Please open the exam when I give the word. There are 200 points available; your score will be your points divided by 200. There are various extra credit points available, but they tend to be harder.

Be sure to write out your algebra before hitting the calculator!

If you're stuck, still write out what you *can* show. Then go on to other parts.

If you are missing a previous answer for a further step, define a symbol for that answer, and calculate the next step *symbolically* in terms of the previous result.

Use the back sides of sheets for scratch paper

Problem #1 [20 max]
Problem #2 [20 max]
Problem #3 [20 max]
Problem #4 [20 max]
Problem #5 [25 max]
Problem #6 [40 max]
Problem #7 [35 max]
Problem #8 [20 max]
Problem #9 [10 EC]
Problem #10[25 EC]

Total ____[200 max]

Useful Constants: $a_0 = 5.29 \text{ E-11 m}$ $\alpha = e^2/4\pi\epsilon_0 \hbar c = .0072874 \sim 1/137$ c = 2.9979 E8 m/s 1 calorie = 4.186 Je = 1.60 E-19 C $1/4\pi\epsilon_0 = 8.99 \text{ E9 Nm}^2/\text{C}^2$ $e^{2}/4\pi\epsilon_{0} = 1.44 \text{ E-9 eV} \cdot \text{m}$ $E_{o}(H) = -13.6 \text{ eV}$ $G = 5.67 \text{ E} \cdot 11 \text{ N} \text{ m}^2 \text{ kg}^{-2}$ $h = 6.63 \text{ E-}34 \text{ J.s} = 4.14 \text{ E-}15 \text{ eV} \cdot \text{s}$ $\hbar = h/2\pi = 1.05 \text{ E-34 J.s} = 6.58 \text{ E-16 eV} \cdot \text{s}$ $hc = 1240 \text{ eV} \cdot nm$ $k_{\rm B} = 1.38 \text{ E-}23 \text{ J/K}$ $\lambda_{c} = h/m_{e}c = 2.42 \text{ E-12 m}$ $m_{\alpha} = 3727.4 \text{ MeV/c}^2 = 4.00151 \text{ u}$ $m_e = 9.1094 \text{ E-}31 \text{ kg} = 511.00 \text{ keV/c}^2 = 5.49\text{E-}4 \text{ u}$ $m_p = 1.6726 \text{ E-}27 \text{ kg} = 938.27 \text{ MeV/c}^2 = 1.00728 \text{ u}$ Msun = 2 E30 kg $\mu_{\rm B} = 5.79 \text{E-5 eV/T}$ $N_A = 6.02 \text{ E}23 \text{ mol}^{-1}$ R = 8.31 J/(K mole) $R_{\infty} = 1.09737 \text{ E7 m}^{-1}$ Rsun = 7 E8 m $\sigma=5.67~E\text{-}8~W/m^2K^4$ $u = 1.66E-27 \text{ kg} = 931.5 \text{ MeV/c}^2$

1. [20] Identify the particle, \mathbf{x} , in the following nuclear interactions:

a.
$$\alpha + {}^{9}\text{Be}_{4} \rightarrow {}^{12}\text{C}_{6} + \mathbf{x}$$

b. $\alpha + {}^{197}\text{Au}_{79} \rightarrow {}^{200}\text{Hg}_{80} + \mathbf{x}$
c. $n + {}^{30}\text{Si}_{14} \rightarrow {}^{31}\text{P}_{15} + \mathbf{x}$
d. $p + {}^{7}\text{Li}_{3} \rightarrow \mathbf{x} + \mathbf{x}$

- 2. Radiocarbon Dating
 - a. [10] When a living body is in equilibrium with the atmosphere, approximately 1 in 10^{12} Carbon atoms are actually ¹⁴C, which has a half-life of 5730 years = 1.8 10^{11} s. For a sample recently cut from a tree, containing1 gram of carbon, calculate D, the number of radioactive decays per second.
 - b. [10] An old sample containing 1g of Carbon is found to have 0.01 decay per second. What is the age of this sample? *Hint*: To get an answer in years, either use the approximation $1y = \pi 10^7$ s, or calculate the decay constant in years.

- 3. To expose photographic film, photons of light disassociate silver bromide (AgBr) molecules. This disassociation requires an energy of 1.2 eV.
 - a. [10] What limitations does this impose on the wavelengths of light that may be recorded by photographic film?
 - b. [5] To increase the range of wavelengths recorded, how should the disassociation energy be changed?
 - c. [5] Could photographic film be exposed if it was bombarded by intense radio waves? Why or why not? What about by X-rays? Why or why not?

- 4. Consider an atom of atomic hydrogen.
 - a. [10] If an atom is in the second excited state, draw a energy level diagram indicating which de-excitation transitions are possible.

- b. [10] List the photon energies for each allowed transition.
- c. Extra Credit [5]: What photon energy (or energies) could ionize an atom in the second excited state?
- 5. We now consider the element Scandium (Sc), which has Z = 21.
 - a. [10] First, find the ground state electronic configuration $(1s^22s^2...)$ for Scandium. The Noble Gas which is at the end of the row before Sc is Ar, which has a configuration of $(1s^22s^22p^63s^23p^6)$. Sc is one of those elements for which a higher n *s* state has lower energy than a lower n, but higher *l* state. So the outer shells for Sc, above those of Ar, are $4s^23x^y$. What are *x* and *y*?
 - b. [5] These values of x and y define the overall angular momentum and spin quantum numbers for Sc (that is, the inner shells contribute 0 to the overall L and S). Using your values for x and y from part a), what are L and S for the Sc ground state?
 - c. [5] This atom has a spin-orbit interaction. Again using your x and y from part a), what are the possible values of J for Sc?
 - d. [5] Finally, again your x and y from part a), what is the lowest energy J state for Sc?

6. Analyze the thermodynamic cycle *a b c* below, performed on one mole of a monoatomic gas (Cv = 3 R/2). The drawing is not quite to scale, but each part of the cycle is a straight line.



In this cycle, let the starting conditions (pressure, volume and temperature) at point *a* be (*p*, *v*, and *t*), with t = 100K. Let the pressure at *c* be p/2, and the volume at *c* be 3v.

Write the quantities below in terms of R t, or a number of kelvins times R (eg 100 R).

- a. [10] Find the temperature at *b* and *c*.
- b. [5] Find the Carnot efficiency of the cycle
- c. [5] Find the work done during *ab*.
- d. [5] Find the work done during *bc*.
- e. [5] Find the work done during the whole cycle.
- f. [5] Find the heat supplied to the gas during the *ab* part of the cycle
- g. [5] Find the efficiency of the cycle
- h. Extra Credit[10] : Find the heat transfer along ca (say whether heat is gained or lost)

7. The Sun.

a. [5] To a reasonable approximation, the sun consists of pure hydrogen. Calculate Np, the number of protons in the sun.

- b. [10] The basic nuclear interaction on the sun is fusion, which in net changes 4 protons into an alpha particle. Calculate the Q value for this fusion process, in MeV. *Note:* here we *ignore* a small correction due positron annihilation, needed to balance charge.
- c. [5] Calculate W, the total amount of fusion energy available to the sun, in Joules, assuming all protons are fused to helium nuclei.
- d. [10] The temperature of the surface of the sun is 5788 K. Find P, the power emitted by the sun. *Hint*: the surface of a sphere is $4\pi R^2$.
- e. [5] Assuming the sun emits power at a constant rate P, calculate the lifetime of the sun (in years) with fusion energy as the power supply. *Hint: 1 year is roughly* $\pi 10^7$ *sec.* Comment: this lifetime is too long, as stars consume only ~10% of their hydrogen "fuel".
- f. Extra Credit [10] Compare this with the time it takes to emit all the thermal energy $(U = 3/5 \text{ G M}^2/\text{R})$ which initially heated the sun during its gravitational collapse from a pre-solar cloud. Why were 1800's physicists were disturbed by this result?

- 8. Neutrons can arrive at earth as cosmic rays. Suppose a neutron were emitted from the center of the Andromeda galaxy, 2 million light years away, with a kinetic energy of 10^{20} eV (about the maximum energy observed for cosmic rays). The neutron has a lifetime of approximately 900 seconds, or tn = 2.8 10^{-6} years. You can use 1 GeV/c² for the neutron mass.
 - *a*. [10] How long does the trip from Andromeda to Earth take, measured in neutron lifetimes (that is, T/tn) ?
 - b. [10] Explain your reasoning from the point of view of an observer riding with the neutron, **and** of an observer on earth, showing that you get the same result.

Extra Credit[5]: What fractions of neutrons survive this journey?

- 9. [Extra Credit] [10] Consider heat flow through pieces of metal of cross sectional area A and heat conductivity k, with a temperature difference ΔT between the source of heat and the heat absorber. Find the ratio of power conducted when connecting the heat source and absorber by the two pieces of metal side by side, compared with connecting them end to end.
- 10. [Extra Credit] The obscure sport of quantum ping pong is played on a square table 10nm on a side. A common strategy is a "quantum knuckleball": serve the "ball" (a hydrogen atom) slowly enough that its wavelength is of the same size as the width of the playing field, rendering it extremely difficult to hit back.
 - a. Extra Credit [10] Calculate the speed of such a serve, and the time it takes to cross the length of the table.
 - b. Extra Credit [5] Estimate the fractional uncertainty of the momentum, assuming that the position uncertainty is given by $\lambda/2$.
 - c. Extra Credit: [5] Calculate the quantum *kinetic* energy of this atom in its ground state, assuming it is confined to the 10 nm x 10 nm table by an infinite potential.
 - d. Extra Credit: [5] Find the kinetic energy expected for a temperature of T=10 K.