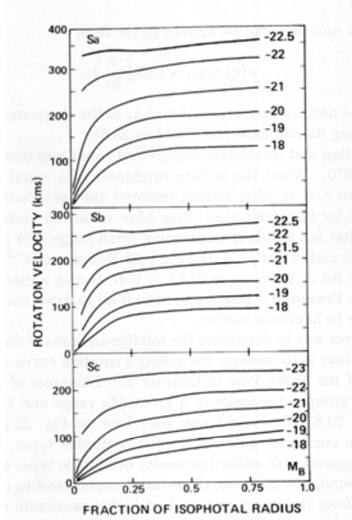
H I Spectra of Spiral Galaxies

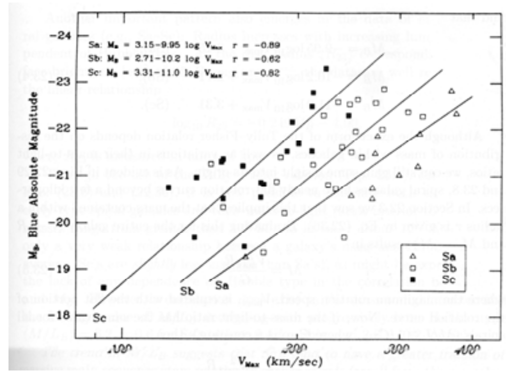




Tully-Fisher Relation

Rotation Curves





- Maximum rotation velocity-Luminosity relation [FIG 25.10]
  - **Tully-Fisher** relation
    - $M_B = -9.95 \log_{10} V_{Max} + 3.15$ (Sa)
    - $M_B = -10.2 \log_{10} V_{Max} + 2.71$ (Sb)
    - $M_B = -11.0 \log_{10} V_{Max} + 3.31$ (Sc)

Semi-derivation of Tully-Fisher Relation:

$M_B = -9.95 \log_{10} V_{max} + 3.15$ (Sa)
$M_B = -10.2 \log_{10} V_{max} + 2.71$ (Sb)
$M_B = -11.0 \log_{10} V_{max} + 3.31$ (Sc)

- Mass interior to outermost *R* where rotation curve can be measured:

$$Mass = \frac{V_{max}^2 R}{G}$$

- Assume $L = Mass / const.$
- "Freeman Law" (observed fact ---maybe):

$$Surf.Bright. = \frac{L}{4\pi R^2} = const.$$

$$L = const \times V_{max}^4$$

Important as a DISTANCE calibrator!

- Convert to Absolute B-band magnitudes:

$$M_B = M_{sun} - 2.5 \log_{10}\left(\frac{L}{L_{sun}}\right) = -10 \log_{10} V_{max} + const.$$

SO FAR:

- Galaxy types
- Ancient history
- Milky Way and spiral galaxy morphology
  - Nuclear bulge
  - Disk
  - Stellar halo
  - Dark matter halo
- Star-forming regions
- Chemical enrichment

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Midterm 1
Wed, Oct 2

Kinematics of spiral galaxies

- Rotation curves ➔ mass distribution
  (includes sidetrack about measuring distances)
- Spiral structure [CO 25.3]

- General properties of S, E, Irr galaxies
- Midterm 1 (Wed. Oct 2)