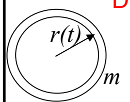
Definitions, results, etc.



* $r = R(t)\,\varpi$

* $H = \frac{1}{R}\frac{dR}{dt}$

Densities:

* Matter: $\rho_m = \rho_{o,m} R^{-3}$

* Radiation: $\rho_r = \rho_{o,r} R^{-4}$

* Dark energy: $\rho_\Lambda = \rho_{o,\Lambda} R^{0}$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

* $\Omega(t) = \frac{\rho(t)}{\rho_c(t)}$

* $\Omega \equiv \Omega_m + \Omega_{rel} + \Omega_\Lambda$

$$\frac{d(R^3\rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

$$q(t) = -\frac{R(t)\left[d^2R(t)/dt^2\right]}{\left[dR(t)/dt\right]^2}$$

* $P = wu = w\rho c^2$

Physics

Per unit mass:

K.E. + potential E. = Total Energy

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$\rho = \frac{u}{c^2}$ *

* Temp. of radiation field: $T_0 = RT(R$

*

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

*

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

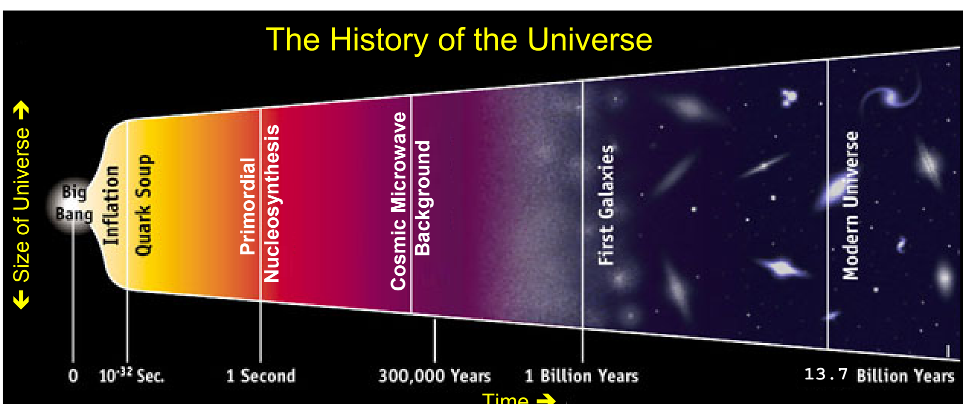
Cosmological Constant (a.k.a. *Dark Energy*)

Curvature $k = \frac{1}{\Re^2} \times \begin{matrix}+1\\0\\-1\end{matrix}$

$dU = -PdV$

$$\frac{d^2R}{dt^2} = \left\{-\frac{4}{3}\pi G\left[\rho_m + \rho_{rel} + \rho_\Lambda + \frac{3(P_m + P_{rel} + P_\Lambda)}{c^2}\right]\right\}R$$

* = you should be able to write these down from memory.

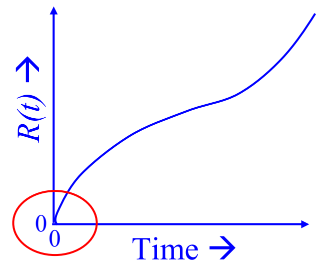


Planck time	Inflation	Formation of H, He, Li	Decoupling of CMB	Galaxy Formation	Now

Hot (10^{32} K) → (10^{9} K) → (3000 K) → Cool (3 K)

High density → Low density

The Planck Time

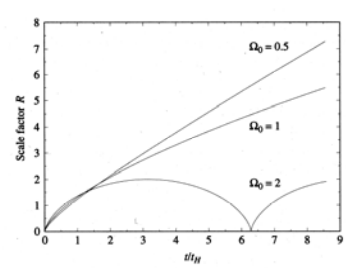


- Dimensional arguments
  - Planck time $t_P = \sqrt{\frac{\hbar G}{c^5}}$ = 5 x 10^{-44} s
  - Planck mass $m_P = \sqrt{\frac{\hbar c}{G}}$ = 2 x 10^{-8} kg
  - Planck length $\ell_P = \sqrt{\frac{\hbar G}{c^3}}$ = 2 x 10^{-35} m
- Before this, everything fuzzed out by uncertainty principle.

Some Problems for Friedmann-Robertson-Walker Universes

- Causality and the particle horizon
- Flatness

- Absence of magnetic monopoles
- Absence of "Domain Walls"



The Horizon Problem

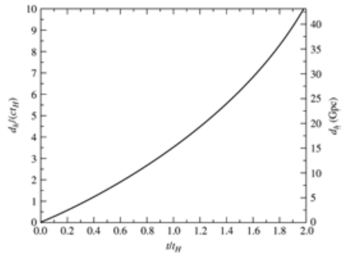


Fig. 29.22
Proper distance from Earth to particle horizon as function of time, including Λ.

The Particle Horizon:

For k = 0, Λ = 0, Ω = 1 example:

- Radiation era: $R(t) \sim t^{1/2}$ $\quad d_h(t) = 2ct \quad \varpi_h(t) = d_h(t)/R(t) \sim t^{1/2}$
- Matter Era: $R(t) \sim t^{2/3}$ $\quad d_h(t) = 3ct \quad \varpi_h(t) = d_h(t)/R(t) \sim t^{1/3}$

As time passes, we can see larger and larger fraction of universe.

➔ *causally connected fraction of universe is constantly growing.*

The Horizon Problem

- Cosmic Microwave Background is smooth to about 1 part in 10^5
- Yet regions in causal contact at time of decoupling should subtend only ~2° on sky.
- How do regions 180° apart know about each other?

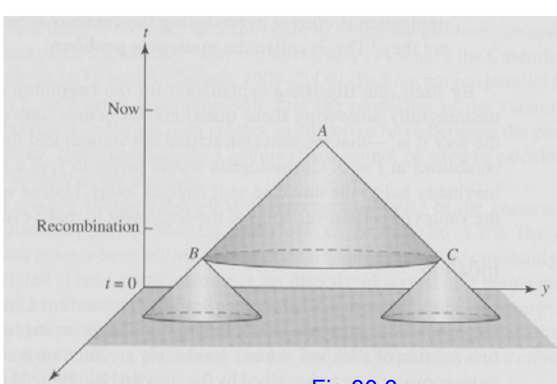
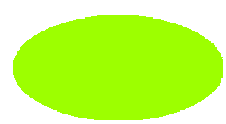
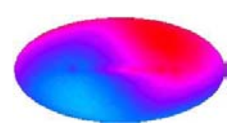


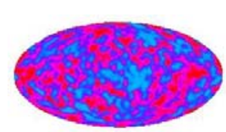
Fig 30.3



Blue = 0°K
Red = 4°K



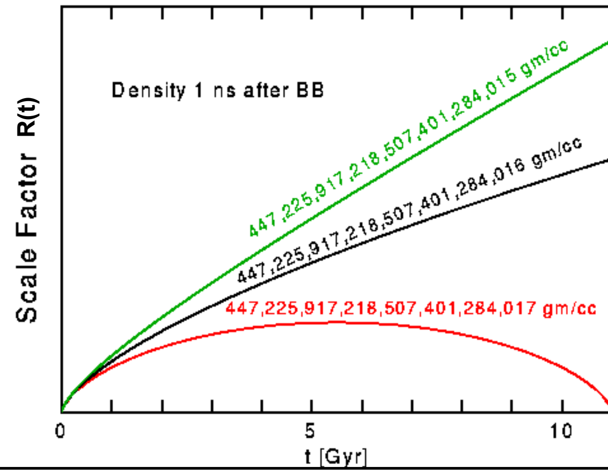
Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
~ 1 part in 300



After removing dipole
Red – blue = 0.0002°K
~ 1 part in 10^5

The Flatness Problem

- Tiny departures from ($\rho = \rho_c$) at small *t* (large *z*) grow into much larger departures than are observed.
- Ω_0 close to 1 at present time.
  - But this requires incredible precision at start (t = 0).
  - ➔ Ω_0 *exactly* = 1



The solution: Inflation

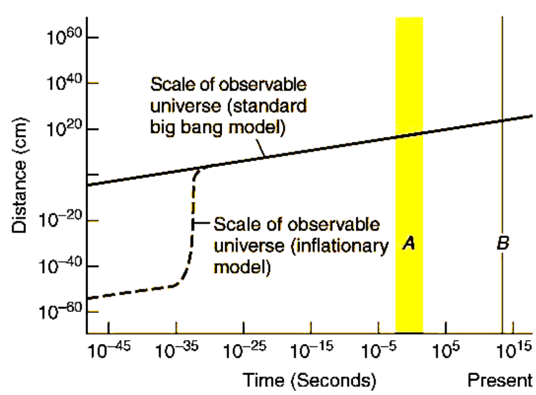
(probably)

(maybe)

Extremely rapid expansion of universe

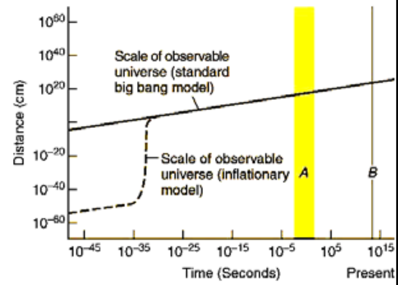
- due to release of energy in "phase change".

Universe became $\sim e^{100} \sim 10^{43}$ times larger within 10^{-34} seconds.



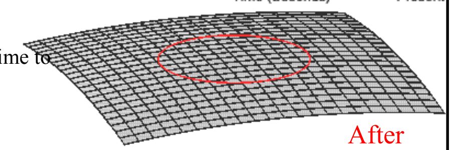
What does inflation predict for geometry of present universe?

Universe became $\sim e^{100} \sim 10^{43}$ times larger within 10^{-34} seconds.



- Predicts a flat universe
  - $\Omega_0 = 1.000000....$
  - As far out as we can see
    - red circle = horizon

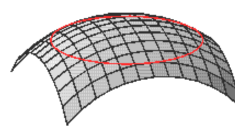
    = most distant place from which light has had time to travel.
- Solves flatness and horizon problems.



After Inflation

Inflation of universe = 10^{43}

$$\frac{\text{Milky Way Disk}}{\text{electron}} = 10^{36}$$



Before Inflation