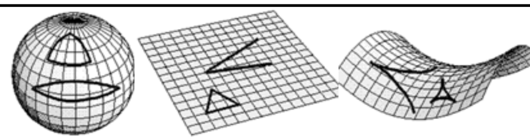
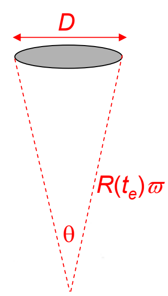
Angular Diameters



RW metric:

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

What is angular size of galaxy at co-moving distance ϖ?



$$dt = d\varpi = d\phi = 0$$

Galaxy's diameter is proper ~~distance~~ linear diameter:

$$D = \int \sqrt{-(ds)^2} = R(t_e)\,\tilde{\omega}_e\,\theta$$

Using ϖ coordinate
➔ Looks like Euclidean result, regardless of curvature of space.

$$\Rightarrow \theta = \frac{D}{R(t_e)\,\tilde{\omega}}$$

but must use $R(t_e)$

$$= \frac{D(1+z)}{\tilde{\omega}} \qquad \text{USING} \quad 1+z = \frac{1}{R(t_e)}$$

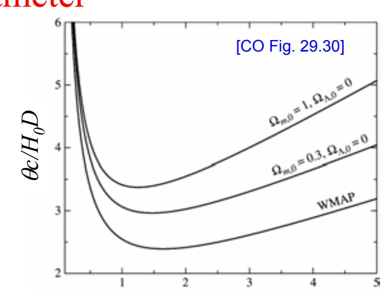
$$\theta = \frac{D(1+z)^2}{d_L}$$

More angular diameter

$$\theta = \frac{D(1+z)^2}{d_L}$$

In practice
(because of that @#$% cosmological constant)

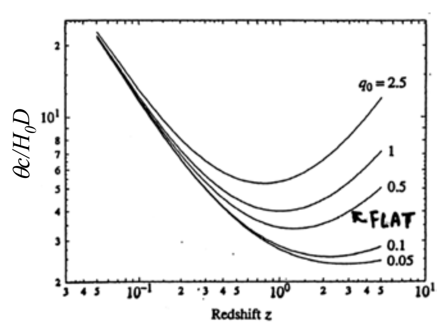
$$\frac{c\theta}{H_0 D} = \frac{(1+z)}{S(z)} \qquad \boxed{29.193}$$



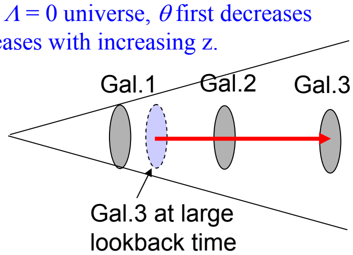
For $\Lambda = 0$:

$$\theta = \frac{H_0 D}{c}\,\frac{q_0^2(1+z)^2}{q_0 z - (1-q_0)(\sqrt{1+2q_0 z} - 1)}$$

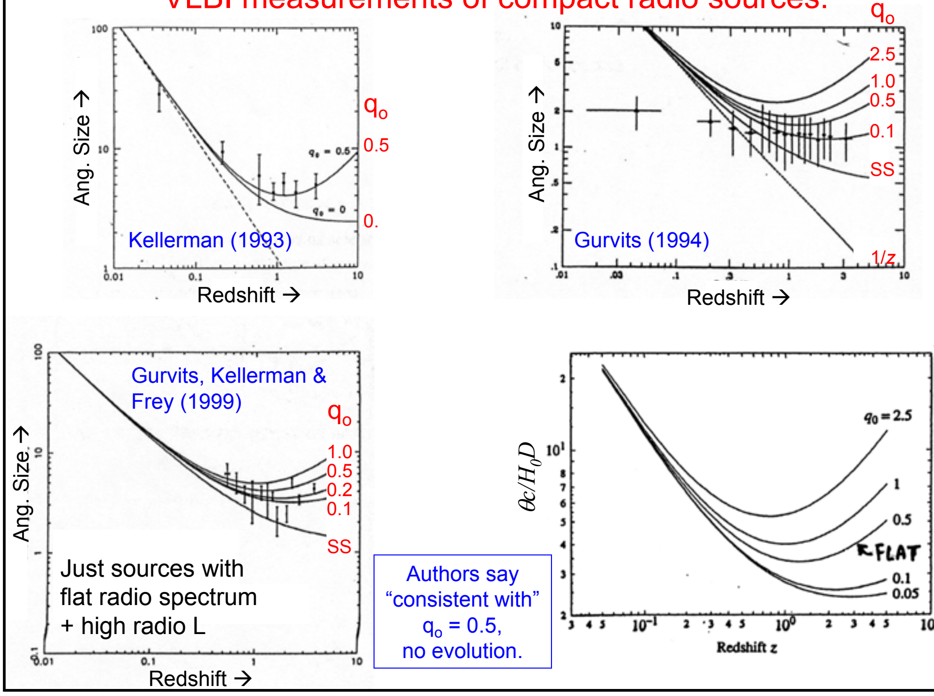
Surprise!
Even for flat, $\Lambda = 0$ universe, θ first decreases but then increases with increasing z.



Competing Effects:
- Distance
- Expansion



VLBI measurements of compact radio sources:



Authors say "consistent with" $q_o = 0.5$, no evolution.

The Concordance Cosmology

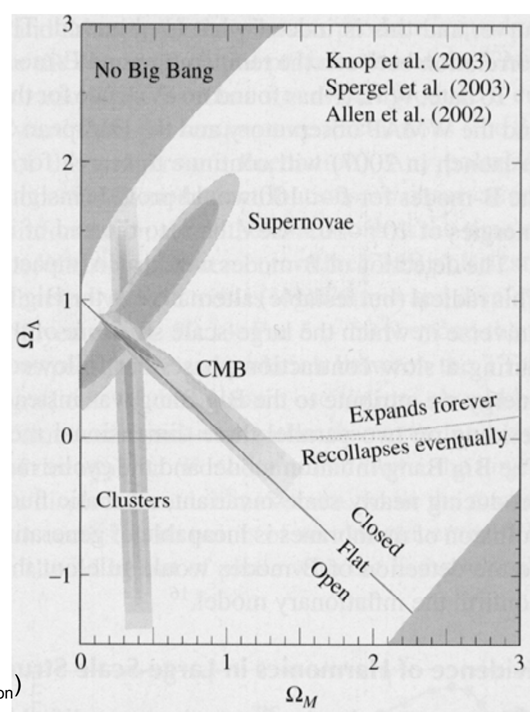
= ΛCDM = LCDM

Dark Energy (Λ) — Cold Dark Matter (CDM)

Concordance between:

- CMB fluctuations.
- Supernovae.
- Galaxy cluster growth rate

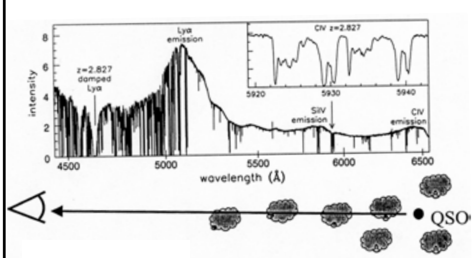
- Globular cluster ages
- Power spectrum of large-scale structure.
- H_0 : HST key project vs. WMAP.
- Baryon density:
  primordial nucleosynthesis
  vs. WMAP.
- Ω_m from $\Omega_{baryon} \times (\rho_{dark\ matter} / \rho_{baryon})$



Big Bang Nucleosynthesis (Oct. 14 lecture)

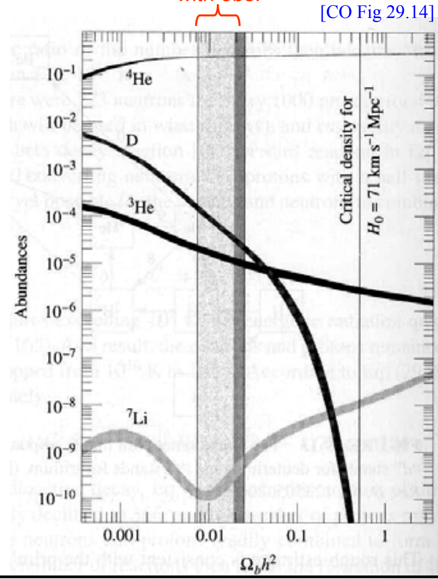
$\Omega_{Baryons}$

- d, ^{7}Li, ^{3}He ➔
  $$\Omega_B = \frac{\rho_{B,o}}{\rho_{c,o}} = 0.02 - 0.05$$
- But better determination now from CMB fluctuations (WMAP)
  $$\Omega_B = 0.044$$



Consistent with obs.

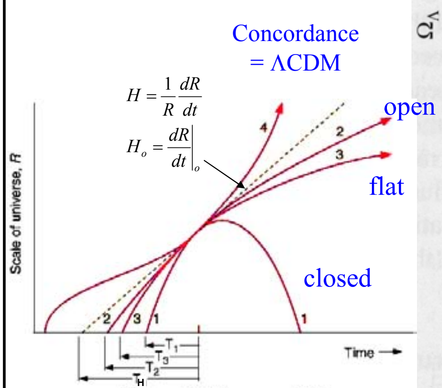
[CO Fig 29.14]

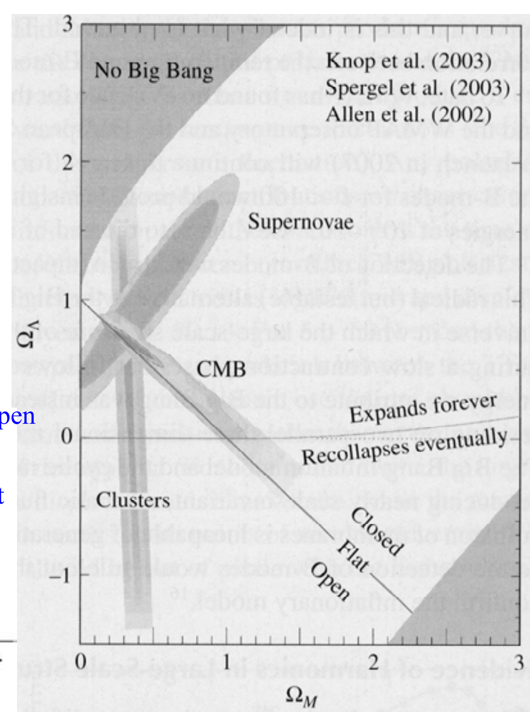


The Concordance Cosmology

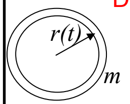
= ΛCDM = LCDM

Dark Energy (Λ), Cold Dark Matter (CDM)





Definitions, results, etc.



* $r = R(t)\ \varpi$

* $H = \frac{1}{R}\frac{dR}{dt}$

Densities:

* Matter: $\rho_m = \rho_{o,m} R^{-3}$

* Radiation: $\rho_r = \rho_{o,r} R^{-4}$

* Dark energy: $\rho_\Lambda = \rho_{o,\Lambda} R^{0}$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

* $\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$

* $\Omega \equiv \Omega_m + \Omega_{\rm rel} + \Omega_\Lambda$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

* $P = wu = w\rho c^2$

Physics

Per unit mass:

K.E. + potential E. = Total Energy

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$\rho = \frac{u}{c^2}$ *

* Temp. of radiation field: $T_0 = RT(R$

*

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

*

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

Cosmological Constant (a.k.a. *Dark Energy*)

Curvature $k = \frac{1}{\Re^2} \times \begin{matrix}+1\\0\\-1\end{matrix}$

$dU = -PdV$

$$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \rho_{\rm rel} + \rho_\Lambda + \frac{3(P_m + P_{\rm rel} + P_\Lambda)}{c^2}\right]\right\}R$$

* = you should be able to write these down from memory.