Relativity

1905 - Albert Einstein:

- Brownian motion

   - atoms.

- Photoelectric effect.

   - Quantum Theory

- "On the Electrodynamics of Moving Bodies"

   - The Special Theory of Relativity

The Luminiferous Ether

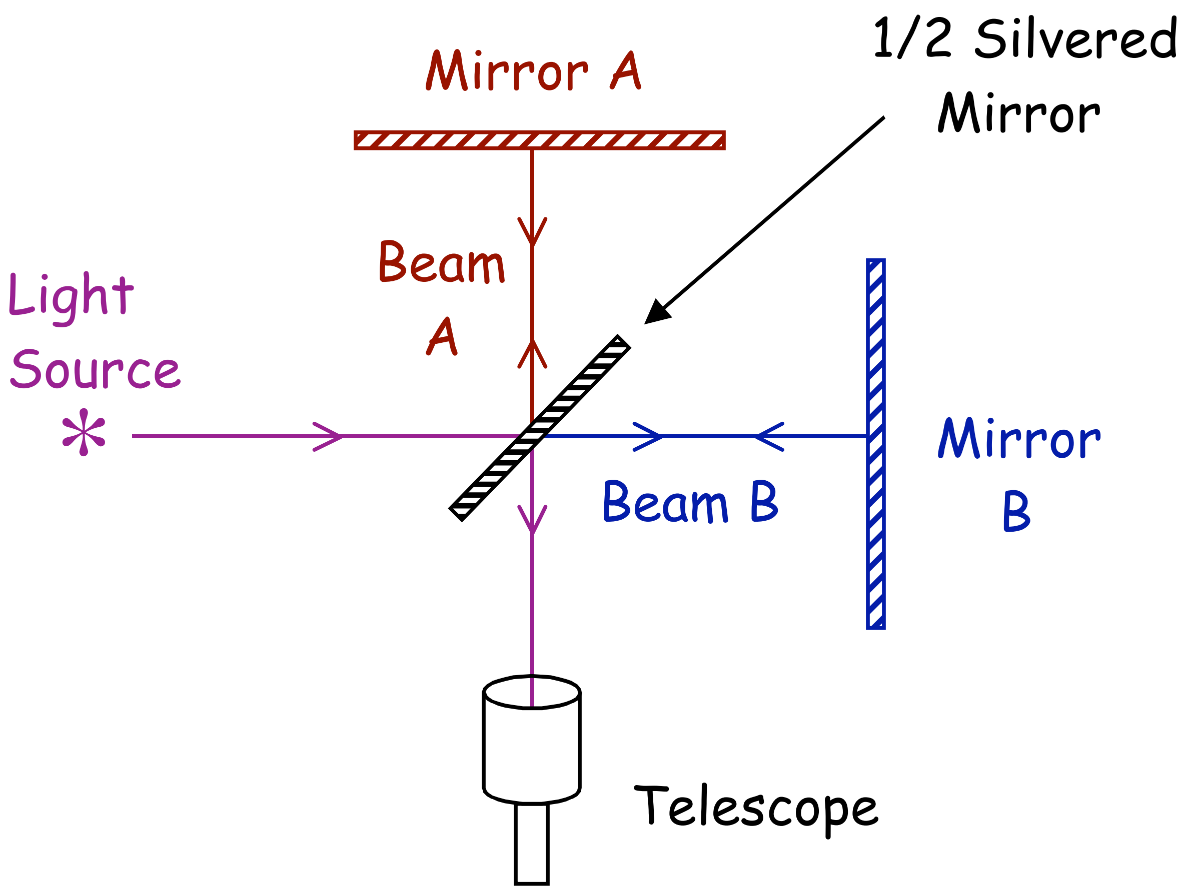
Hypothesis: EM waves (light) travel through some medium - The Ether

Speed of light: $c = 3 \times 10^8$ m/s
w.r.t fixed ether.

The earth moves at $v = 3 \times 10^4$ m/s
w.r.t fixed ether.

 Speed of light w.r.t earth should depend on direction.

The Michelson-Morley Experiment



An interferometer

The interference fringes should shift.

But no effect was observed!

What was wrong?

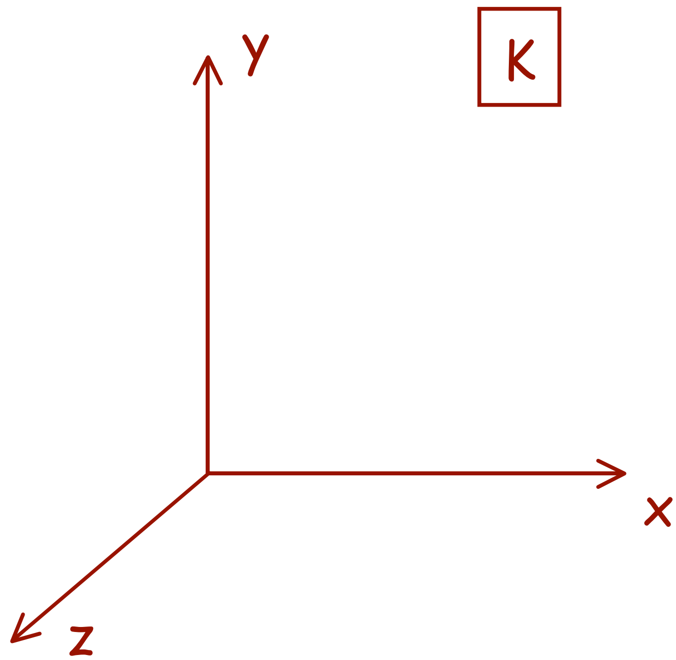
The Lorentz-Fitzgerald Contraction

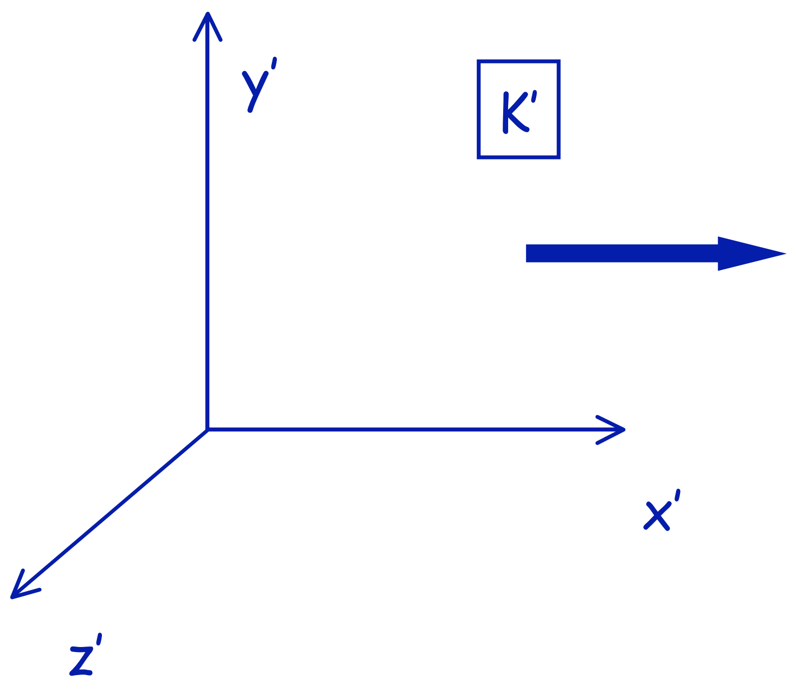
Suppose that the ether squashes any object moving through it?

To counteract the change in light speed, we need:

$$d' = d\sqrt{1 - v^2/c^2}$$

Galilean Transformations.





$$t' = t$$
$$z' = z$$
$$y' = y$$
$$x' = x - vt$$

From Head on Collision to Collision at Rest by changing Frames

Start from known ("Obvious"):
equal-mass head-on elastic collision

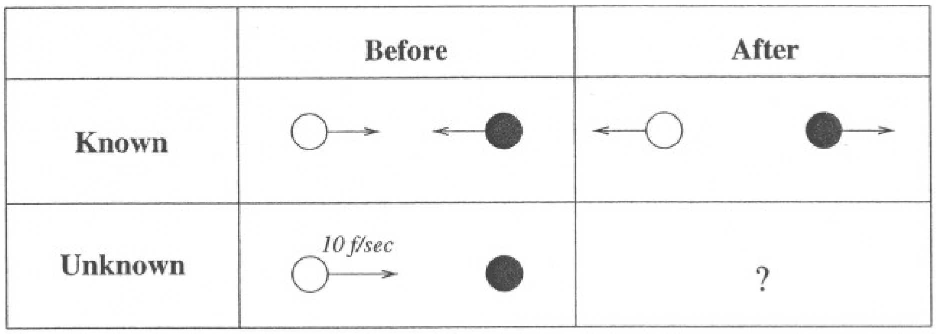


Figure 1.1

Relate to elastic collision with one at rest
View train frame (5f/s right):
transforms into known situation

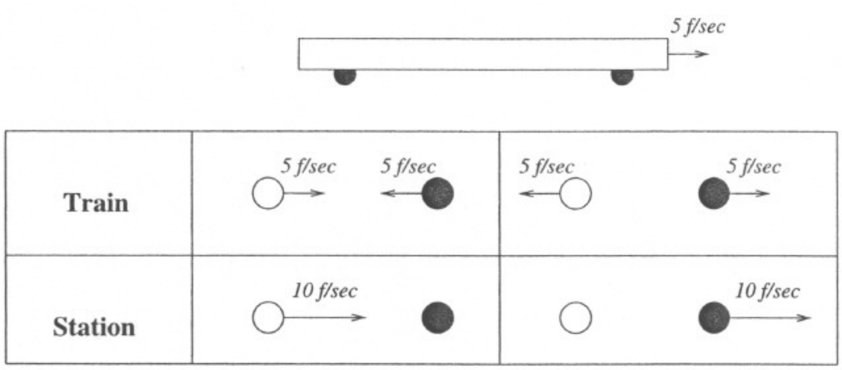


Figure 1.2

Ut = Us – V   (+ u,v to right): Ut = Us - 5

Then transform back to station (5f/s left):

Us = Ut + V: U's = U't + 5

Result: cue ball stops, target ball rolls on

Mermin 2005

Equal-mass totally inelastic collision

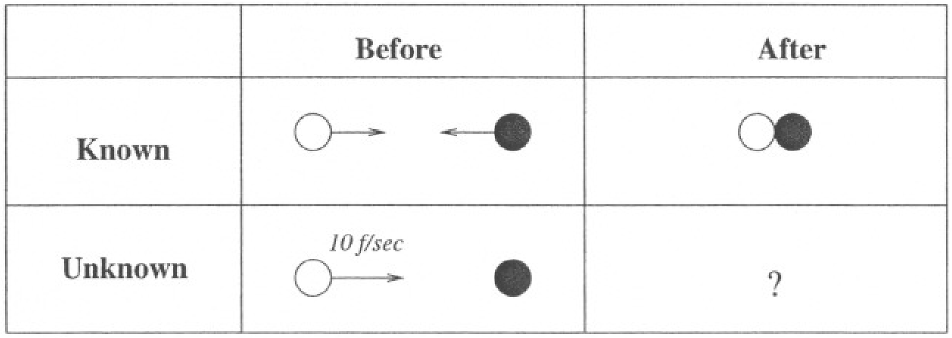


Figure 1.3

Relate to inelastic collision with one at rest

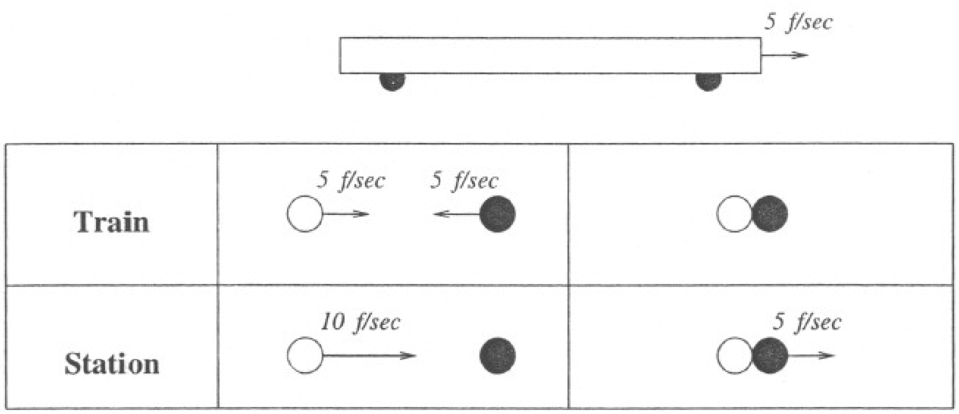


Figure 1.4

$$U_t = U_s - 5$$

$$U'_s = U'_t + 5$$

Result: combined mass moves at half speed of incident

(Very) Asymmetric Elastic Collision

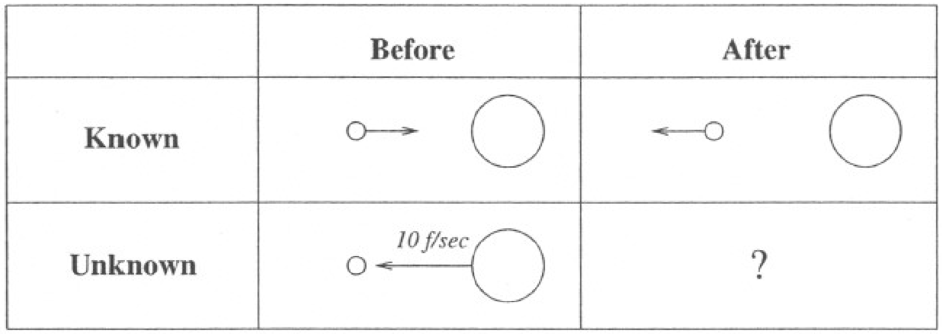


Figure 1.5

Choose Train Frame to put big mass at rest:

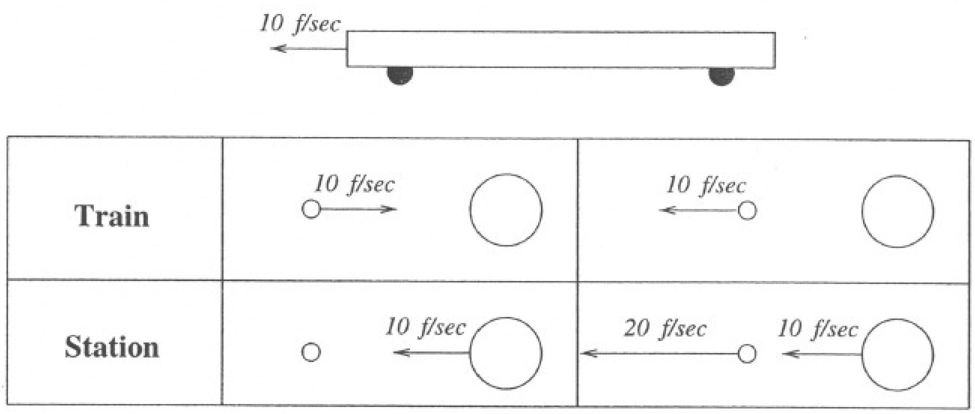


Figure 1.6

$$U_t = U_s - (-10) = U_s + 10$$
$$U'_s = U'_t - 10$$

Result: light ball (nearly) twice speed of heavy ball; heavy ball (nearly) unaffected

Finally, use to solve for
Asymmetric head-on Elastic Collision

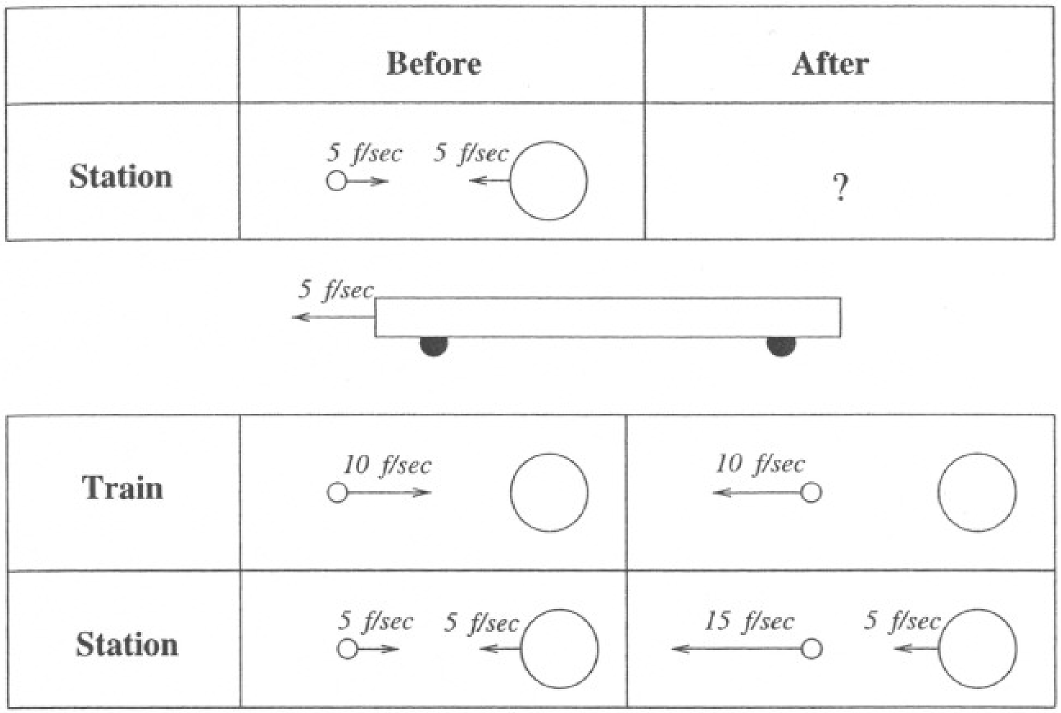


Figure 1.7

Again Choose Train Frame to put big mass at rest:

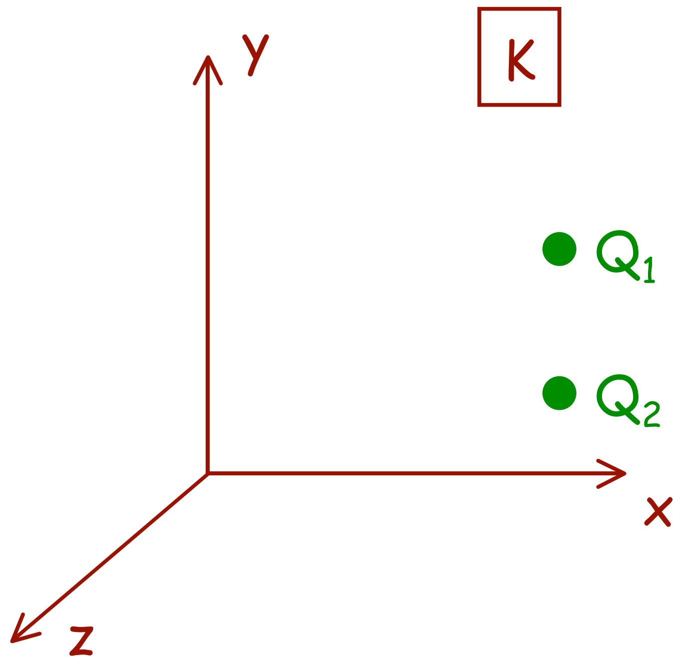
$$U_t = U_s + 5$$
$$U'_s = U'_t - 5$$

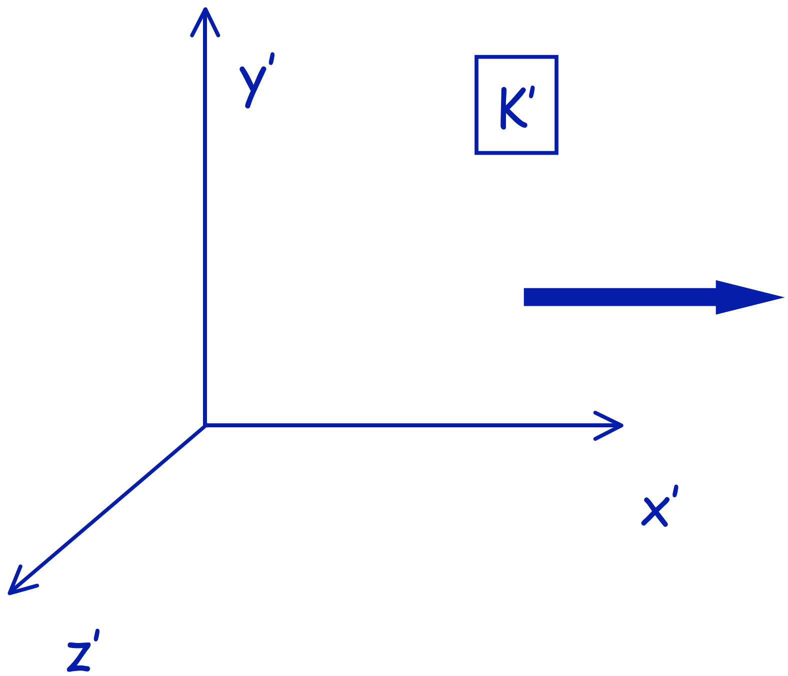
Result: light ball (nearly) three times speed of heavy ball;
heavy ball (nearly) unaffected
Example: drop tennis ball on top of basketball
rebound matches this situation

Lessons from changing frames:

- An exercise in Gallilean transform for velocities
- Analysis from the simplest point of view
- Well-chosen transformation can give non-trivial results

In frame K, two charges at rest. Force is given by Coulomb's law.





In moving frame K', two charges are moving.
Since moving charges are currents,
Force is Coulomb + Magnetism.

Principle of relativity:

"The laws of nature are the same in all inertial reference frames"

Something is wrong!

- Maxwell's Equations?
- The Principle of Relativity?
- Gallilean Transformations?

Einstein decided

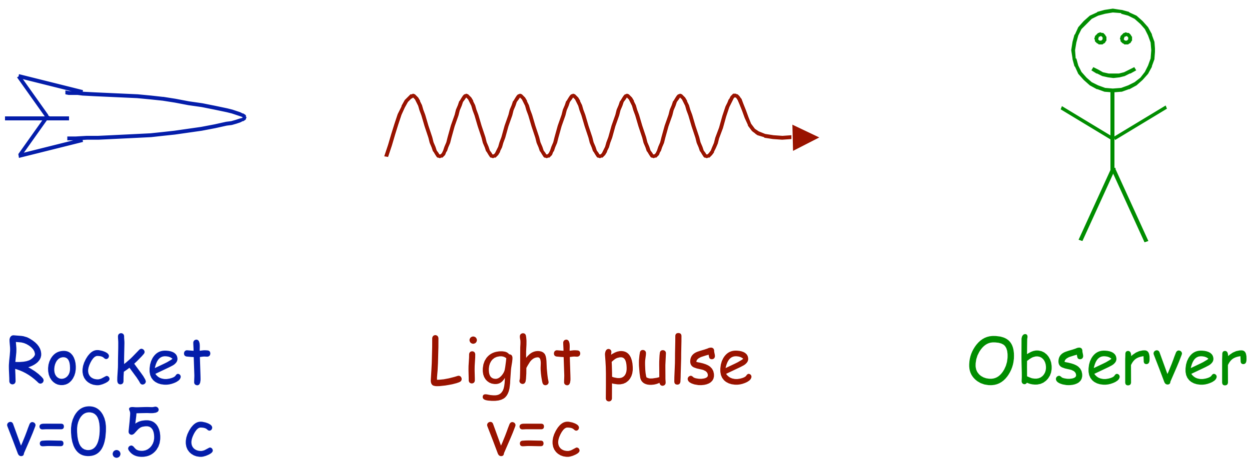
- Galilean Transformations are the problem.

Einstein's two postulates:

1. The principle of relativity is correct. The laws of physics are the same in all inertial reference frames.

2. The speed of light in vacuum is the same in all inertial reference frames ($c = 3 \times 10^8$ m/s regardless of motion of the source or observer).

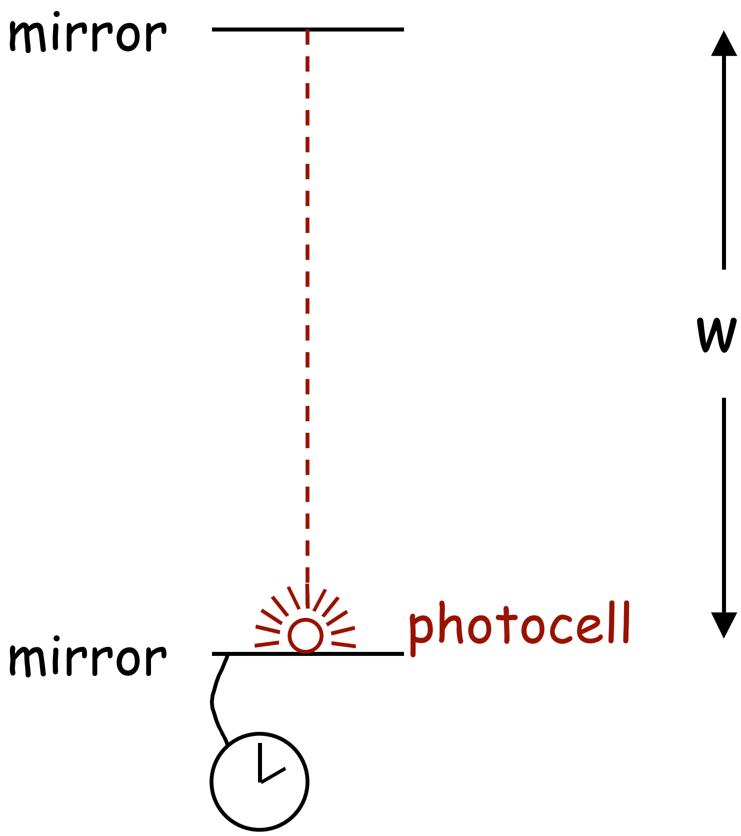
The second postulate seems to violate everyday common sense!



Einstein says: observer measures the light as traveling at speed c, not 1.5c.

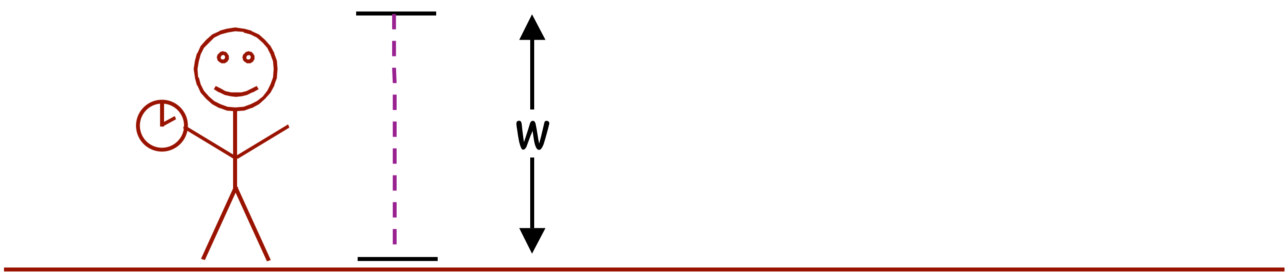
Gedanken Experiments

A light clock:

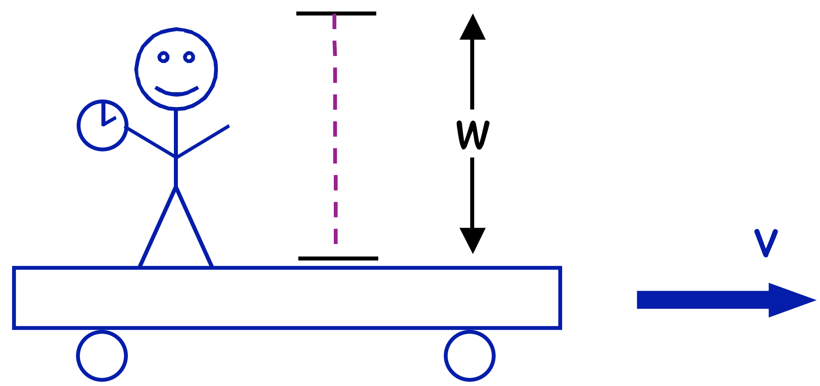


It ticks every $\Delta t = 2\, w/c$ seconds.
One can synchronize ordinary clocks with it.

Time Dilation



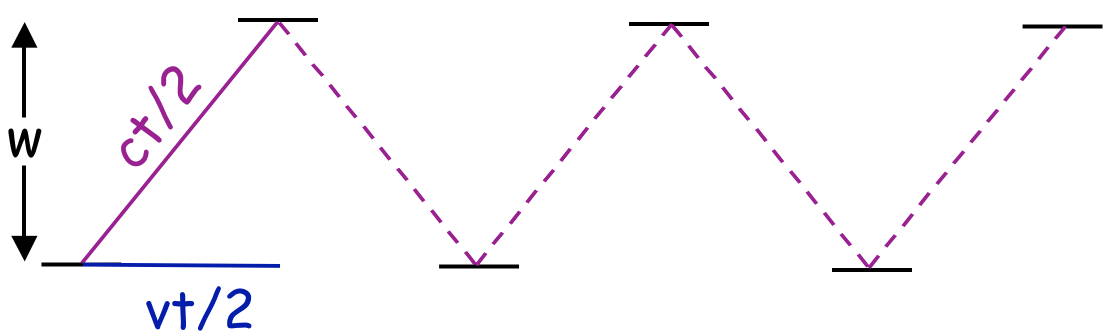
O_G: Observer on Ground



O_T: Observer on Truck

O_T's clock as seen from the ground:

$$c = 3 \times 10^8 \text{ m/s}$$



$$(ct/2)^2 - (vt/2)^2 = w^2$$

Time for one round trip of light, as seen from the ground:

$$t = (2\,w/c) \Big/ \sqrt{1 - v^2/c^2}$$

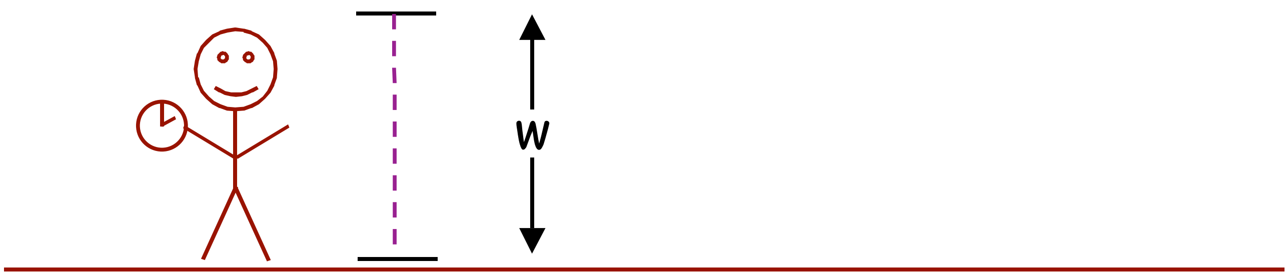
For $v = 0.6c$, $t = (2\,w/c) \times 1.25$

All of O_T's processes slow down compared to O_G as seen by O_G.

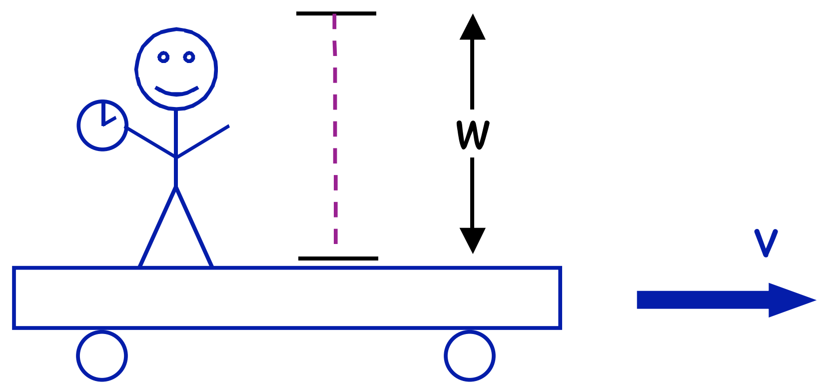
Similarly,

All of O_G's processes slow down compared to O_T as seen by O_T.

Length Contraction



O_G: Observer on Ground



O_T: Observer on Truck

Device on truck makes mark on track each time clock ticks.

As seen from ground:

Distance between marks

= (time between ticks) x v

$$= \left[(2\,w/c)\Big/\sqrt{1 - v^2/c^2}\,\right] v$$

As seen from truck:

Distance between marks

= (time between ticks) x v

= (2 w/c) v

(To the person on the truck the time between ticks is (2 w/c).)

(Distance measured on truck)

$$= \sqrt{1 - v^2/c^2}$$

x (distance measured on ground)

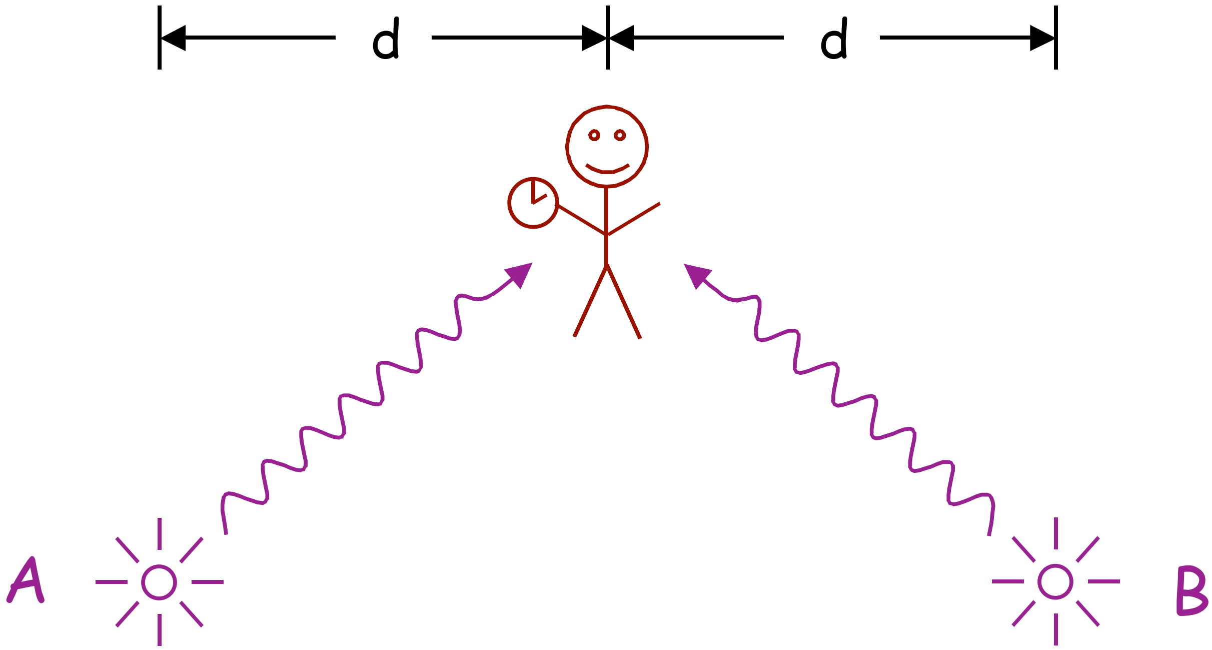
As seen from a moving frame, rest distances contract.

(L-F contraction)

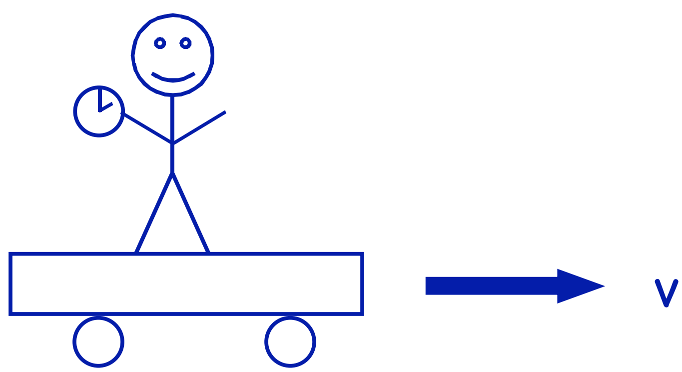
Simultaneity

Events occur at a well defined position and a time (x,y,z,t).

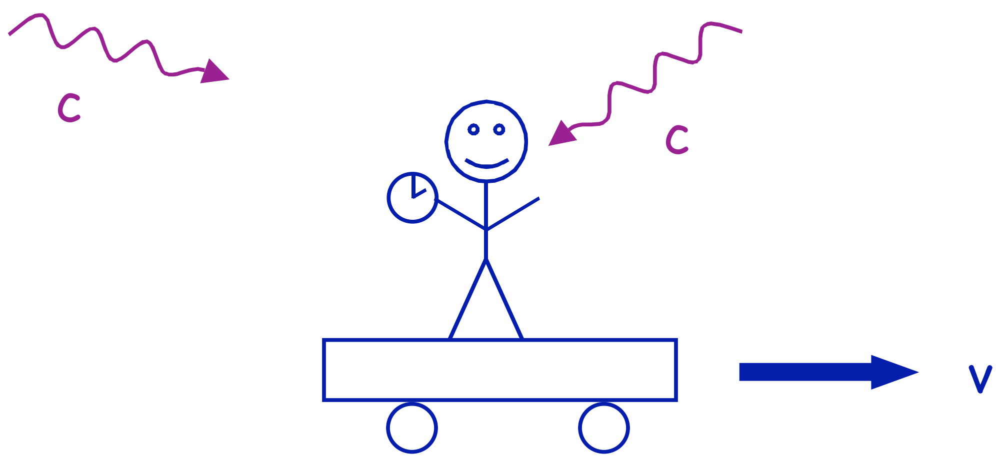
But events that are simultaneous (same t) in one inertial frame are not necessarily simultaneous in another frame.



The light from the two flashes reach O_G at the same time. He sees them as simultaneous.

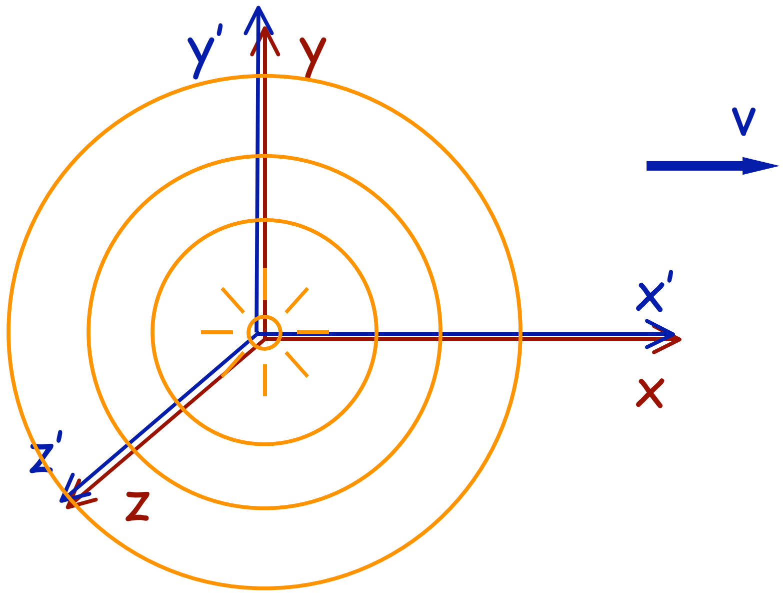


O_T passes O_G just as the lights flash.



But light from B reaches O_T first. Since both light beams started the same distance from her, and both travel at speed c, she concludes that B must have flashed before A.

Lorentz Transformations



- Flashbulb at origin just as both axes are coincident.
- Wavefronts in both systems must be spherical:

$$x^2 + y^2 + z^2 = c^2t^2 \quad \text{and}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2$$

- Inconsistent with a Galilean transformation
- Also cannot assume $t=t'$.

Assuming:

- Principle of relativity
- linear transformation $(x,y,z,t) \rightarrow (x',y',z',t')$

Lorentz Transformations (section 2.4)

$$x' = \gamma (x - v t)$$

$$y' = y$$

$$z' = z$$

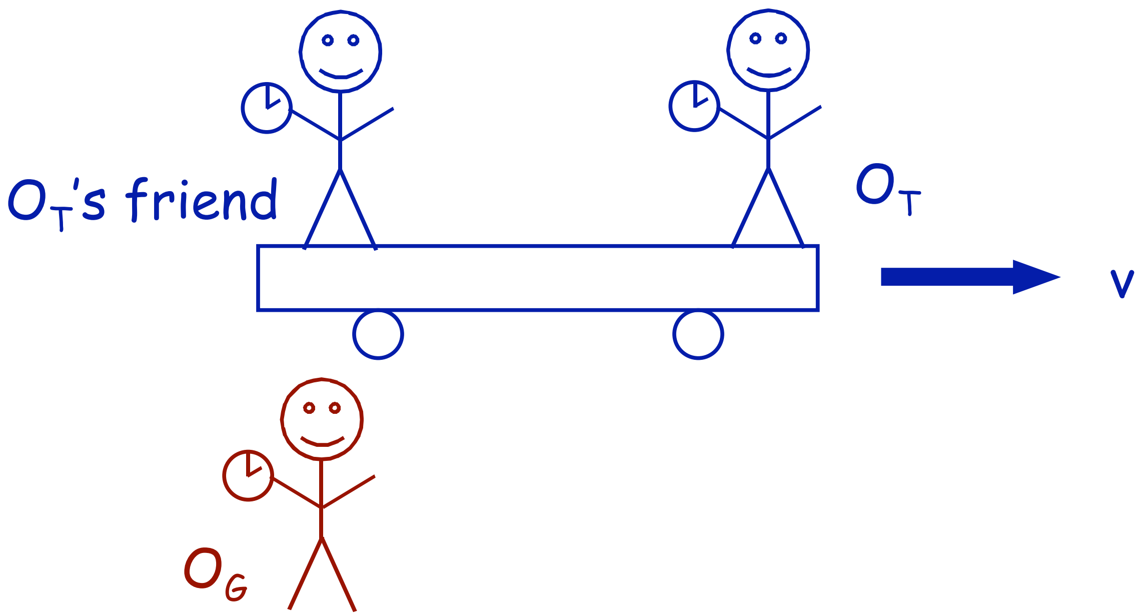
$$t' = \gamma (t - v x / c^2)$$

With $\gamma = 1 / \sqrt{1 - v^2/c^2}$.

(Often also define $\beta = v / c$.)

Time Dilation (again)

Proper time: time T_0 measured between two events at the same position in an inertial frame.



O_G's clock: $T_0 = t_2 - t_1,$ $(x_2 - x_1 = 0)$

O_T's clock: $T' = t'_2 - t'_1$

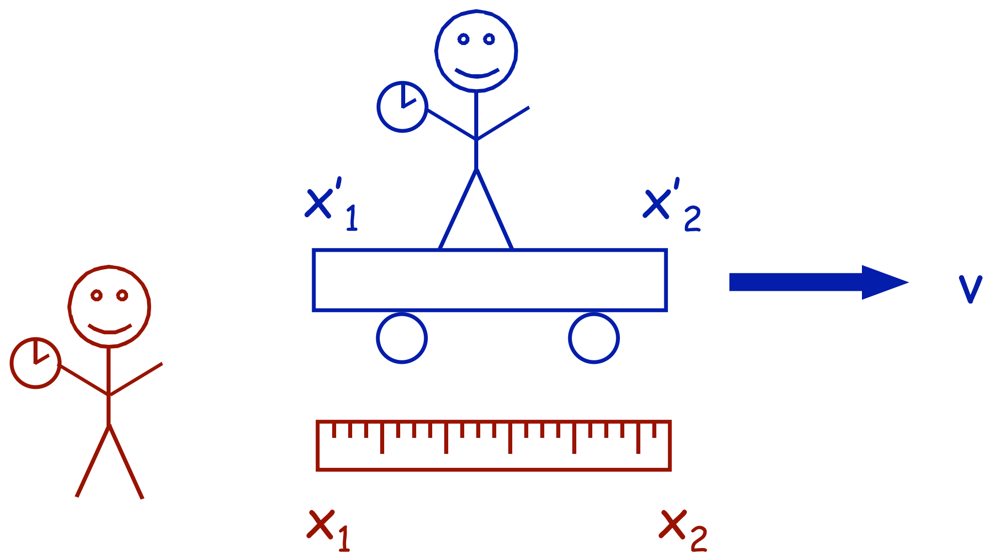
$$t'_2 - t'_1 = \square(t_2 - t_1 - v/c^2\,(x_2 - x_1))$$

$$T' = \square\, T_0 > T_0$$

Clocks, as seen by observers moving at a relative velocity, run slow.

Length Contraction (again)

Proper length: distance L_0 between points that are at rest in an inertial frame.



O_T on truck measures its length to be $L_0 = x'_2 - x'_1$. This is its proper length.
O_G on ground measures its length to be $L = x_2 - x_1$, using a meter stick at rest ($t_2 = t_1$).

Then

$$L_0 = x'_2 - x'_1 = \gamma(x_2 - x_1 - v\,(t_2 - t_1))$$
$$= \gamma L$$

O_G measures $L = L_0/\gamma < L_0$.

Truck appears contracted to O_G.

An application

Muon decays with the formula:

$$N = N_0 \, e^{-t/\tau}$$

N_0 = number of muons at time t=0.
N = number of muons at time t seconds later.
τ= 2.19 x 10^{-6} seconds is mean lifetime of muon.

Suppose 1000 muons start at top of mountain d=2000 m high and travel at speed v=0.98c towards the ground. What is the expected number that reach earth?

Time to reach earth:
$t = d/v = 2000\text{m}/(0.98 \times 3 \times 10^8 \text{ m/s})$
$= 6.8 \times 10^{-6}$ s

Expect $N = 1000 \, e^{-6.8/2.19} = 45$ muons.

But experimentally we see 540 muons!
What did we do wrong?

Time dilation: The moving muon's internal clock runs slow. It has only gone through

$$t' = 6.8 \times 10^{-6} \sqrt{1 - 0.98^2} \text{ s}$$
$$= 1.35 \times 10^{-6} \text{ s}$$

So $N = 1000\ e^{-1.35/2.19} = 540$ muons survive.

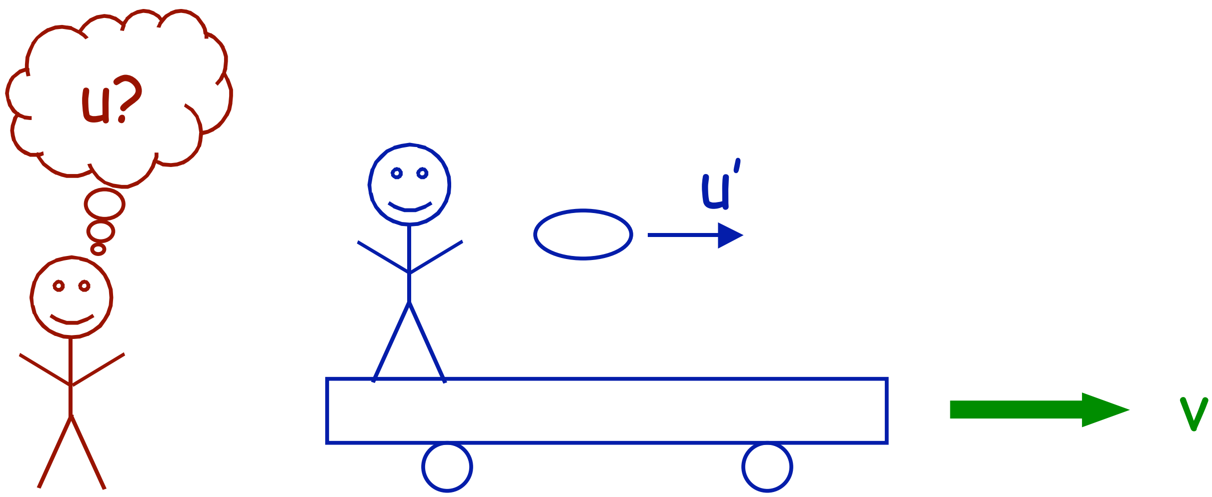
Alternate explanation: From muon's viewpoint, the mountain is contracted. Get same result.

Addition of velocities

Galilean formula ($u=u'+v$) is wrong.

Consider object, velocity u' as seen in frame of O_T who is on a truck moving with velocity v w.r.t the ground.

What is velocity u of the object as measured by O_G on the ground?



Recall $u = \Delta x/\Delta t$, $u' = \Delta x'/\Delta t'$.
Inverse Lorentz transformation formulae:

$$\Delta x = \gamma(\Delta x' + v \Delta t')$$

$$\Delta t = \gamma(\Delta t' + v \Delta x' / c^2)$$

$$u = \frac{\Delta x}{\Delta t} = \frac{\gamma(\Delta x' + v\,\Delta t')}{\gamma(\Delta t' + v\,\Delta x' / c^2)}$$

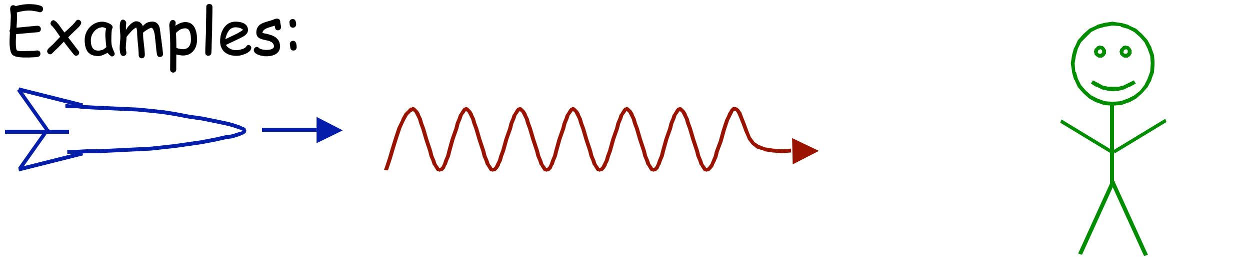
$$u = \frac{u' + v}{1 + v\,u'/c^2}$$

For u' and v much less than c:

$$u \approx u' + v$$

Velocities in y and z directions are also modified (due to $t' \neq t$, see section 2.6)

Examples:



Rocket
$v=0.5\,c$

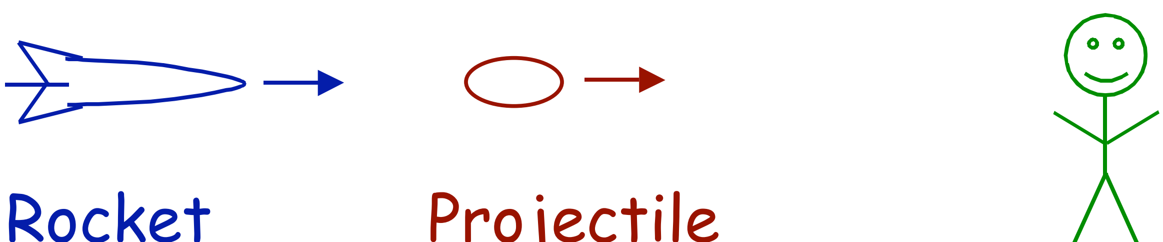
Light pulse
$u'=c$

Observer

Observer sees light move at

$$u = \frac{0.5c + c}{1+(0.5c)(c)/c^2} = c$$

Light moves at $c=3\times10^8$ m/s <u>in all frames</u>.



Rocket
$v=0.8\,c$

Projectile
$u'=0.5c$

Observer

Observer sees projectile move at

$$u = \frac{0.5c + 0.8c}{1+(0.5)(0.8)} = 0.93c$$

Massive objects always move at speeds $< c$.

The Twin Paradox

Suppose there are two twins, Henry and Albert. Henry takes a rocket ship, going near the speed of light, to a nearby star, and then returns. Albert stays at home on earth.

Albert says that Henry's clocks are running slow, so that when Henry returns he will still be young, whereas Albert is an old man.

But Henry could just as well say that Albert is the one moving rapidly, so Albert should be younger after Henry returns!

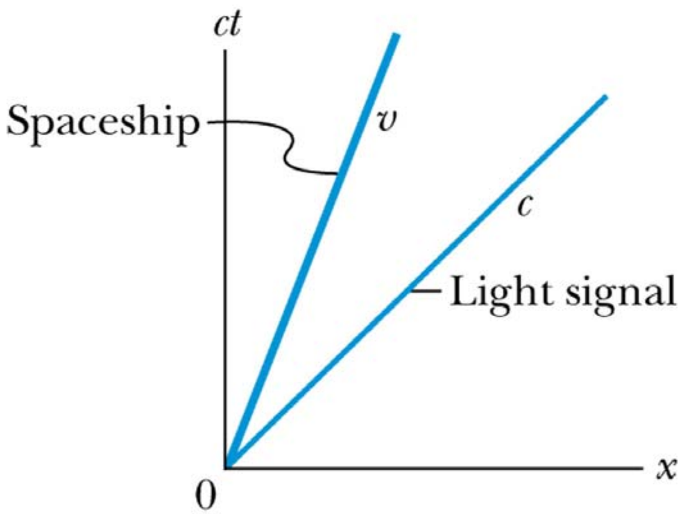
Who is right?

The first scenario is the correct one.

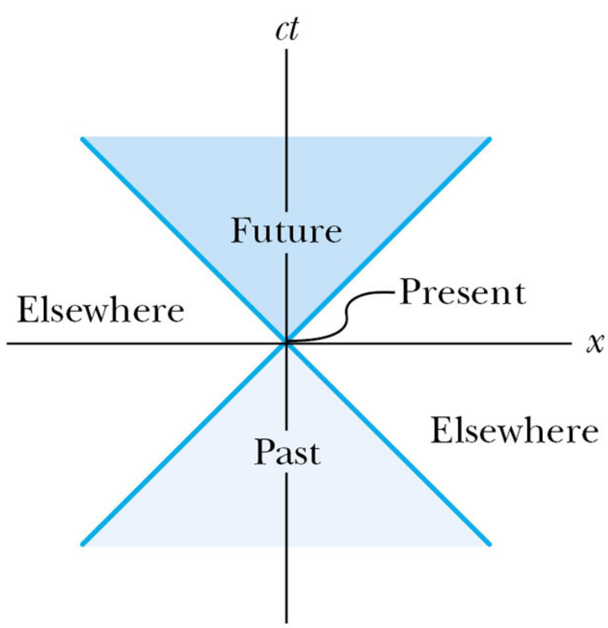
The situation is not symmetric, because the rocket has to decelerate, turn around and accelerate again to return to earth. Thus, Henry is not in an inertial frame throughout the trip. He does return younger than Albert.

Spacetime Diagrams: Minkowski

Put axes in same units:
x in, say, light minutes or lightyears



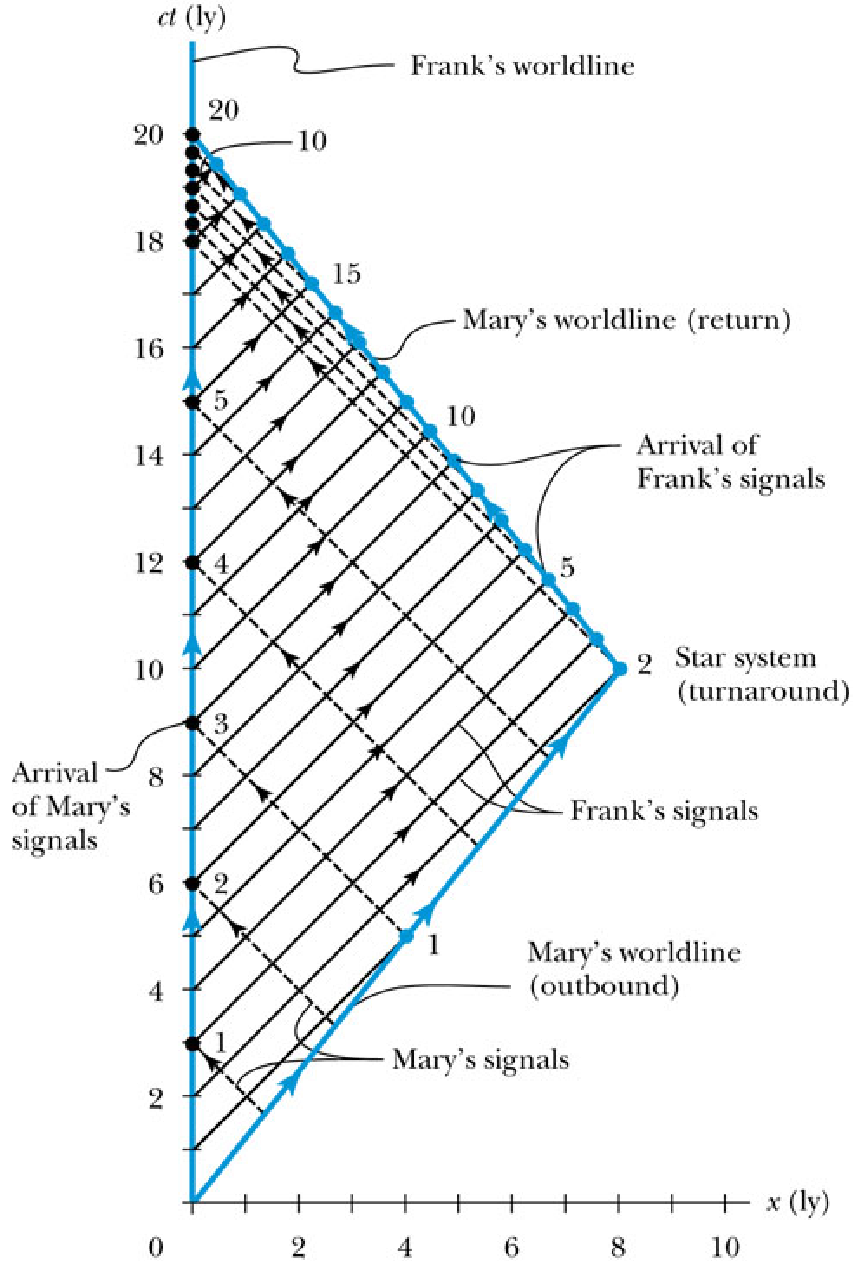
Slope higher for slower rocket, 45 degrees for light



(a)

"Past" can get a signal to Present, but "Elsewhere" can't
(light is too slow)
Present can affect future, but not "Elsewhere"

The Twin Paradox: two inertial frames!



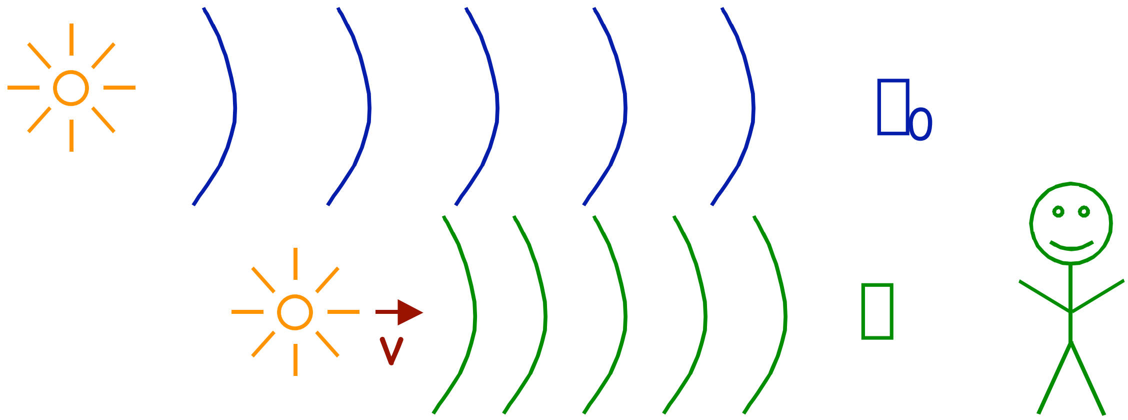
$v = .8\ c$, $\gamma = 1/.6$

Frank: T = 2 * 8ly /.8c = 2 * 10 = 20 y

Mary: T = 2 * 10 y/ γ = 2 * 6y = 12 y

Relativistic Doppler Effect

Light source and observer approach each other with relative velocity, v.
Light is emitted at frequency ν_0.



Observer sees light at a higher frequency:

$$\nu = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\,\nu_0 \quad \text{with } \beta = v/c$$

- If source is receding, the formula still holds but now β is <u>negative.</u>

We know that the universe is expanding, because light from distance galaxies is red-shifted, indicating motion away from us.

Comparison of Relativistic and NR Doppler Effect:

Relativistic (for light), source moving at β

$$\nu = \nu_o \sqrt{ (1+\beta) / (1-\beta) } = \nu_o (1-\beta^2)^{1/2} / (1-\beta)$$

multiply top and bottom by $(1-\beta)$

Nonrelativistic (for light, u=c, source moving at β)

$$\nu = \nu_o \times 1 / (1-\beta)$$

They agree whenever β is small
same lowest order shift—from denominator
numerator is higher order correction:

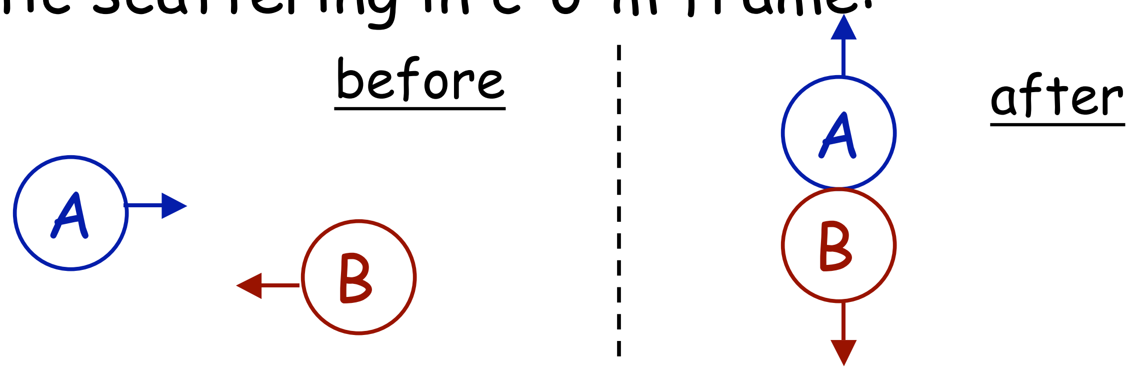
$$(1-\beta^2)^{1/2} \sim 1 - \tfrac{1}{2}\beta^2 \sim 1 \text{ for } \beta \ll 1$$

Remark: $c = \lambda \nu$ Always, both NR and Rel:
It's fundamental to mathematics of waves.

Relativistic Momentum

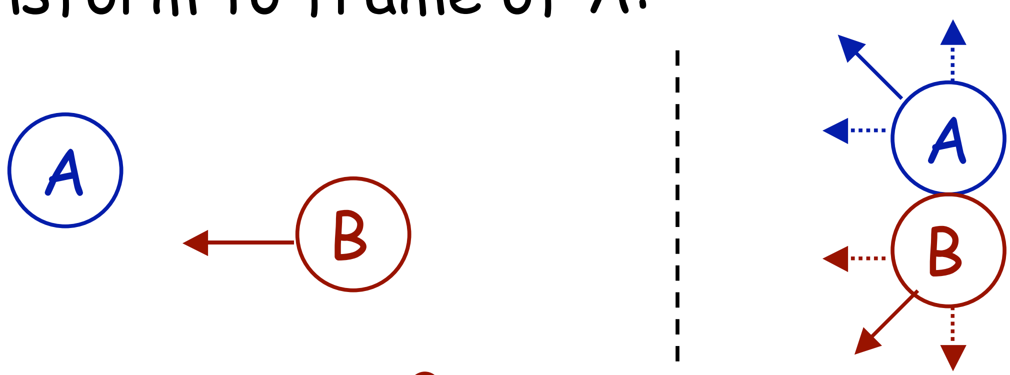
Requirement: momentum is conserved in all inertial frames.
Assume: $\vec{p} = m\,\vec{v}$.

Elastic scattering in c-o-m frame:



$$p_x: \quad mu + m(-u) = 0$$

Transform to frame of *A*:



$$p_x: \quad 0 + m\left(\frac{-2u}{1+u^2/c^2}\right) \neq m(-2u)$$

It doesn't work!

Relativistic momentum:

$$\vec{p} = \gamma m \vec{v} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$$

Relativistic Kinetic Energy:

$$K = (\gamma - 1)\, mc^2$$

$$= \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right) mc^2$$

For small velocities, $v/c \ll 1$:

$$K = (\,1 + 1/2\,(v/c)^2 + \ldots \qquad - 1)\, mc^2$$

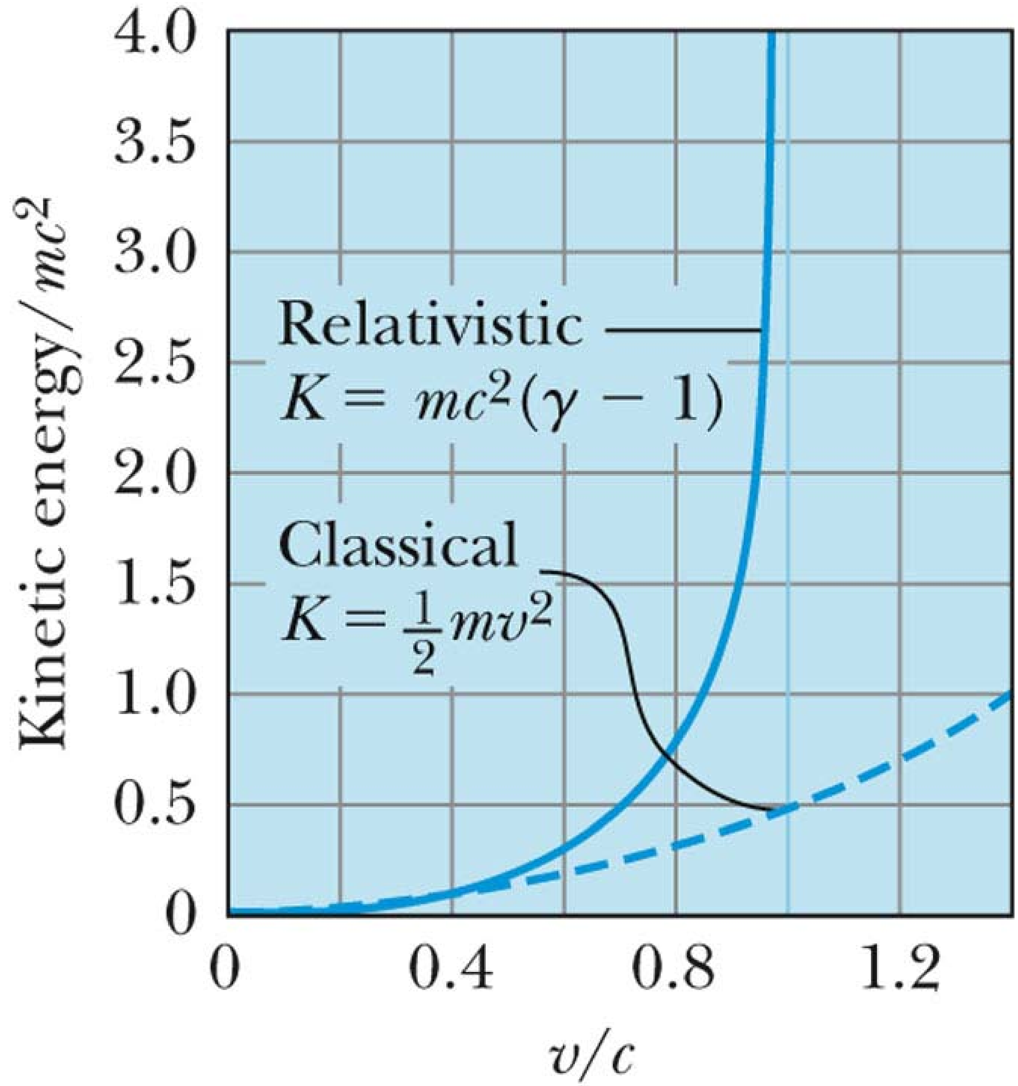
$$\approx 1/2\ m\,v^2$$

For large velocities $v \rightarrow c$:

$$K \longrightarrow$$

Massive objects always travel at speeds less than c.

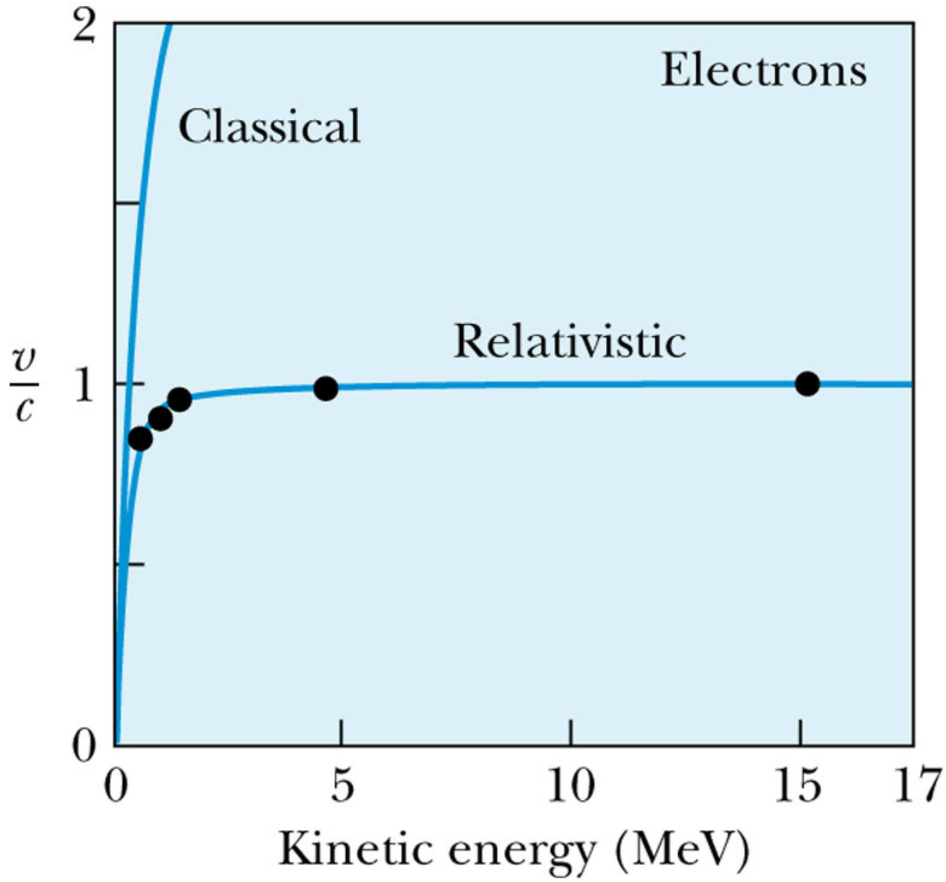
KE and velocity: Relativistic vs. Classical



Noticeable departures for v/c > .4 or so

Starting from v=0, takes *infinite* KE to get to v=c

Velocity nearly stops changing after KE ~ 4 mc^2



$KE = (\gamma - 1)\, mc^2$

$\gamma = 2$ (KE = rest) happens for $\beta = \sqrt{(1 - 1/\gamma^2)} = .87$

electron has $mc^2 = .511$ MeV

Relativistic Energy

According to Einstein, even a mass at rest has energy:

$$E_0 = m\,c^2 \qquad \text{(rest energy)}$$

Thus, the total energy of a moving object is

$$E = K + E_0$$

$$= (\gamma - 1)\,mc^2 + mc^2$$

$$= \gamma mc^2$$

It is straightforward to show:

$$E^2 - p^2c^2 = m^2c^4$$

For a massless particle (e.g. a photon):

$$E = |\vec{p}|\,c$$

In general

$$v = \frac{|\vec{p}|\, c^2}{E} = \frac{\gamma m v\, c^2}{\gamma m c^2}$$

For a massless particle this gives

$$v = c$$

Massless particles travel at the speed of light c.

Conservation Laws and $E = mc^2$

	NR	Relativistic
Mass	Always	Elastic Only
Momentum	Always	Always
Energy	KE: Elastic Only	Always

After relativistic redefinitions

Trade Conservation of Mass (NR)
for Conservation of Relativistic Energy
NR conservation of mass: just a very good approximation

$E = \gamma mc^2$ is a convention, though a very sensible one.
$E = KE + E_{rest}$, $E_{rest} = mc^2$

The physics (the "real" $E = mc^2$) is in
$\Delta E = \Delta mc^2$

Changes in energy show up as immeasurably tiny changes in mass, for everyday cases like heating up an object.

But: if you change mass more substantially,
it releases a LOT of Energy: typically kinetic

Or: use lots of energy (inelastic relativistic collision)
to create new particles (more mass, less KE)

Collisions of equal masses

Calculate either initial KE, or final effective mass

Fixed target, moving projectile

NR result

$$ko = \frac{1}{2}\, m\, u^2$$

$$Kcm = 2 \times \frac{1}{2}\, m\, (u/2)^2 = ko / 2 \quad \text{(linear)}$$

In cm: -u/2, u/2 are velocities, momentum sums to 0

Relativistic result (let c = 1…)

$$M^2 = E_i^2 - p_i^2 \qquad (= E_f^2 - p_f^2 \text{ since relativistic invariant})$$

$$= (e_i + m)^2 - p_i^2 \quad = \quad e_i^2 - p_i^2 + 2\, e_i\, m + m^2$$

Using $e_i^2 - p_i^2 = m^2$

$$M^2 = 2\, m\, (e_i + m)$$

$$M = 2m\{1 + k/2m\}^{1/2}$$

Using $e_i = k + m$

NR: $k \sim ko \ll 2m$, so $M = 2m$ as expected

Relativistic: KE(cm) = M – 2m (conserve E, not KE)

Can swap this KE of initial 2m, for less KE, more mass in final state

Highly Relativisic case: when $k \gg 2m$,

$$M = \sqrt{(2km)}$$

So only grows as square root, not linearly in initial k

Most of initial KE wasted in motion of compound cm object M

If collide equal masses head on, no such waste!

Much more M for similar electricity bill

For k>>m, for each mass:

$$M^2 = E_i^2 - p_i^2 = (2e_i)^2 - 0 \quad \text{(p sums to 0)}$$

$$M = 2\,(k + m) \sim 2k \text{ (linear):} \quad \text{use a collider!}$$

Examples: head on collision of proton on proton (m)

Tevatron (collider): k = 1000 m ;

m = proton mass ~ 1 GeV = 10^9 eV, so k ~ 1 TeV = 10^{12} eV

M = 2000 m

Up to 1000 pairs of proton/antiprotons could be produced

LHC (collider): k = 7000 m ~ 7 TeV

M = 14000 m

Examples of fixed target:

Highest Energy cosmic rays colliding on proton in nucleus of air atom

k = 10^{20} eV = 10^{12} m (~10^8 higher than LHC beam)

$$M = \surd\,(2km) = m\,\surd\,(2 \times 10^{12}) \sim 10^6 \text{ m}$$

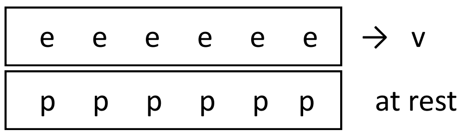
Still ~ 10^2 higher than LHC but nowhere near 10^8

1 LHC beam as fixed target

$$M = \surd\,(2km) = m\,\surd\,(14000) \sim 120 \text{ m} \ll 14000 \text{ m}$$

B and E are Related by Relativity

F Test charge q at rest wrt neutral wire carrying current



Lab frame
= + ion (p) frame

. q

$E=0$, $F_E=0$ $F_B=0$ since $u=0$ in $\underline{F} = q\,(\underline{u} \times \underline{B})$
so $F = 0$

Related situation in the same frame:

M As F, but q moves right at $u = v$
$F_E = 0$ $F_B = xxx$ purely magnetic

M′ SAME as M, but in frame of e (and thus q)
$u' = 0$ so $F_B' = 0$

But: *Lorentz contraction* of charge separation:
$F_E' = xxx$ $F_B' = 0$ purely electric?!

interpretation depends on frame!

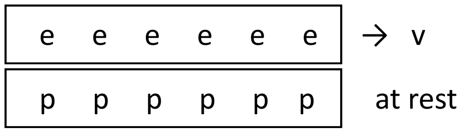
Original situation, different frame:

F′ As F, but e frame
$F_E'=yyy$ $F_B'=-yyy$ so can have $F' = 0$
(Relativity requires!)

Forces in M and M′ same to 10^{-24}

M Charge q moving wrt neutral wire carrying current

q . → u = v

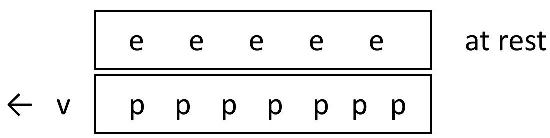


Lab frame
= + ion (p) frame

Charges separated by $d_e = d_p = d$
Net Charge line density $\lambda = e/d_p - e/d_e = 0$ so $F_E=0$
$F_B = q\, v\mu_o i/(2\pi r)$ purely magnetic

M′ As M, but e frame

q .   (at rest) so $F_B=0$



$d_e' = \gamma d_e = \gamma d$ and $d_p' = d_p/\gamma = d/\gamma$

$\lambda' = e/d_p' - e/d_e' = e/d\,(\gamma - 1/\gamma)$ ($\lambda' > 0$!)

$F_E' = q\,\lambda'/(2\varepsilon_o \pi r)$ purely electric

$F_E'/F_B = e/d\,(\gamma - 1/\gamma)\,/v\varepsilon_o\mu_o i$ $i = dQ/dt;\ Q = L\lambda_p;\ v = dL/dt$

So $i = v\,e/d_p = v\,e/d$.   Using $\varepsilon_o\mu_o = 1/c^2$,

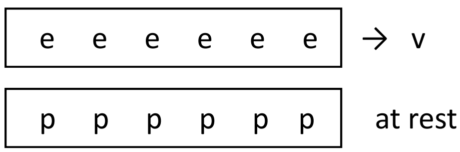
$F_E'/F_B = (1 - 1/\gamma^2)\,/\,\gamma\,\beta^2 = 1/\gamma$

The magnetic field can be interpreted as a residual E field from relativity; B ↔ E under LT
even though $\beta = 10^{-4}/10^8 = 10^{-12}$

The Directions are Also Correct

M In p frame:

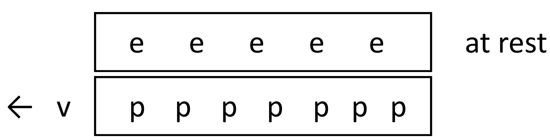
q . → v



I to left B into page F_B is up

M′ In e frame

q . (at rest) so $F_B=0$



$\lambda' > 0$

+ charge excess so F_E is up

Comments

- $\beta = 10^{-12}$ too small to matter? No!

  $\beta^2 = 10^{-24}$ is the fractional charge excess λ' / λ_p'

  Small relativistic effect revealed by a cancellation can become 1st order effect

- Assumed Q a relativistic invariant (like m, c)

  It is: if charge were v-dependent, heated matter would get charge due to $m_e < m_{proton}$

- The factor of γ between F and F′ is how transverse forces transform (complex topic)

  here $\gamma \sim 1 + \beta^2 = 1 + 10^{-24}$

  Caused by time dilation:

  Transverse momenta are *invariant* across frames

  From F = dp/dt, $dp_y = F_y\, dt = F_y'\, dt' = dp_y'$

Simplified from:

Thornton & Rex Modern Physics

Piccioni, Physics Teacher 45 (2007) 152

Feynman Lectures II-13-6