All Universes ~ "flat" ($\rho \sim \rho_c$) at early times.

- Homework problem 29.9 will show:

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{kc^2}{(dR/dt)^2} \qquad (29.194)$$

Homework:
[CO 29.9]
Due Oct. 26

and that $dR/dt \to \infty$ as $t \to 0$

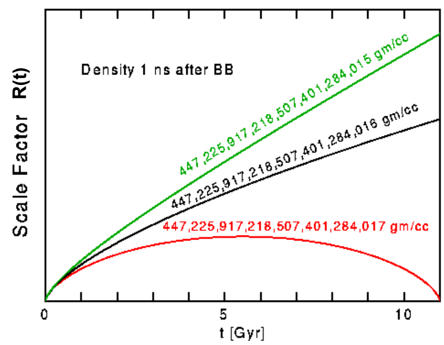
implying $\rho(t) \to \rho_c(t)$ as $t \to 0$ for all values of k.

Consequences:

1. For small t, it is OK to use:

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = 0$$

2. Even tiny departures from flatness ($\rho = \rho_c$) at small t would have grown into impossibly large departures from flatness by present time.



Including Pressure

[pp. 1160-1161]

- For a fluid undergoing *adiabatic* expansion (no transfer of heat):

Homework:
[CO 29.12]
= derive acceleration eqn.
Due Oct. 26

Work done is $dU = -PdV$

$$\frac{dU}{dt} = -\frac{4}{3}\pi P \frac{d(r^3)}{dt}$$

$$u = \frac{U}{\frac{4}{3}\pi r^3} \quad \Rightarrow \quad \frac{d(r^3 u)}{dt} = -P\frac{d(r^3)}{dt}$$

$$\rho = \frac{u}{c^2} \quad \Rightarrow \quad \frac{d(r^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(r^3)}{dt}$$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

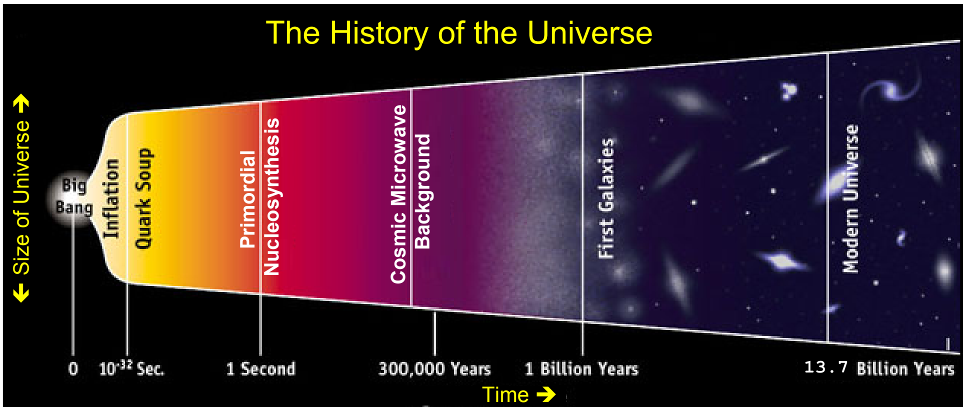
Friedman Equation (Energy)

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$$

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^3 = -kc^2 R$$

Time derivative + algebra

Acceleration Equation (Force): $\dfrac{d^2R}{dt^2} = -\dfrac{4}{3}\pi G\left(\rho + \dfrac{3P}{c^2}\right)R$



Planck time | Inflation | Formation of H, He, Li | Decoupling of CMB | Galaxy Formation | Now

Hot (10^{32} K) → (10^9 K) → (3000 K) → Cool (3 K)

High density → Low density

Cosmology in 1946

- Big-Bang Nucleosynthesis
  - Cooling of Universe (Alpher & Herman, 1948)
    - Radiation energy density $u_{rad} \propto \frac{1}{R(t)^4}$

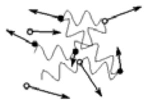
because $E_{phot}(t) = \frac{hc}{\lambda(t)} = \frac{hc}{\lambda_o R(t)}$

λ_o means as observed at present time!

  - Big-Bang Nucleosynthesis
    - Alpher, Bethe & Gamow (αβγ) paper --- all elements built in Big Bang?
    - Later found: can't get much past ^{4}He
- Steady State Model
  - Bondi, Gold & Hoyle
  - "Perfect" Cosmological Principle – universe same at all points and at all times
    - U has always been here.
  - Nucleosynthesis in stars
    - B^2FH

Also… "tired light"

Black-Body Radiation



$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

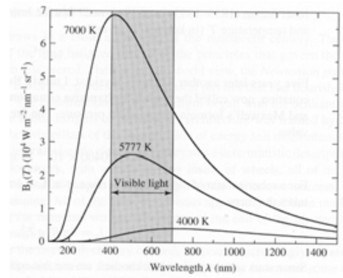


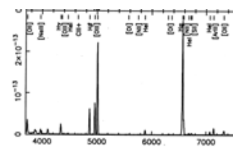
FIGURE 3.8 Blackbody spectrum [Planck function $B_\lambda(T)$].

[CO Sect. 3.4]

Black body

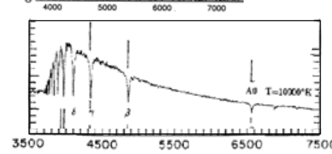
$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

$$L = A\sigma T^4 \qquad \lambda_{\max} T = 0.002897755 \text{ m K}$$



for *large* λ : (Rayleigh-Jeans tail)

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$



also… dilute black body

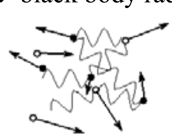
A Prediction

λ_o means observed at present time!

$$u_{rad} \propto \frac{1}{R(t)^4}$$

$$E_{phot}(t) = \frac{hc}{\lambda(t)} = \frac{hc}{\lambda_o R(t)}$$

- Hot universe ➔ filled with free electrons
- Electron opacity ➔ black body radiation field



$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

- Cooling universe: at some point, $e^- + H^+$ ➔ H^0
- Universe becomes transparent.
- ➔ relic of black body radiation field should be observable today.

Redshifted radiation ➔ black body radiation field for a lower temperature

$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

u_o , λ_o, T = present (observed) values
$u(R)$, λ, $T(R)$ = values when $R=R(t)$

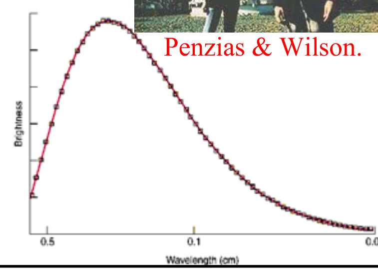
$\lambda = R\lambda_o$
$d\lambda = Rd\lambda o$

$$u_0 \, d\lambda_0 = R^4 u(R) \, d\lambda(R)$$
$$= R^4 \frac{8\pi hc/R^5\lambda_0^5}{e^{hc/R\lambda_0 kT(R)} - 1} R d\lambda_0$$
$$= \frac{8\pi hc/\lambda_0^5}{e^{hc/\lambda_0 k[RT(R)]} - 1} \, d\lambda_0.$$

Penzias & Wilson.

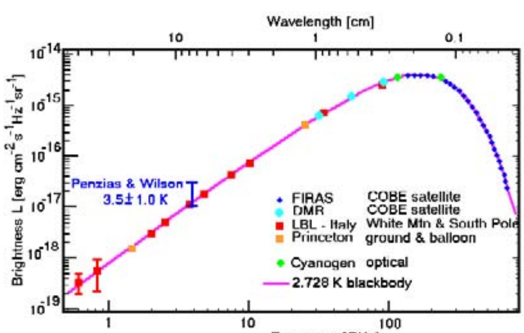
- Both shape *and* energy density are predicted.

$$T_0 = RT(R)$$



How COBE Changed Things

- COBE satellite (1991).



Nobel Prizes

 Penzias & Wilson, 1978

 Mather & Smoot, 2006

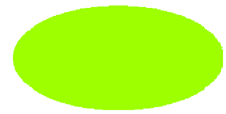
Isotropy of the Cosmic Microwave Background

Black body

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

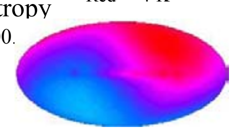
for *large λ*

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$



Blue = 0°K
Red = 4°K

Dipole Anisotropy
~ 1 part in 300.



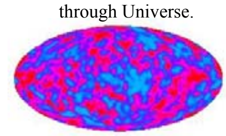
$$T_{\text{moving}} = \frac{T_{\text{rest}}\sqrt{1 - v^2/c^2}}{1 - (v/c)\cos\theta}$$

for $v << c$

$$T_{\text{moving}} \simeq T_{\text{rest}}\left(1 + \frac{v}{c}\cos\theta\right)$$

(29.62)

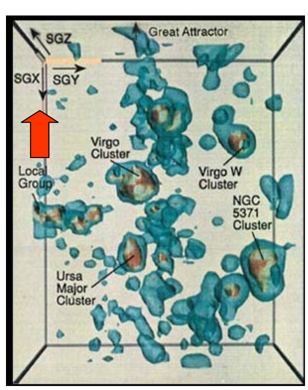
Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
→motion of Sun through Universe.



After removing dipole
Red – blue = 0.0002°K

Isotropy of the Cosmic Microwave Background

Local Group



Black body

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

for *large λ*

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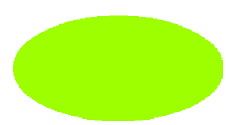
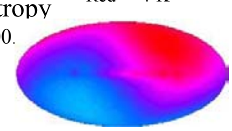


Blue = 0°K
Red = 4°K

Dipole Anisotropy
~ 1 part in 300.



$$T_{\text{moving}} = \frac{T_{\text{rest}}\sqrt{1 - v^2/c^2}}{1 - (v/c)\cos\theta}$$

for $v << c$

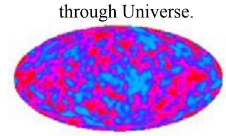
$$T_{\text{moving}} \simeq T_{\text{rest}}\left(1 + \frac{v}{c}\cos\theta\right)$$

(29.62)

Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
→motion of Sun through Universe.

- Dipole Anistropy ~ 1 part in 300
  - 600 km/sec motion of Local Group in grav. field of larger scale mass concentrations.

Homework:
[CO 29.21]
Derive above eqn.
Due Oct. 26



After removing dipole
Red – blue = 0.0002°K