Homework Set 2

Exercises; due Monday 9/14

E4. (a) What is Re $e^{\iota\theta}$? (b) What is Im $e^{\iota\theta}$?

Problems; due Monday 9/14

P6. Phase velocity and group velocity. The frequency of an electromagnetic wave in a wave guide is

$$\omega = \sqrt{\left(\frac{\pi c}{b}\right)^2 + c^2 k^2} \quad \text{(dispersion relation)}$$

where k is the wave vector.
(a) Calculate the phase velocity.
(b) Calculate the group velocity.
(c) Sketch a graph of phase velocity and group velocity versus ω. Your graph should be qualitatively correct, and any special values of ω or v should be labeled on the axes.

P7. Consider a rectangular wave guide whose dimensions are 5 cm by 5 cm. In what part of the electromagnetic spectrum are waves that would propagate in this wave guide? Explain.

P8. Consider a car entering a tunnel of dimensions 15 m wide and 4 m high. Assuming the walls are good conductors, can AM radio waves propagate in the tunnel? Explain.

P9. Metals are opaque to light. Even a thin foil of metal will block light.

Analyze the propagation of an electromagnetic wave in a metal. The field equations are

$$\nabla \cdot \mathbf{E} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{and} \quad \nabla \times \mathbf{B} = \mu_0 \sigma \mathbf{E} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

where $\mathbf{J} = \sigma \mathbf{E}$ (Ohm's law). Consider solutions of the form

$$\mathbf{E}(\mathbf{x}, t) = \hat{\mathbf{i}} E_0 e^{i(kz - \omega t)}$$

(The physical wave is the real part of this complex wave.)

(a) Derive the relation between ω and k.

(b) Show that the wave amplitude decreases exponentially with z.

P9 continues… (c) Determine the attenuation length.

P10. Essay question of 1 to 2 pages typed. What is the ionosphere? Who discovered it, when and how? What is its structure? Why is it important? *[Instructions: You may discuss the essay with other students, and get information from the Internet. However, you must write your own essay, in your own words. Do not cut and paste from the Internet. Do not quote from the Internet. Put all information in your own words.]*