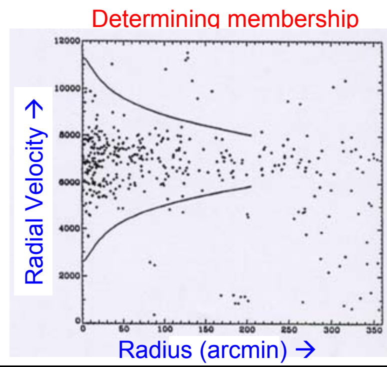
Virial Theorem for Clusters

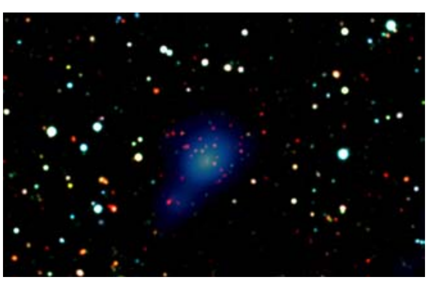
- Galaxy clusters – "fair samples" of the universe.
- Coma is closest relaxed cluster
- Original mass measurement was by Zwicky (1933).

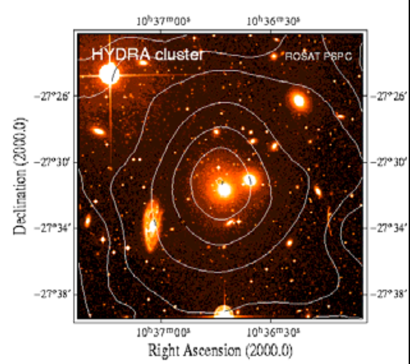
$$M = \frac{5\sigma_r^2 R}{G} = 3\times10^{15} M_\odot$$

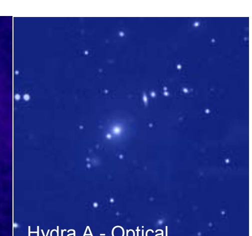
- Measure $n(r)$, $\sigma_v(r)$
  $n(r)$ = # of galaxies,
  $\sigma_v(r)$ = vel. Dispersion
- Fit to models based on
  collisionless Boltzmann eq.
  ~ isothermal, non-spherical.
- Coma: $M = 2\times10^{15}\,M_\odot$
  $M/L = 360h$ (+0, -180h)
- Perseus: $M/L = 600h$



X-ray emitting gas in clusters

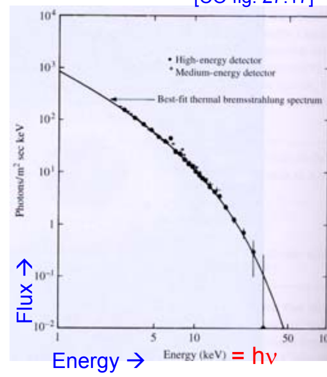






X-ray emitting gas in clusters

[CO fig. 27.17]

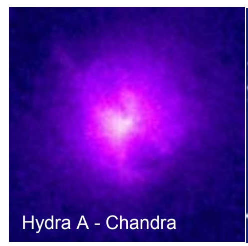


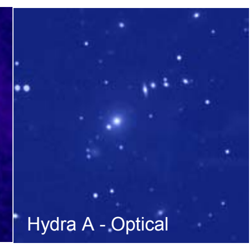
- **gas is important mass component of cluster**
  - emission by thermal bremsstrahlung (free-free).
  - $L_X \sim 10^{43}$ - 10^{45} erg/s (5x10^{44} erg/s for Coma)

$$\ell_\nu \, d\nu = 5.44 \times 10^{-52} \overbrace{(4\pi n_e^2)}^{\text{amplitude}} T^{-1/2} \overbrace{e^{-h\nu/kT}}^{\text{freq. distr.}} d\nu \text{ W m}^{-3}$$

T ~ 10^7 K. *Why?*

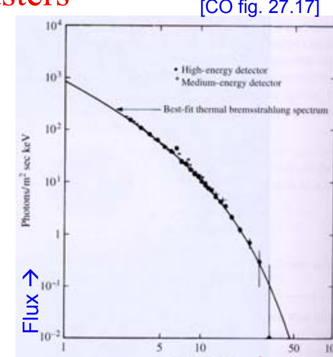
- Heated by shocks: infall, radio jets, SNe, etc.
- Temp. set by (heating rate) = (cooling rate).
- Cooling rate depends on $n_e n_p T^{-1/2}$
  - low density ➔ high T


Hydra A - Chandra


Hydra A - Optical

X-ray emitting gas in clusters

[CO fig. 27.17]



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$$L_{total} = \frac{4}{3}\pi R^3 \int \ell_\nu d\nu = 1.42 \times 10^{-40} n_e^2 T^{1/2} \text{ W m}^{-2}$$

[CO eq. 27.19]

Measure L_X, R, T
Solve for n_e = electron density (electrons m^{-3})
= H nuclei m^{-3}

Mass = $n_e \times m_H \times$ *volume*

- M_{gas} = (4/3) $\pi R^3 \, n_e m_H$ = 3x10^{14} $M_\odot$
- M_{stars} = $(M/L)_{Local} \, L_V$ = 2x10^{13} $M_\odot$

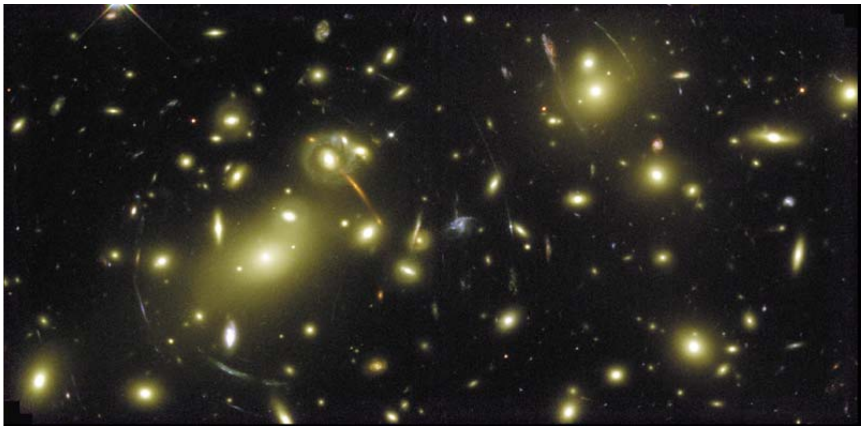
10x more baryons in hot intergalactic gas than in stars

But still factor of ~10 short…


Hydra A - Chandra


Hydra A - Optical

Gravitational Lensing



- Foreground cluster distorts images of numerous background galaxies.
- Use to determine total mass of foreground cluster.
- Shows that 85% of mass is Dark Matter.

Gravitational Lensing

The Schwarzschild metric:

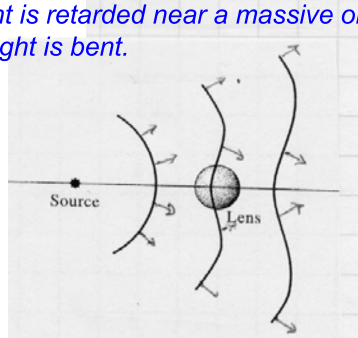
$$(ds)^2 = \left(c\,dt\sqrt{1-2GM/rc^2}\right)^2 - \left(\frac{dr}{\sqrt{1-2GM/rc^2}}\right)^2 - (r\,d\theta)^2 - (r\sin\theta\,d\phi)^2$$

For light: $ds = 0$

$$\frac{dr}{dt} = c\left(1-\frac{2GM}{rc^2}\right) \qquad \text{[CO 17.28]}$$

➔ *Wavefront is retarded near a massive object.*

➔ *path of light is bent.*



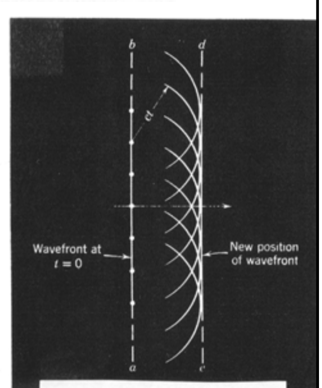
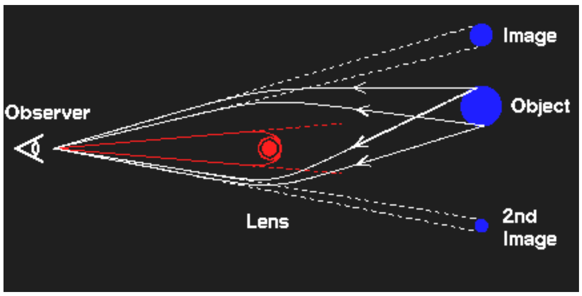
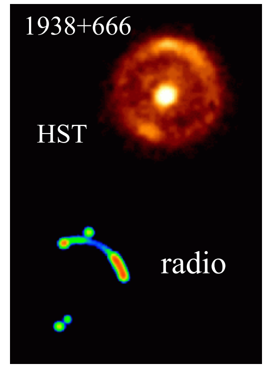
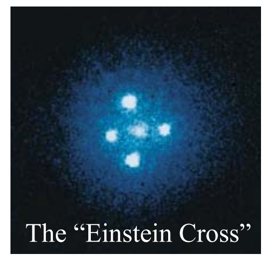


Fig. 41–7 The propagation of a plane wave in free space is described by the Huygens construction. Note that the ray (horizontal arrow) representing the wave is perpendicular to the wavefronts.

Gravitational Lenses



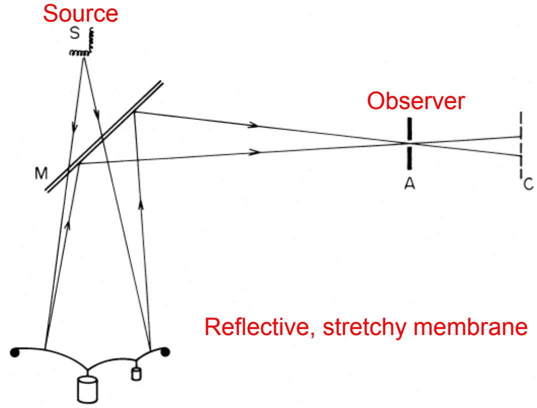




Galaxy at center causes 4 images of same quasar.

Gravitational Lens Simulator

Blandford & Narayan 1986 ApJ, 310, 568



Gravitational Lensing by a Point Mass

[CO Sect. 28.4]

Angle of deflection of photon: $\phi = \frac{4GM}{r_0 c^2}$ (28.20) ← From Schw. Metric.

$\Rightarrow \theta^2 - \beta\theta - \frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right) = 0$ (from trig) (28.21)

The Quadratic Eqn.

$ax^2 + bx + c = 0$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic eq. in $\theta \Rightarrow$ 2 solutions θ_1, θ_2

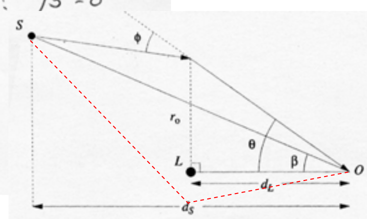
$\beta = \theta_1 + \theta_2$

$M = -\frac{\theta_1 \theta_2 c^2}{4G}\left(\frac{d_S d_L}{d_S - d_L}\right)$

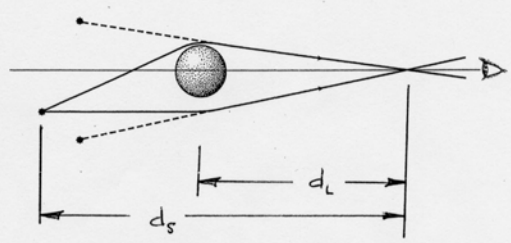
If lens is exactly on line of sight to source: $\beta = 0$

image is Einstein Ring

(28.24) with $\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)}$



Point mass forms two images (or ring)



– For sun: rays intersect at $d_L \sim 50$ ly

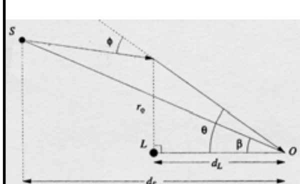
- For $d_S >> d_L$

$$\theta_E = \sqrt{\frac{4GM}{c^2}\left(\frac{d_S - d_L}{d_S d_L}\right)} = \sqrt{\frac{4GM}{c^2 d_L}}$$

- For stars in Milky Way:
  – $M = 1\ M_{sun}$, $d_L = 10^4$ ly ==> $\theta_E \sim 2 \times 10^{-3}$ arcsec

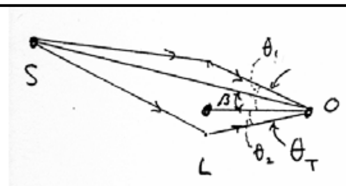
- For external galaxies
  – $M = 10^{11}\ M_{sun}$, $d_L = 10^{10}$ ly ==> $\theta_E \sim 1$ arcsec

- Need $\beta < \theta_E$ to see multiple images (strong lensing)



Effect of Lensing on Flux

See Refsdal (1964) MNRAS 128, 295



Use [CO] notation, but also define

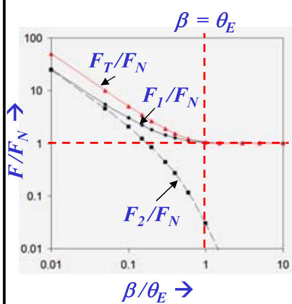
$\theta_T = |\theta_1| + |\theta_2|$ = total separation between images

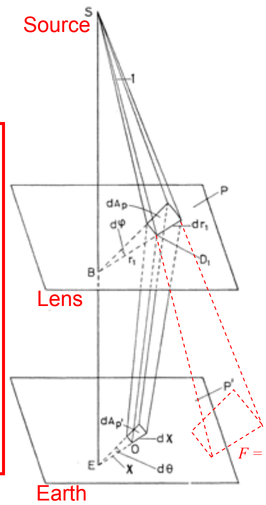
$$\theta_T = \sqrt{\theta_E^2 + \beta^2}$$

$$F_1 = \frac{1}{4}\left(2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

$$F_2 = \frac{1}{4}\left(-2 + \frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right) F_N$$

$$F_{TOTAL} = F_1 + F_2 = \frac{1}{2}\left[\frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right] F_N$$

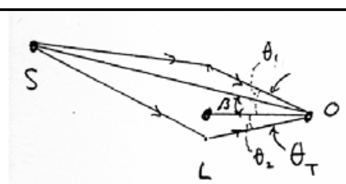




	β/θ_E	F_1/F_N	F_2/F_N	F_T/F_N
Not aligned	10	1·0000063	0·0000063	1·000013
	5	1·0001	0·0001	1·0002
	3	1·0014	0·0014	1·0028
	1·5	1·0084	0·0084	1·017
$\beta = \theta_E$	1	1·030	0·030	1·06
	0·6	1·116	0·116	1·23
	0·4	1·27	0·27	1·54
	0·3	1·44	0·44	1·88
	0·2	1·83	0·83	2·66
	0·15	2·23	1·23	3·46
	0·1	3·04	2·04	5·08
Close	0·05	5·52	4·52	10·0
alignment	0·01	25·5	24·5	50·0

Effect of Lensing on Flux

See Refsdal (1964) MNRAS 128, 295



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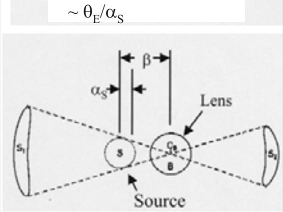
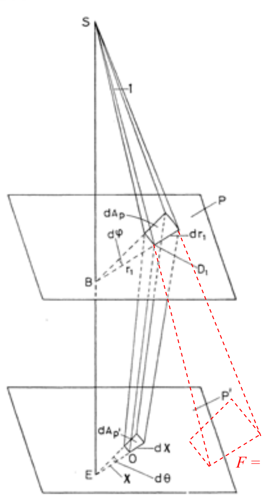
$$F_{TOTAL} = F_1 + F_2 = \frac{1}{2}\left[\frac{\theta_T}{\beta} + \frac{\beta}{\theta_T}\right] F_N$$

Lensing of Extended Sources

- Image has same surface brightness as unlensed image, but more area.
- Ring if $\beta < \alpha_S$
- Arcs if $\beta > \alpha_S$
- Max amplification when $\beta = 0$ $\sim \theta_E/\alpha_S$





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	3	1·0014	0·0014	1·0028
	1·5	1·0084	0·0084	1·017
$\beta = \theta_E$	1	1·030	0·030	1·06
	0·6	1·116	0·116	1·23
	0·4	1·27	0·27	1·54
	0·3	1·44	0·44	1·88
	0·2	1·83	0·83	2·66
	0·15	2·23	1·23	3·46
	0·1	3·04	2·04	5·08
Close	0·05	5·52	4·52	10·0
alignment	0·01	25·5	24·5	50·0