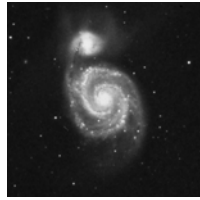
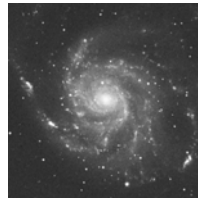


Hwk Set 3
 Due Wed Sept. 30
 [CO 25.13, 25.14, 25.16, 25.20]

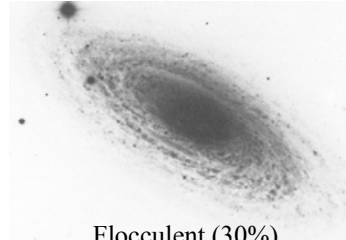
Spiral Structure [CO 25.3]



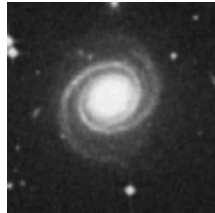
Grand design (10%)
 M51



Multi-arm (60%)
 M101



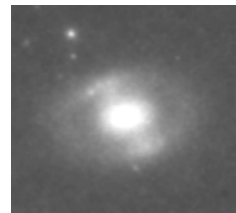
Flocculent (30%)
 NGC 2841



Inner rings
 NGC 7096



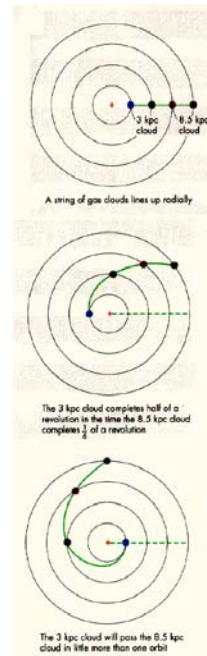
M81

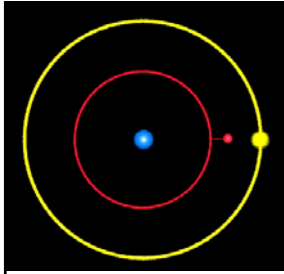


Outer Ring
 NGC 4340

What causes spiral structure?

- Winding up of arms
 - Due to differential rotation
- Stochastic, Self-Propagating Star Formation
 - Chain-reaction star formation
 - SN shells → shock fronts → density enhancements → star formation → more SN
 - Differential rotation then winds these regions up into spiral patterns
- Density Waves
 - Wave in gravitational potential
 - Orbital velocity of stars different than pattern speed
 - Stars, gas bunch up at position of spiral arms
 - Causes higher grav. potential
 - Unclear if self-sustaining or forced.



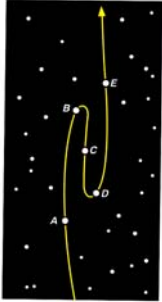
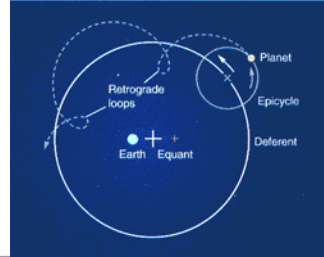


Retrograde Motion & Ptolemy's Epicycles

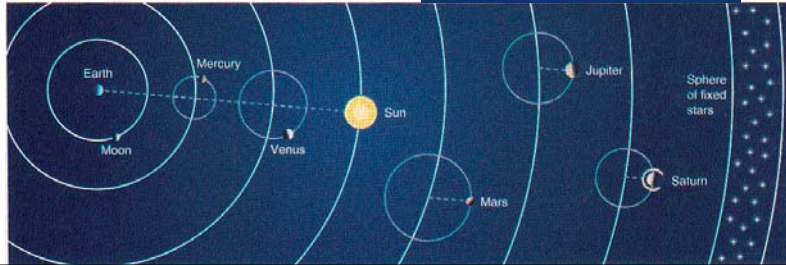


Ptolemy
Alexandria, 140AD

- Trying to place *Earth* at center.
- Using only circular motions.
- Led to very complicated system.



Path of Mars, etc. as seen from Earth

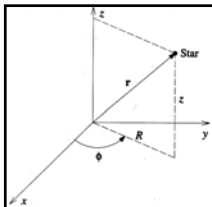


Epicycles... the short form.

For lurid details, see [CO 25.3]



Ptolemy
Alexandria, 140AD



$U = \text{potential energy}$
 $\Phi = U/M$

$$M \frac{d^2 \mathbf{r}}{dt^2} = -\nabla U(R, z) = -\frac{\partial U}{\partial R} \hat{\mathbf{e}}_R - \frac{1}{R} \frac{\partial U}{\partial \phi} \hat{\mathbf{e}}_\phi - \frac{\partial U}{\partial z} \hat{\mathbf{e}}_z$$

Circular symmetry \rightarrow independent of ϕ

$$\frac{d^2 \mathbf{r}}{dt^2} = -\frac{\partial \Phi}{\partial R} \hat{\mathbf{e}}_R - \frac{\partial \Phi}{\partial z} \hat{\mathbf{e}}_z \quad (25.17)$$

$$\frac{d^2 \mathbf{r}}{dt^2} = (\ddot{R} - R\dot{\phi}^2) \hat{\mathbf{e}}_R + \frac{1}{R} \frac{\partial (R^2 \dot{\phi})}{\partial t} \hat{\mathbf{e}}_\phi + \ddot{z} \hat{\mathbf{e}}_z$$

Separate $d^2 \mathbf{r}/dt^2$ into R, ϕ, z components

\rightarrow 3 equations. (25.19-25.21)

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R}$$

$$\frac{1}{R} \frac{\partial (R^2 \dot{\phi})}{\partial t} = 0$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}$$

Conservation of specific angular momentum
 $J_z = R^2 d\phi/dt$

Define an effective potential:

$$\Phi_{\text{eff}}(R, z) \equiv \Phi(R, z) + \frac{J_z^2}{2R^2}$$

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$$

Conservation of J_z
 \rightarrow acceleration in ϕ direction when r changes.

Taylor series expansion around position of minimum Φ_{eff} (circular orbit):

$$\rho = R - R_m$$

$$\Phi_{\text{eff}}(R, z) = \Phi_{\text{eff},m} + \frac{\partial \Phi_{\text{eff}}}{\partial R} \Big|_m \rho + \frac{\partial \Phi_{\text{eff}}}{\partial z} \Big|_m z + \frac{1}{2} \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \Big|_m \rho^2 + \frac{1}{2} \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \Big|_m z^2 + \dots$$

$$\Phi_{\text{eff}}(R, z) \simeq \Phi_{\text{eff},m} + \frac{1}{2} \kappa^2 \rho^2 + \frac{1}{2} \nu^2 z^2 \quad (25.31)$$

$$\ddot{\rho} \simeq -\kappa^2 \rho$$

$$\kappa^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \Big|_m$$

$$\ddot{z} \simeq -\nu^2 z$$

$$\nu^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \Big|_m$$

(25.32-25.33)

$$\ddot{\rho} \simeq -\kappa^2 \rho$$

$$\ddot{z} \simeq -\nu^2 z$$

Harmonic oscillation in R , ϕ , z about circular orbit
(Epicycles)



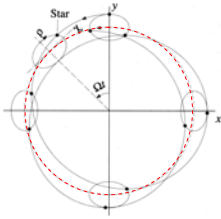
$$\rho(t) = R(t) - R_m = A_R \sin \kappa t$$

$R_m = R$ at min. Φ_{eff}
 $\Omega =$ circular ang. vel.

$$z(t) = A_z \sin(\nu t + \zeta)$$

$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2} t + \frac{2J_z}{\kappa R_m^3} A_R \cos \kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t$$

In inertial frame:



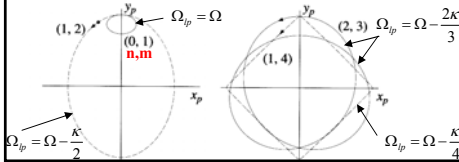
"local pattern speed"

Orbits closed if:

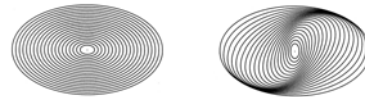
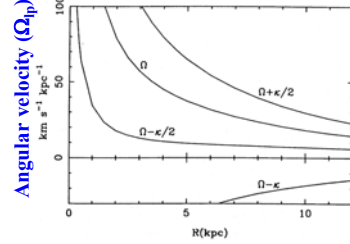
$$m(\Omega - \Omega_{lp}) = n\kappa$$

$$\Omega_{lp}(R) = \Omega(R) - \frac{n}{m}\kappa(R)$$

Viewed from frame rotating with Ω_{lp} :

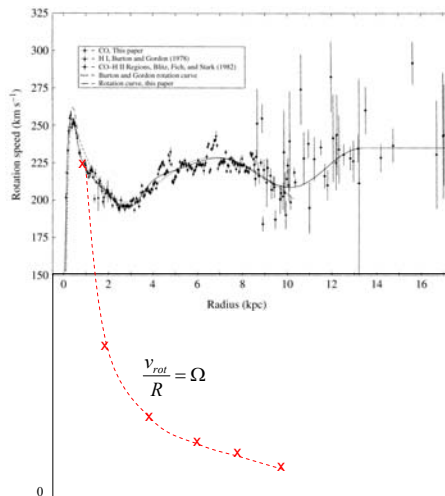


$\Omega =$ the rotation curve for
Milky Way:



Two ways to line up closed elliptical orbits
(as seen from frame rotating with Ω_{lp})

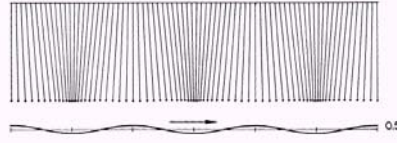
Flat rotation curves



Basic nature of a density wave

From: Toomre, Annual Review of Astronomy & Astrophysics, 1977 Vol. 15, 437.

Pendulum example



Rotation is opposite to the example on previous slide:

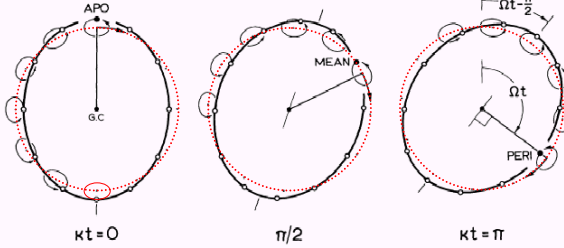
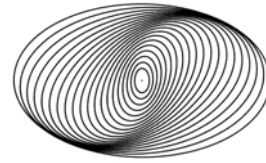


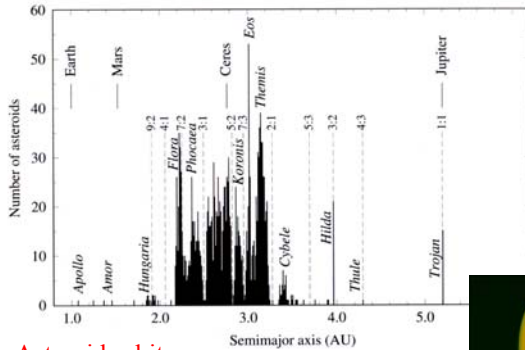
Figure 2 Slow $m = 2$ kinematic wave on a ring of test particles, all revolving clockwise (like the 12 shown) with mean angular speed Ω in strictly similar and nearly circular orbits. The small elliptical "epicycles," traversed counterclockwise in the above sequence of snapshots separated in time by exactly one-quarter of the period $2\pi/\kappa$ of radial travel along each orbit, depict the apparent motions of these particles relative to their mean orbital positions or "guiding centers." Drawn for the case $\kappa = \sqrt{2}\Omega$ —or one where the rotation speed $V(r) = r\Omega(r) = \text{const}$ at neighboring radii—the diagram emphasizes that the oval locus of such independent orbiters advances in longitude considerably more slowly than the particles themselves. That precession rate equals $\Omega - \kappa/2$, as one can verify at once by comparing the last frame with the first.

- At each R_m , stars' positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this R_m forms an ellipse rotating at pattern speed.



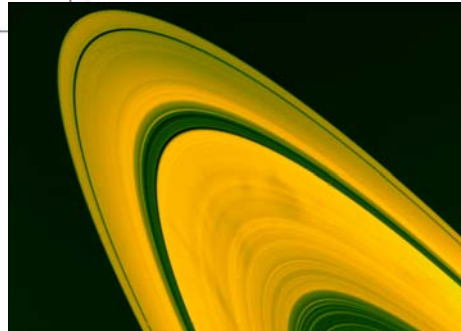
- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.

Some Solar System Resonance Phenomena

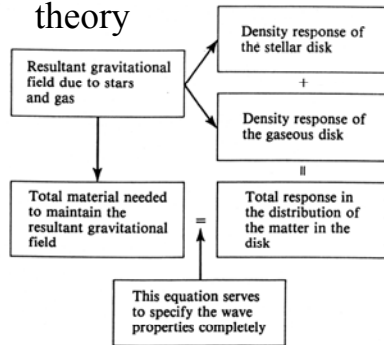


Asteroid orbits

Gaps in Saturn's rings



Lin & Shu's theory



Perturbed form of collisionless Boltzmann equation. "quite complicated"

Hydrodynamics

$$\kappa^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2} \Big|_m$$

$$\nu^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \Big|_m$$

Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin.)

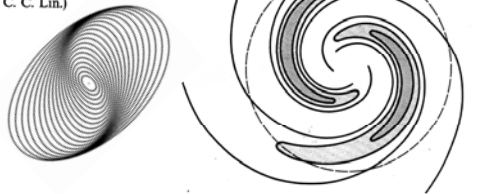
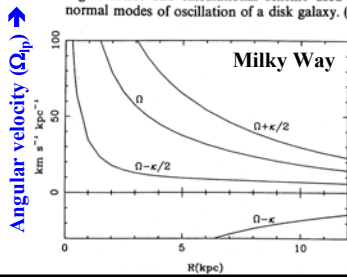
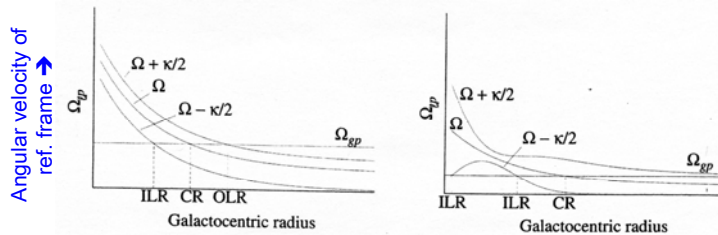


Figure 12.27. Contours of equal density excesses above the average value around a circle in a typical spiral mode. The dashed circle gives the radius where the material rotates at the same speed as the wave pattern.

Inner Lindblad Resonance (ILR)
Co-rotation Radius
Outer Lindblad Resonance (OLR)

Important in all disk galaxies

Hwk Set 3
Due Wed Sept. 30
[CO 25.13, 25.14, 25.16, 25.20]



Density waves cannot propagate across ILR or OLR

Density wave theory interprets most spirals as 2-armed

- 4-armed pattern is $n / m = 1 / 4$
- exists over a narrow range of radius.
 - less likely to be seen.

