$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$$H^2(1-\Omega)R^2 = -kc^2$$

$$H_0^2(1-\Omega_0) = -kc^2$$

Some Universes

Open *vs.* Closed:

$$k = 0 \;\Rightarrow\; \Omega_0 = \Omega_{m,0} + \cancel{\Omega_{\mathrm{rel},0}}^{\sim 0} + \Omega_{\Lambda,0} = 1$$

Accelerating *vs.* Decelerating:

$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

Deceleration parameter

For $\Lambda = 0$: $\quad = \frac{1}{2}\Omega(t)$

$$q_0 = \frac{1}{2}\Omega_0$$

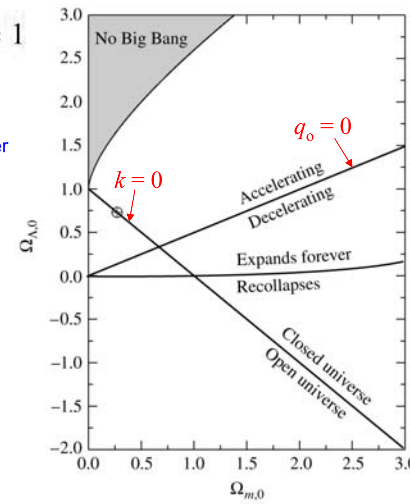
For $\Lambda = 0$:	
$q_o = 0$	empty
< 0.5	open
$= 0.5$	flat
> 0.5	closed

For $\Lambda \neq 0$: $\quad q(t) = \frac{1}{2}\sum_i (1+3w_i)\Omega_i(t)$

$$q(t) = \frac{1}{2}\Omega_m(t) + \cancel{\Omega_{\mathrm{rel}}}^{\sim 0}(t) - \Omega_\Lambda(t)$$

Expands Forever *vs.* Recollapses:

Does dR/dt ever = 0? See [29.135]



All Universes ~ "flat" ($\rho \sim \rho_c$) at early times.

- Homework problem 29.9 will show:

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{kc^2}{(dR/dt)^2} \qquad (29.194)$$

and that $dR/dt \to \infty$ as $t \to 0$

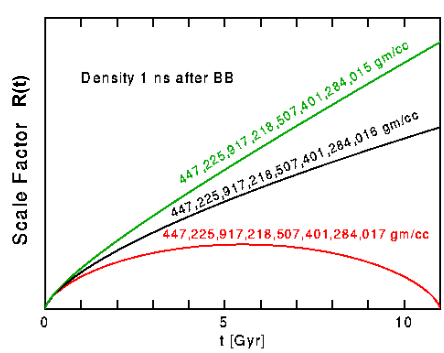
implying $\rho(t) \to \rho_c(t)$ *as* $t \to 0$ for all values of k.

Consequences:

1. For small t, it is OK to use:

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = 0$$

2. Even tiny departures from flatness ($\rho = \rho_c$) at small t would have grown into impossibly large departures from flatness by present time.



Proper distance

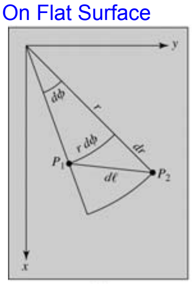
= the *current* distance to a distant object.

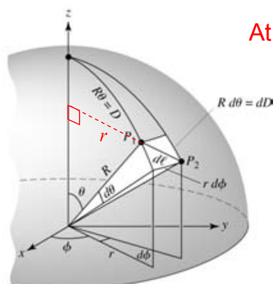
$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

$dt = 0$, proper distance $d_p(t)$ = sqrt($-ds^2$)

$$d_p(t) = R(t)\int_0^{\varpi} \frac{d\varpi'}{\sqrt{1-k\varpi'^2}}$$

On Flat Surface





On Surface of Ball

At the current time (using $R(t_0) = 1$):

Flat: $d_{p,0} = \varpi$

Closed: $d_{p,0} = \frac{1}{\sqrt{k}}\sin^{-1}(\varpi\sqrt{k})$

Open: $d_{p,0} = \frac{1}{\sqrt{|k|}}\sinh^{-1}(\varpi\sqrt{|k|})$

The particle horizon

Horizon distance = distance a photon has traveled since $t = 0$.

$$\int_{t1}^{t2}\frac{cdt}{R(t)} = -\int_{\varpi 1}^{\varpi 2}\frac{d\varpi}{\sqrt{1-k\varpi^2}} = \int_{\varpi 2}^{\varpi 1}\frac{d\varpi}{\sqrt{1-k\varpi^2}}$$ ← For a *photon*

$$d_p(t) = R(t)\int_0^{\varpi}\frac{d\varpi'}{\sqrt{1-k\varpi'^2}} = R(t)\int_{t_e}^{t_0}\frac{c\,dt'}{R(t')}$$

$$\boxed{d_h(t) = R(t)\int_0^t \frac{c\,dt'}{R(t')}}$$

Radiation dominated flat universe: $R \propto t^{1/2}$ → $d_h(t) = 2ct$

Matter dominated flat universe: $R \propto t^{2/3}$ → $d_h(t) = 3ct$

Matter dominated flat universe in terms of *redshift* → $d_h(z) = \frac{2c}{H_0\sqrt{\Omega_{m,0}}}\frac{1}{(1+z)^{3/2}}$

Including Ω_Λ → $d_h(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3}\sinh^{2/3}\left(\frac{3}{2}H_0 t\sqrt{\Omega_{\Lambda,0}}\right)\int_0^t \frac{c\,dt'}{\left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3}\sinh^{2/3}\left(\frac{3}{2}H_0 t'\sqrt{\Omega_{\Lambda,0}}\right)}$

= 14.6 Gpc (WMAP)

[29.158]

The paths of photons in terms of proper distance.

$$(\cancelto{0}{ds})^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,\cancelto{0}{d\theta})^2 + (\varpi\,\cancelto{0}{\sin\theta\,d\phi})^2\right]$$

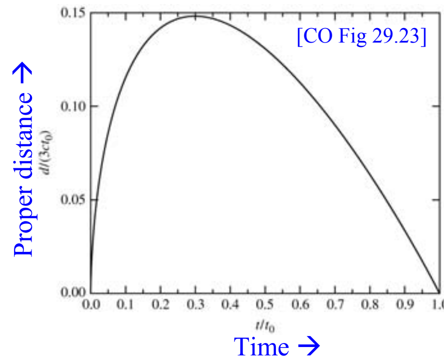
Matter dominated flat universe:

$$\int_0^t \frac{c\,dt'}{R(t')} = \int_{\varpi}^{\varpi_e} d\varpi'$$

$$R(t) = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}} = \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\varpi = \varpi_e - 3ct_0\left(\frac{t}{t_0}\right)^{1/3}.$$

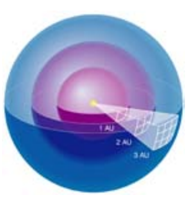
At $t = t_o$, $\varpi = 0$ ➔ $\varpi_e = 3ct_0$



Proper distance:

$$R(t)\varpi = \quad d_p(t) = 3ct_0\left[\left(\frac{t}{t_0}\right)^{2/3} - \left(\frac{t}{t_0}\right)\right] \qquad \boxed{[29.165]}$$

Luminosity Distance



$$F = \frac{L}{4\pi d^2}$$

$$F = \frac{L}{4\pi\varpi^2(1+z)^2}$$

Redshift ➔ $(1+z)$
Time dilation ➔ $(1+z)$

$$d_L = \varpi(1+z)$$

In practice
(because of that @#$% cosmological constant)

$$d_L(z) = \frac{c}{H_0}(1+z)S(z)$$

$$\begin{aligned} S(z) &\equiv I(z) && (\Omega_0 = 1) \\ &\equiv \frac{1}{\sqrt{\Omega_0 - 1}}\sin\left[I(z)\sqrt{\Omega_0 - 1}\right] && (\Omega_0 > 1) \\ &\equiv \frac{1}{\sqrt{1-\Omega_0}}\sinh\left[I(z)\sqrt{1-\Omega_0}\right] && (\Omega_0 < 1), \end{aligned}$$

From previous slide:

About right…

$$\varpi \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right] \quad (\text{for } z \ll 1).$$

$$d_L(z) \simeq \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)z\right] \quad (\text{for } z \ll 1)$$

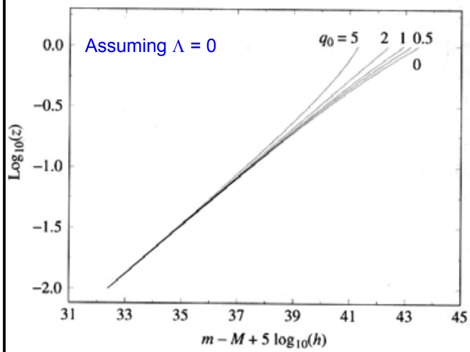
$$\boxed{m - M = 5\log_{10}(d_L/10\text{ pc})}$$

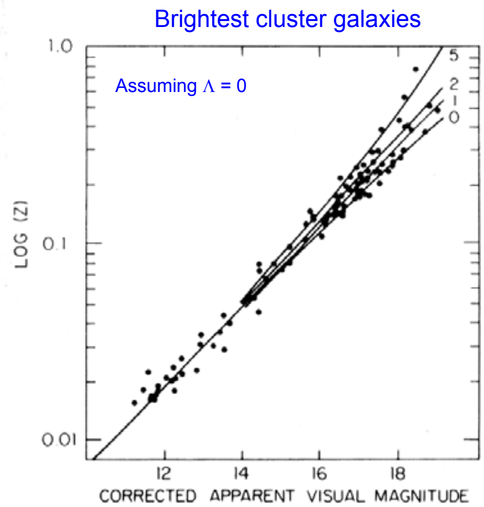
$$m - M \simeq 5\log_{10}\left[\frac{c}{(100\text{ km s}^{-1}\text{ Mpc}^{-1})(10\text{ pc})}\right] - 5\log_{10}(h) + 5\log_{10}(z) + 5\log_{10}\left[1 + \frac{1}{2}(1-q_0)z\right] \quad (\text{for } z \ll 1)$$

$$\begin{aligned} m - M &= 5\log_{10}\left[\frac{c}{(100\text{ km s}^{-1}\text{ Mpc}^{-1})(10\text{ pc})}\right] - 5\log_{10}(h) \\ &\quad + 5\log_{10}(1+z) + 5\log_{10}[S(z)] \\ &= 42.38 - 5\log_{10}(h) + 5\log_{10}(1+z) + 5\log_{10}[S(z)]. \end{aligned}$$

$$\boxed{m - M \simeq 42.38 - 5\log_{10}(h) + 5\log_{10}(z) + 1.086(1-q_0)z \quad (\text{for } z \ll 1). \quad [\text{CO } 29.188]}$$

Old Redshift-Magnitude Results



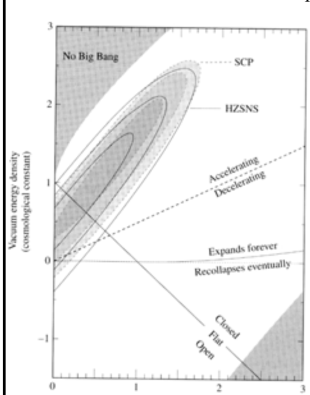


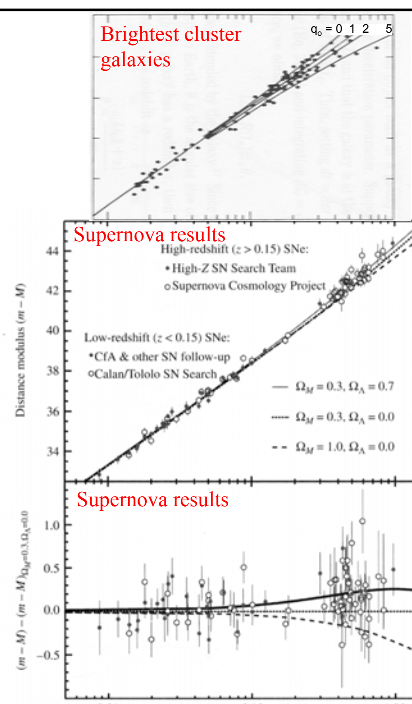
- Curves diverge at larger z
  - ➔ use very luminous objects which are observable at large z.
- Best bet = brightest cluster galaxies
  - But big correction for evolution of stellar population.

$$m - M \simeq 42.38 - 5\log_{10}(h) + 5\log_{10}(z) + 1.086(1-q_0)z \quad (\text{for } z \ll 1).$$ [CO 29.188]

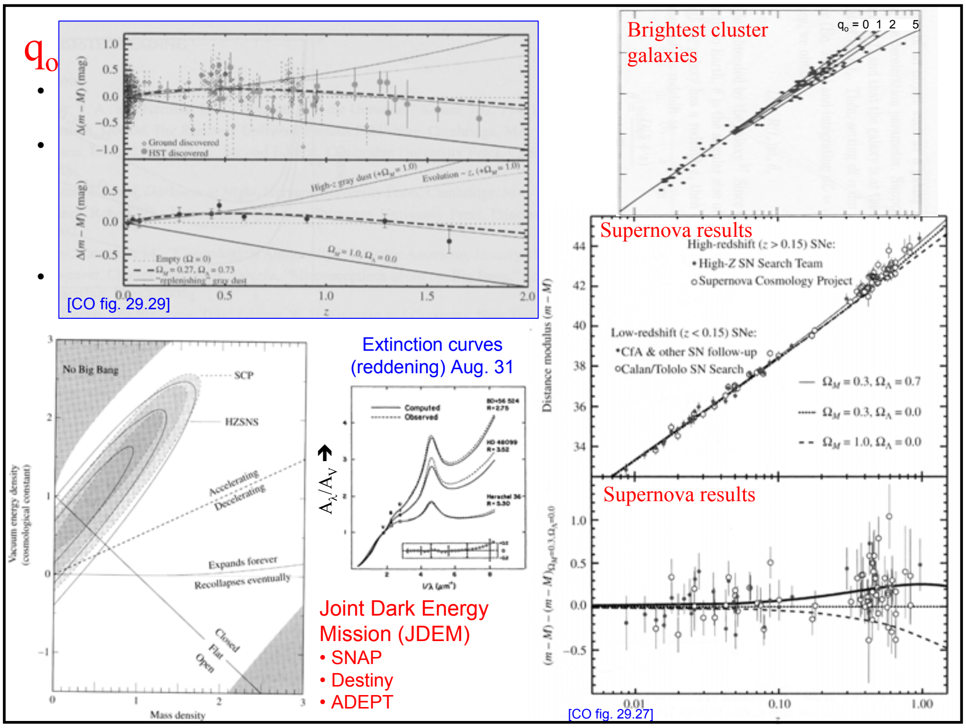
q_o – the accelerating universe

- Type Ia supernovae are best standard candles.
  - Least scatter in luminosity
- 2 independent groups get same answer
  - High-z Supernova Search
    - Garnavich et al. 1998, ApJ 509, 74
  - Supernova Cosmology Project
    - Perlmutter et al. 1999, ApJ 517, 565
- Found *acceleration*
  - Not deceleration as expected.

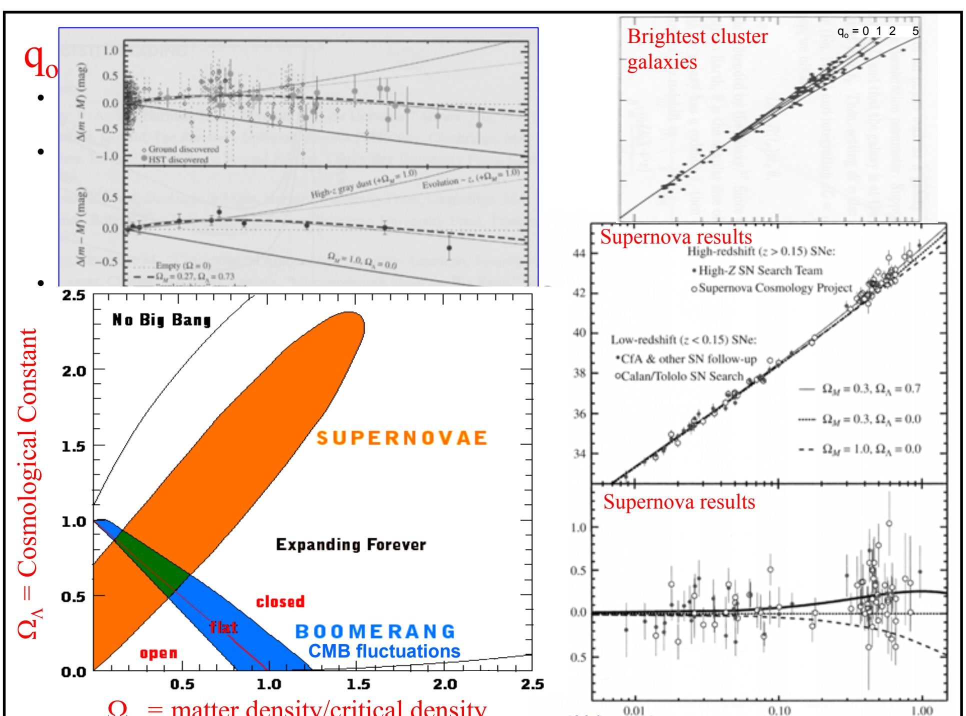




[CO fig. 29.27]



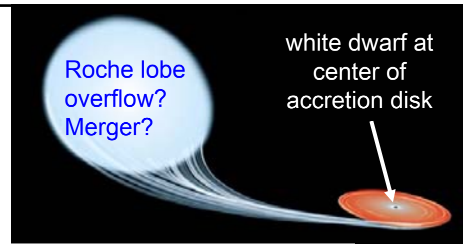
q$_o$
[CO fig. 29.29]
Ground discovered
HST discovered
Empty (Ω = 0)
Ω_M = 0.27, Ω_Λ = 0.73
No Big Bang
SCP
HZSNS
Accelerating
Decelerating
Expands forever
Recollapses eventually
Closed
Flat
Open
Mass density
Vacuum energy density (cosmological constant)
Extinction curves (reddening) Aug. 31
A_λ/A_V
Computed
Observed
Joint Dark Energy Mission (JDEM)
• SNAP
• Destiny
• ADEPT
Brightest cluster galaxies
q$_o$ = 0 1 2 5
Supernova results
High-redshift (z > 0.15) SNe:
• High-Z SN Search Team
○ Supernova Cosmology Project
Low-redshift (z < 0.15) SNe:
• CfA & other SN follow-up
○ Calan/Tololo SN Search
Ω_M = 0.3, Ω_Λ = 0.7
Ω_M = 0.3, Ω_Λ = 0.0
Ω_M = 1.0, Ω_Λ = 0.0
Distance modulus (m − M)
Supernova results
[CO fig. 29.27]



q$_o$
No Big Bang
SUPERNOVAE
Expanding Forever
closed
flat
open
BOOMERANG
CMB fluctuations
Ω_Λ = Cosmological Constant
Ω_m = matter density/critical density
Brightest cluster galaxies
q$_o$ = 0 1 2 5
Supernova results
High-redshift (z > 0.15) SNe:
• High-Z SN Search Team
○ Supernova Cosmology Project
Low-redshift (z < 0.15) SNe:
• CfA & other SN follow-up
○ Calan/Tololo SN Search
Ω_M = 0.3, Ω_Λ = 0.7
Ω_M = 0.3, Ω_Λ = 0.0
Ω_M = 1.0, Ω_Λ = 0.0
Supernova results
[CO fig. 29.27]

Type Ia Supernovae

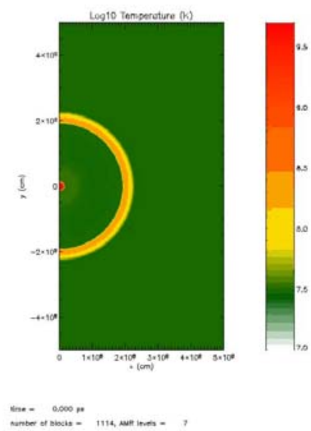
- Something dumps too much mass onto white dwarf.
- Increased density ➔ runaway heating through C + C burning
- Heating rate faster than dynamical timescale
  - White dwarf cannot peacefully respond to pressure increase.
- *Deflagration*
  - leading to *detonation*?



Type Ia Supernovae as "standard candles".

- Always happens when mass goes just past limit for heating-cooling balance.

  ➔ Supernova always has ~ same luminosity (factor 10).
- Get distance from $\text{Flux} = \dfrac{L}{4\pi r^2}$



Deflagration simulation