Nuclear Reactions

Thornton and Rex, Ch. 13
Reaction Kinematics

Consider a general reaction,

\[ A (x, y) B \quad \text{or} \quad A + x \rightarrow y + B \]

with target \( A \) at rest.

Ex. \( ^9\text{Be}_4 + ^4\text{n} \rightarrow ^1\text{n}_0 + ^{12}\text{C}_6 \)

or equivalently,

\( ^9\text{Be}_4 (\text{n},n) ^{12}\text{C}_6 \)

Conservation of energy gives:

\[
M_A c^2 + m_x c^2 + K_x = m_y c^2 + K_y + M_B c^2 + K_B
\]

The difference between final and initial kinetic energies is called the \textbf{Q-value}.

\[
Q = K_y + K_B - K_x = M_A c^2 + m_x c^2 - (m_y c^2 + M_B c^2)
\]
If Energy is released, $Q > 0$

- **Exothermic**

If Energy is converted to mass, $Q < 0$

- **Endothermic**

Two of the most important exothermic reactions are **Fission** and **Fusion**.
Neutron Activation

Neutrons - uncharged, can penetrate close to the nucleus, can induce reactions (Neutron Activation).

1930’s - Enrico Fermi bombarded elements from Hydrogen to Uranium with neutrons. On Uranium, Fermi was unable to identify the final products of the reactions. He expected elements heavier than Uranium, such as in the expected process:

\[
^{238}U_{92} + ^1n_0 \rightarrow ^{239}U_{92} \rightarrow ^{239}Np_{93} + ^0e_{-1} + \square
\]
Otto Hahn and the chemist Fritz Strassmann eventually identified one of the product nuclei as Barium ($^{141}\text{Ba}_{56}$), an element much lighter than Uranium.

1938 - Meitner and Frisch gave an interpretation of the reaction and coined the term Nuclear Fission.

They used a model (due to Bohr and others) in which the nucleus acts like a liquid drop with surface tension.
Comments on Fission

• The observed reaction is one of many possible reactions:

\[ ^{235}\text{U}_{92} + {}^1\text{n}_0 \rightarrow ^{141}\text{Ba}_{56} + ^{92}\text{Kr}_{36} + 3\ {}^1\text{n}_0 \]

• \(^{235}\text{U}_{92}\) undergoes fission. \(^{238}\text{U}_{92}\) does not. Uranium ore contains 99.3\% \text{U}\,-238 and only 0.7\% \text{U}\,-235.

• Fission occurs more easily if the neutron is slow (allowing more time for the reaction to occur.)
• Q > 0, so energy released (~200 MeV). (Cf. Binding Energy vs. Atomic Weight)

• Since heavier nuclei are more neutron rich, the fission process results in the release of extra neutrons. (Cf. plot of N vs. Z)
• Under the right conditions, the extra neutrons could cause more U-235 to fission. This would release even more neutrons, etc., resulting in a chain reaction.

(The idea of chain reaction was patented by Leo Szilard in 1933, before he had any idea what nuclei might participate!)
Two technical problems that had to be solved in order to achieve a chain reaction in Uranium:

1) Fission occurs if U-235 captures a slow neutron, but the neutrons emitted in fission are fast.

The device must contain a substance which slows the neutrons down. A “moderator” is an element whose nuclei don’t absorb neutrons and which are relatively light so that in collisions with neutrons they will absorb energy and thus slow the neutrons down. Typical moderators are water, heavy water (D$_2$O), or Carbon (graphite).

2) The sample must be enriched with enough U-235 relative to U-238.
An additional technical problem must be overcome to achieve controlled nuclear fission, as for power generation.

A variable amount of an additional material that is highly efficient in capturing neutrons must be inserted in the Uranium. => Cadmium rods

The first self-sustaining nuclear reactor, using graphite as moderator, was built under the stands of the football stadium at the University of Chicago by Enrico Fermi on December 2, 1942.
<table>
<thead>
<tr>
<th>What</th>
<th>Amount density</th>
<th>Mass</th>
<th>Energy (J)</th>
<th>MJ/m³</th>
<th>% Mc²</th>
<th>MJ/kg</th>
<th>1000 MW annual</th>
<th>1000 MW daily</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>1kg</td>
<td>1</td>
<td>0.001</td>
<td>3.00E+07</td>
<td>3.00E+10</td>
<td>3.3E-08</td>
<td>30.00</td>
<td>8.0E+06</td>
<td>2.78E+09</td>
</tr>
<tr>
<td>Oil</td>
<td>1 barrel</td>
<td>0.9</td>
<td>0.16</td>
<td>6.00E+09</td>
<td>3.75E+10</td>
<td>4.6E-08</td>
<td>41.67</td>
<td>5.8E+06</td>
<td>2.78E+09</td>
</tr>
<tr>
<td>Natural gas</td>
<td>cu ft</td>
<td>0.8</td>
<td>0.0283</td>
<td>1.00E+06</td>
<td>3.53E+07</td>
<td>5.0E-08</td>
<td>44.64</td>
<td>5.6E+06</td>
<td>2.89E+09</td>
</tr>
<tr>
<td>Wood</td>
<td>1kg</td>
<td>1</td>
<td>0.001</td>
<td>1.00E+07</td>
<td>1.00E+10</td>
<td>1.1E-08</td>
<td>10.00</td>
<td>2.4E+07</td>
<td>2.78E+09</td>
</tr>
<tr>
<td>Gasoline</td>
<td>gallon</td>
<td>0.75</td>
<td>0.003785</td>
<td>1.32E+08</td>
<td>3.49E+10</td>
<td>5.1E-08</td>
<td>46.32</td>
<td>5.2E+06</td>
<td>2.78E+09</td>
</tr>
<tr>
<td>Uranium</td>
<td>1kg</td>
<td>18.9</td>
<td>5.29E-05</td>
<td>1.00E+14</td>
<td>1.89E+18</td>
<td>1.1E-08</td>
<td>1.0E+08</td>
<td>3</td>
<td>3.47E+09</td>
</tr>
</tbody>
</table>

\( c = 3.00E+08 \)
Fusion

Combining smaller nuclei up to Fe increases stability, and releases energy. Also note sharp peak at $^4\text{He}$.
(Cf. Binding Energy vs. Atomic Weight)

Source of energy in the sun.
The predominant process is the pp chain:

1) $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu$

2) $^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \nu$

3) $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$

4) $e^+ + e^- \rightarrow \nu + \nu$
• Net result is conversion of 4 protons and 2 electrons into a $^4$He nucleus and $Q=26.7$ MeV of energy. (reactions 1, 2, and 4 occur twice)

• Most of the energy is in KE of products and in the energy of the photons, and a small amount in the neutrino.

• Photons take millions of years to escape the center of the sun, the neutrino escapes immediately.
• The slowest reaction in the chain is step 1, which keeps the sun burning for a long time (~10 billion years). After that it will collapse further, heat up and begin to build heavier elements, up to Fe.

• All elements on earth (up to Fe) were originally built in earlier generations of stars.

• There are other chains, including the CNO cycle, which uses Carbon to catalyse Fusion. It is more important in massive stars.
Fusion on earth

A hope for virtually unlimited source of clean energy in Fusion reactions, such as:

\[ {}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + {}^1\text{n} \]

which releases 17.6 MeV of energy.

The difficulty of controlled fusion is the large energies required by the nuclei to overcome the Coulomb force and react.

The main effort has been to use a hot ionized gas called a **plasma**. The nuclei are kept at high energy and compressed together by large electric and magnetic fields. The magnets are usually in the form of a toroid (donut), called a **Tokamak**.
At present no reactor has yet produced more energy from fusion than was required to produce and contain the plasma.

Another scheme is to use powerful lasers to irradiate tiny pellets of deuterium ($^2\text{H}$) and tritium ($^3\text{H}$) from all sides, resulting in their vaporization and compression, again producing fusion.

Estimates for commercial exploitation of fusion reactors range from 30 to 50 years.
Accelerators

Other nuclear reactions are studied by collisions of fast moving particles on a nuclear target.

Accelerators use electric and magnetic fields to give energy to the charged particles.

One type of accelerator is the cyclotron, which is used at the National Superconducting Cyclotron Laboratory (NSCL) here at MSU. Giant magnets contain the ion beams in circles, while electric fields are applied in phase to accelerate the beam.
NSCL coupled cyclotron

- Ions stripped from heavy nuclei

- K500 accelerates up to 20 MeV/nucleon (B=3-5 T)

- Carbon foil strips more ions (2.5 x charge)

- K1200 accelerates up to 200 MeV/nucleon

- Beam hits metal target, making unstable isotopes

- A1900 fragment separator selects particular isotopes

- Isotopes hit target and are studied in various detectors.
• Properties of unstable isotopes are studied. What is origin of elements beyond Iron? (thought to be made in supernova explosion through neutron capture processes).

• General properties of nuclei, such as:

**Compound Nucleus:**  
(excited nuclear resonance in reaction)

\[ p + ^{15}\text{N} \rightarrow ^{16}\text{O}^* \rightarrow p + ^{15}\text{N} \]