The Virial Theorem [CO 2.4]

- For gravitationally bound systems *in equilibrium*
  - Time-averaged kinetic energy = - ½ time-averaged potential energy.

  E = total energy
  U = potential energy.
  K = kinetic energy.

  $E = K + U$

- Can show from Newton's 3 laws + law of gravity:
  - $\frac{1}{2}(d^2I/dt^2) - 2K = U$ where $I = \Sigma m_i r_i^2$ = moment of inertia.
  - Time average $\langle d^2I/dt^2 \rangle = 0$, or at least ~ 0.
  - Virial theorem ➔ $-2\langle K\rangle = \langle U\rangle$

    $\langle K\rangle = -\frac{1}{2}\langle U\rangle$

    $\langle E\rangle = \langle K\rangle + \langle U\rangle$ ➔

    $\langle E\rangle = \frac{1}{2}\langle U\rangle$

Mass determinations from absorption line widths

- Virial Theorem
  $2K = -U$
  $\langle v^2\rangle = 3\langle v_r^2\rangle$

  $$U = -\frac{3}{5}\frac{GM^2}{R}$$

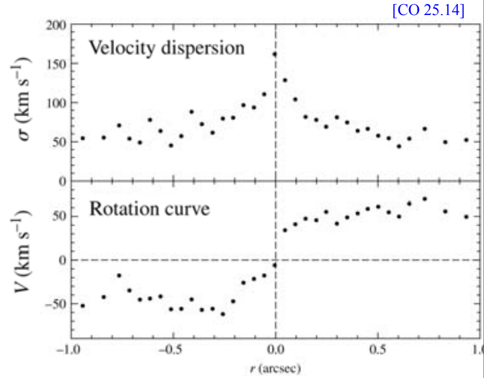
  ➔ $$M_{virial} = \frac{5R\sigma_r^2}{G}$$

- See pp. 959-962, + Sect. 2.4

- Applied to nuclei of spirals
  ➔ presence of massive black holes

- Also often applied to
  - E galaxies
  - Galaxy clusters

[CO 25.14]



M32

Mass determinations from absorption line widths

- Virial Theorem
  $2K = -U$
  $\langle v^2 \rangle = 3 \langle v_r^2 \rangle$

$$U = -\frac{3}{5}\frac{GM^2}{R}$$

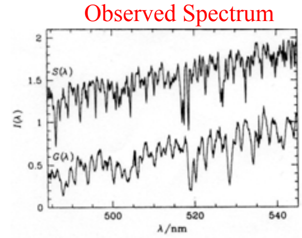
$$\Rightarrow \boxed{M_{virial} = \frac{5R\sigma_r^2}{G}}$$

- See pp. 959-962, + Sect. 2.4

- Applied to nuclei of spirals
  ➔ presence of massive black holes

- Also often applied to
  - E galaxies
  - Galaxy clusters

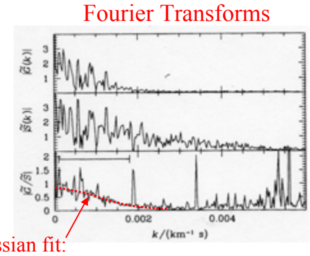
Observed Spectrum



K star

E galaxy = K star convolved with Gaussian velocity distribution of stars.

Fourier Transforms



Star

Galaxy

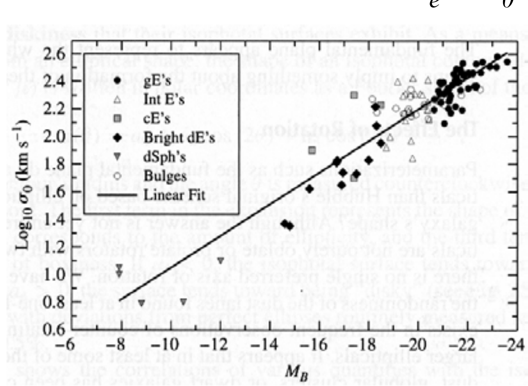
Ratio

Gaussian fit:
- Convolution turns into multiplication in F.T. space.
- F.T. of a Gaussian is a Gaussian.

$$I(R) = I_e 10^{-3.33\left[\left(\frac{R}{R_e}\right)^{1/4} - 1\right]}$$

I_e = surface brightness at R_e
L_e = luminosity *within* R_e

Faber-Jackson relation: $L_e \sim \sigma_0^4$



(Absolute magnitude)

Mass-Luminosity relationships

- Faber-Jackson relation: $L_e \sim \sigma_0^4$
- D_n - σ_0 correlation.
  - D_n = diameter within which $<I>$ = 20.75 μ_B
- Fundamental plane in log R_e, $<I>_e$, log σ_0 space
  - R_e = scale factor in $R^{1/4}$ law
  - $<I>_e$ = mean surface brightness within R_e *Different from* I_e *!*
  - *Intro. to Principle Component Analysis*: astro-ph/9905079

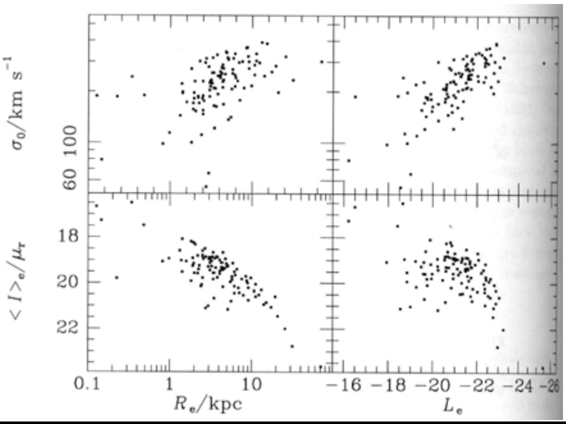
$$I(R) = I_e 10^{-3.33\left[\left(\frac{R}{R_e}\right)^{1/4} - 1\right]}$$

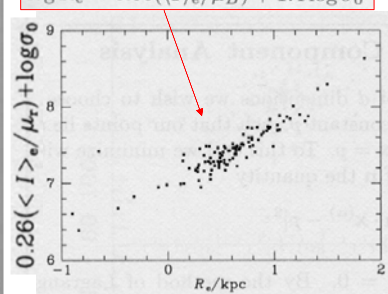
CO give different coefficients???

$r_e \propto \sigma_0^{1.24} I_e^{-0.82}$

$L \propto \sigma_0^{2.65} r_e^{0.65}$

$$\log R_e = 0.36(\langle I \rangle_e / \mu_B) + 1.4 \log \sigma_0$$



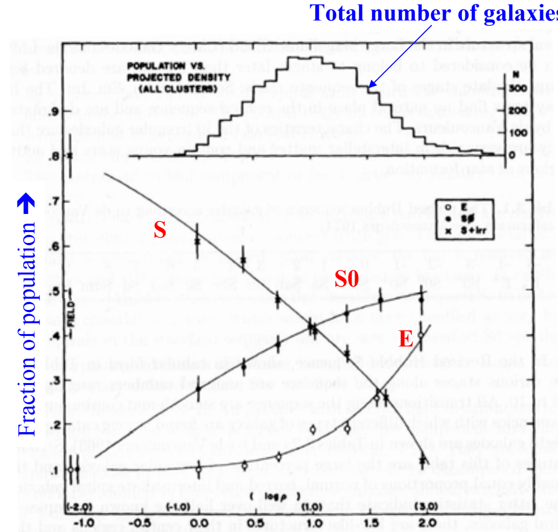


From Binney & Merrifield, *Galactic Astronomy*

Distribution of galaxy types

- Dense regions (cluster centers) predominantly ellipticals.
- Field galaxies predominantly spirals.
- On average, roughly even split between E and S.

Total number of galaxies



Dressler 1980

Log (Projected surface density of galaxies) ➔

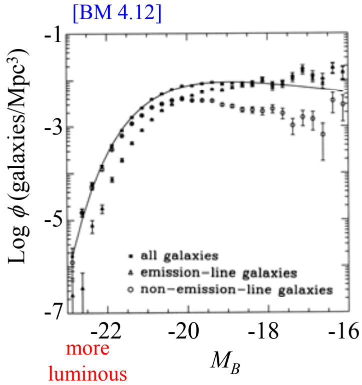
Schechter Luminosity Function

$$\phi\,(L)dL = L^{\alpha}\, e^{-L/L^*}\, dL$$

$$\phi(M)dM = 10^{\,-0.4(\alpha+1)M}\, e^{\,-10^{\,0.4(M^* - M)}} dM$$

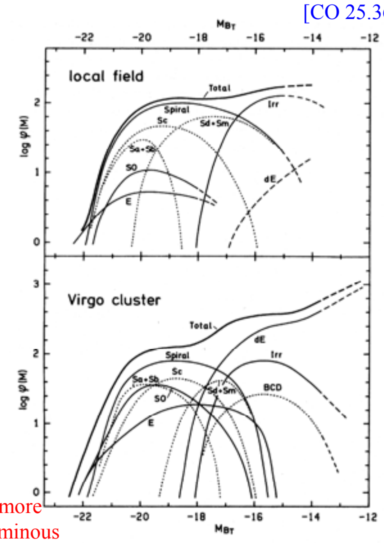
- The Milky Way is an L^* galaxy.

[BM 4.12]



more luminous

[CO 25.36]



more luminous