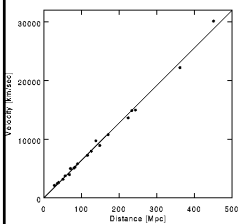
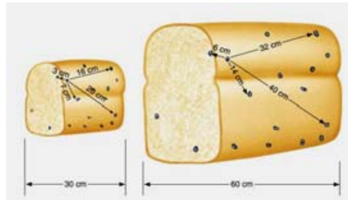
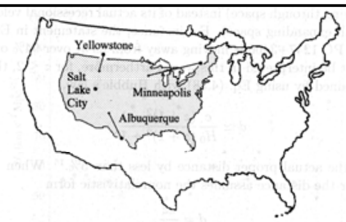


Redshift vs distance



As seen from a Wonder Bread raisen

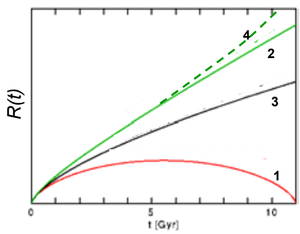


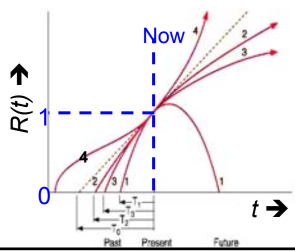
As seen from Utah

The Expanding Universe [CO 29.1]

$R(t)$ = scale factor = 1/(1+z)

$R(t)$ vs. t





The Cosmological Principle:

At any given time, the universe is the same everywhere.

Universe is

- Homogeneous
- Isotropic

Cosmological Principle: Universe is homogeneous & isotropic

Newtonian Cosmology

- Energy:

Kinetic + Potential = Total

$$\frac{1}{2}mv^2 - \frac{GM_r m}{r} = -\frac{1}{2}mkc^2\varpi^2 \quad [29.1]$$

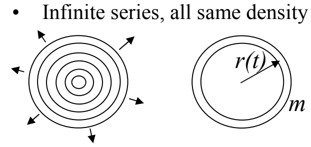
$$\frac{1}{2}mv^2 - \frac{G\frac{4}{3}\pi r^3 \rho m}{r} = -\frac{1}{2}mkc^2\varpi^2$$

$$\frac{1}{2}m\left(\frac{dR}{dt}\varpi\right)^2 - \frac{4}{3}\pi G\rho m(R\varpi)^2 = -\frac{1}{2}mkc^2\varpi^2$$

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2 \quad [29.10]$$

Friedman eq'n:

- Nested, expanding shells
  - Infinite series, all same density $\rho(t)$



  - Follow single shell, mass m

[29.3]

- **$r(t)$ = (Scale factor) x (co-moving coordinate)**

$$r(t) = R(t)\varpi$$

$$\frac{dr(t)}{dt} = v(t) = \frac{dR(t)}{dt}\varpi$$

- **Define: Total Energy** $= -\frac{1}{2}mkc^2\varpi^2$

Why???

Cosmological principle ➔

For bound universe, each nested shell must simultaneously have KE➔0

$$E = -\frac{4}{3}\pi G\rho m R^2\varpi^2 \propto m\varpi^2$$

Other forms of the Friedman Equation:

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$ [29.11]

$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.9]

- Define: $R(t_o) = 1$
- Conservation of mass:

$R^3(t)\rho(t) = R^3(t_o)\rho(t_o) = \rho_o$ [29.5]

- Hubble's law: [29.7]

$v(t) = H(t)\, r(t)$

But also: $r(t) = R(t)\, \varpi$

$$v(t) = \frac{dR(t)}{dt}\varpi$$

$$H(t) = \frac{v(t)}{r(t)} = \frac{\frac{dR(t)}{dt}\varpi}{R(t)\varpi} = \frac{1}{R}\frac{dR(t)}{dt}$$ [29.8]

The Critical Density

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$ [29.11]

$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.9]

$$E = -\frac{1}{2}mkc^2\varpi^2$$

$$\text{Energy per unit mass} = -\frac{1}{2}kc^2\varpi^2$$

$k > 0$ ➔ negative E, shells will collapse back

$k = 0$ ➔ E = 0, each shell has exactly escape velocity.

$k < 0$ ➔ positive E, shells expand forever

Critical density

$$k = 0 \quad ➔ \quad H^2 = \frac{8}{3}\pi G\rho$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$ [29.15]

$\rho_{c,o} = 1.88 \times 10^{-26}\, h^2$ kg m^{-3} [27.15]

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \boxed{\Omega_o = \frac{\rho_o}{\rho_{c,o}}}$$ [29.18] [29.19]

The Critical Density

Kinetic + Potential = Total

[29.10]
$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

[29.11]
$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$

For k=0:
$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho_{c,0}}{3R}$$

$$\int_0^R \sqrt{R'}\,dR' = \sqrt{\frac{8\pi G\rho_{c,0}}{3}}\int_0^t dt'$$

$$R = \left(\frac{3}{2}\right)^{\frac{2}{3}}\sqrt{\frac{8\pi G\rho_{c,0}}{3}}^{\frac{2}{3}} t^{\frac{2}{3}}$$

$$= \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}$$

$$E = -\frac{1}{2}mkc^2\varpi^2$$

Energy per unit mass $= -\frac{1}{2}kc^2\varpi^2$

$k > 0$ ➔ negative E, shells will collapse back
$k = 0$ ➔ E = 0, each shell has exactly escape velocity.
$k < 0$ ➔ positive E, shells expand forever

Critical density

$k = 0$ ➔ $H^2 = \frac{8}{3}\pi G\rho$

$$\frac{8\pi G\rho_{c,0}}{3} = H_o^2 = \frac{1}{t_H^2}$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$ [29.15]

$\rho_{c,o} = 1.88 \times 10^{-26}\ h^2$ kg m^{-3} [27.15]

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \Omega_o = \frac{\rho_o}{\rho_{c,o}}$$ [29.18] [29.19]

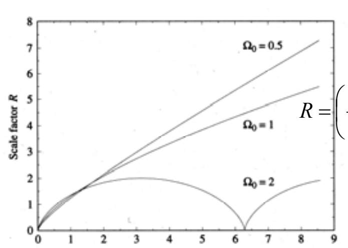
Homework: [CO 29.7]
= max size + lifetime of closed U.
Due Oct. 26

The Critical Density

Kinetic + Potential = Total

[29.10]
$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

[29.11]
$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$



[Fig. 29.5]

$$R = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}}$$

For Ω≠1, see parametric solutions in CO [29.32-29.39]

$$E = -\frac{1}{2}mkc^2\varpi^2$$

Energy per unit mass $= -\frac{1}{2}kc^2\varpi^2$

$k > 0$ ➔ negative E, shells will collapse back
$k = 0$ ➔ E = 0, each shell has exactly escape velocity.
$k < 0$ ➔ positive E, shells expand forever

Critical density

$k = 0$ ➔ $H^2 = \frac{8}{3}\pi G\rho$

$$\frac{8\pi G\rho_{c,0}}{3} = H_o^2 = \frac{1}{t_H^2}$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$ [29.15]

$\rho_{c,o} = 1.88 \times 10^{-26}\ h^2$ kg m^{-3} [27.15]

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \Omega_o = \frac{\rho_o}{\rho_{c,o}}$$ [29.18] [29.19]

All Universes ~ "flat" ($\rho \sim \rho_c$) at early times.

- Homework problem 29.9 will show:

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{kc^2}{(dR/dt)^2} \qquad (29.194)$$

Homework:
[CO 29.9]
Due Oct. 26

and that $dR/dt \to \infty$ as $t \to 0$

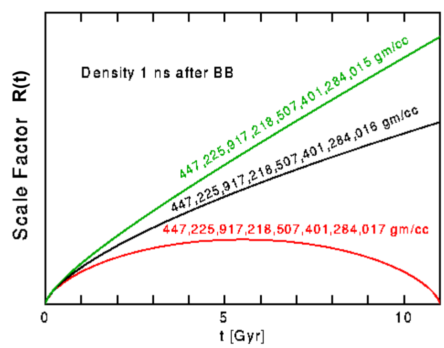
implying $\rho(t) \to \rho_c(t)$ as $t \to 0$ for all values of k.

Consequences:

1. For small t, it is OK to use:

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = 0$$

2. Even tiny departures from flatness ($\rho = \rho_c$) at small t would have grown into impossibly large departures from flatness by present time.



Including Pressure

[pp. 1160-1161]

- For a fluid undergoing *adiabatic* expansion (no transfer of heat):

Homework:
[CO 29.12]
= derive acceleration eqn.
Due Oct. 26

Work done is $dU = -PdV$

$$\frac{dU}{dt} = -\frac{4}{3}\pi P \frac{d(r^3)}{dt}$$

$$u = \frac{U}{\frac{4}{3}\pi r^3} \quad \Rightarrow \quad \frac{d(r^3 u)}{dt} = -P\frac{d(r^3)}{dt}$$

$$\rho = \frac{u}{c^2} \quad \Rightarrow \quad \frac{d(r^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(r^3)}{dt}$$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

Friedman Equation (Energy)

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$$

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^3 = -kc^2 R$$

Time derivative + algebra

Acceleration Equation (Force): $\displaystyle \frac{d^2R}{dt^2} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right)R$