Homework Set 3

Exercises; due Friday 9/18

E5. Hand in a computer generated graph of sin θ versus θ.

E6. Write a short essay (2 paragraphs) on Synthetic Aperature Radar. The 1st paragraph should explain what it is. The 2nd paragraph should show an example – an actual picture from the internet – and explain how the image was obtained.

Problems; due Monday 9/21

P11. A soap film with air on both sides has index of refraction n = 1.36. A region of the film appears red (w.l. = 633 nm) when white light is reflected from normal incidence. What is the thickness of the soap film in that region?

P12. One flat glass plate is horizontal; a second plate rests on top of the first plate at an angle α. (Index of refraction of the glass = 1.5.) A parallel beam of monochromatic light (w.l. = 550 nm) is incident normally on the top plate. Calculate the angle α if the fringe separation is 0.25 cm.

P13. As an example of thin film interference, consider light reflecting from an oil slick floating on water. Assume the thickness of the oil film is 500 molecular layers = 500 x 0.3 nm = 150 nm.
(a) Assume n = 1.4 for the oil. Plot, accurately, a graph of the angle for the primary bright fringe versus the wavelength of light. [Use Mathematica or some other computer program to make the graph.]
(b) For incident white light, what color would you see at grazing incidence?

P14. Consider two-slit interference with these parameters:
a = separation of the two slits = 0.02 mm; D = 2 m = distance from the slits to the observation screen; λ = wavelength = 550 nm.
(a) Plot the intensity I(x) as a function of position x on the screen. (x = distance from the center to the point.)
(b) Calculate the full width of the central bright fringe, i.e., between the two positions of minimum intensity.

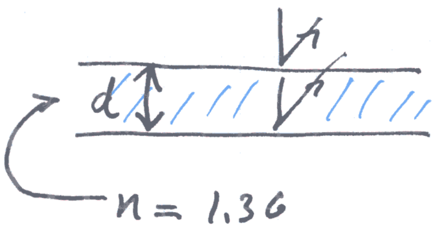
P15. Newton's rings. Suppose the diameter of the 5th bright ring is 5 mm.
(a) Calculate the diameter of the 10th bright ring.
(b) Calculate the distance (along a radial line) from the 10th bright ring to the 20th bright ring.

P16. Newton's rings. Use computer graphics. Plot the bright rings --- a series of concentric rings with the correct radii --- for these parameters: R = 1 m, λ = 550 nm.

Solutions H.W. Set 3

SOLUTION + GRADING KEY

P.11. SOAP FILM



Constructive interference occurs if

$$k' \cdot 2d \pm \pi = 2\pi m$$

when $k' = \frac{2\pi}{\lambda'}$, $\lambda' = \frac{\lambda}{n}$, and

m is an integer.

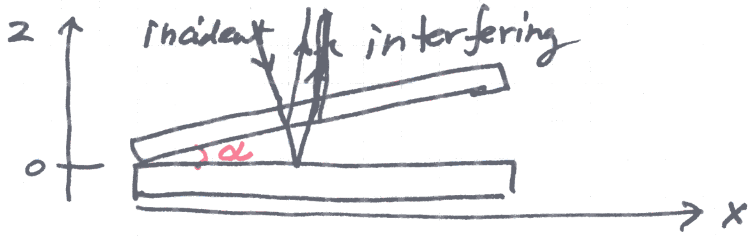
Thus,
$$d_m = \frac{\lambda}{4n}(2m-1)$$

For $\lambda = 633$ nm, different thicknesses will produce constructive interference

m	1	2	3
d_m	116 nm	349 nm	582 nm

2 points

P. 12 OPTICAL FLATS at ANGLE α



$\lambda = 550$ nm

Path difference $\Delta z = 2z = 2x\tan\alpha$

Phase difference $\Delta\phi = k\,\Delta z \pm \pi$ where $k = \frac{2\pi}{\lambda}$

Constructive interference $\Longrightarrow$ $\Delta\phi = 2\pi m$

(m an integer)

$$\therefore \quad x_m \tan\alpha = \frac{\lambda}{2}\left(m - \tfrac{1}{2}\right)$$

$$/ \; 2\pi m = \frac{2\pi}{\lambda}\, 2x\tan\alpha + \pi \; /$$

We are given that $x_{m+1} - x_m = 0.25$ cm

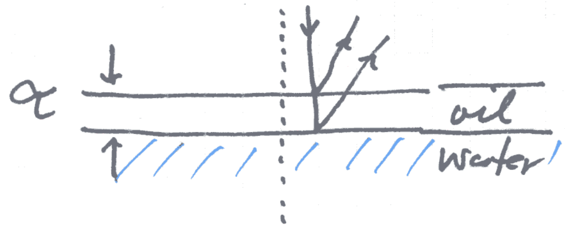
Thus $\tan\alpha = \dfrac{\lambda}{2\,(x_{m+1} - x_m)} = 1.10 \times 10^{-4}$

$\alpha = 1.10 \times 10^{-4}$ radians or 6.3×10^{-3} degrees

or 22.7 seconds of arc.

2 points

P13. OIL SLICK



$n = 1.40$
$\tau = 150$ nm

Constructive interference occurs if

$$\lambda = \frac{4\tau}{2m-1}\sqrt{n^2 - \sin^2\theta_m}$$

for m an integer ($m = 1, 2, 3, \ldots$).
The primary bright fringe ($m=1$) has

$$\sin^2\theta_1 = n^2 - \left(\frac{\lambda}{4\tau}\right)^2$$

$$/ \quad \frac{\lambda}{4\tau} = \sqrt{n^2 - \sin^2\theta_1} \quad /$$

(a) Plot θ_1 versus λ for $n = 1.40$ and $\tau = 150$ nm.

2 points

(b) At grazing incidence ($\theta_1 = 90$ degrees)

$$\lambda = 4\tau\sqrt{n^2 - 1} = 588 \text{ nm}$$

(yellow-orange)

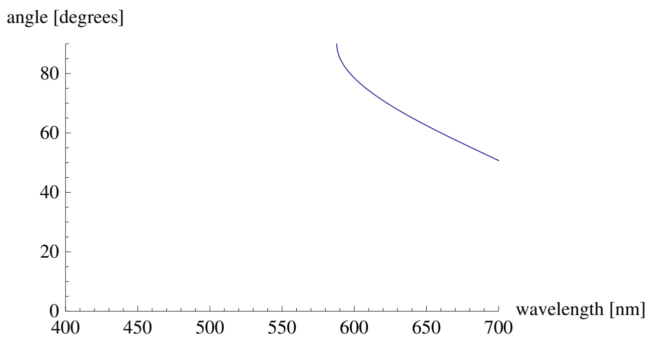
1 point

P. 13. Oil Slick

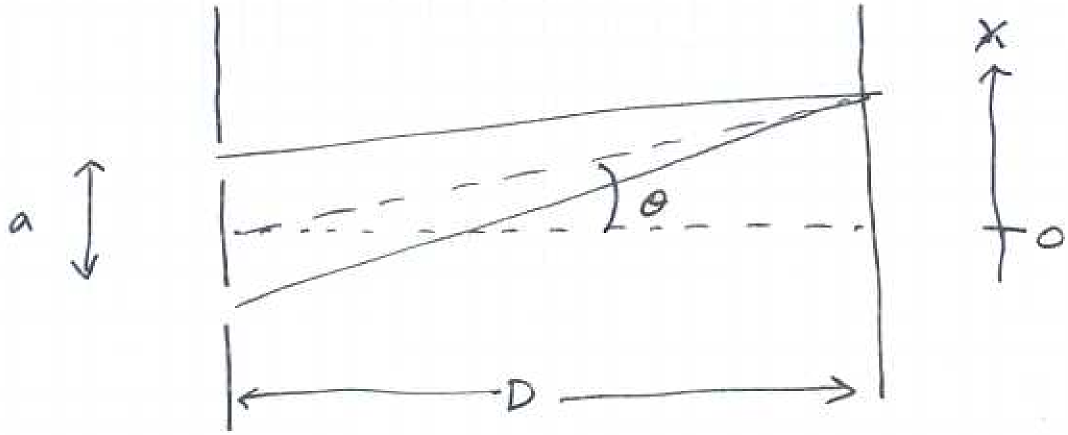
In[5]:=
```
{n, tau} = {1.40, 150}
theta1[lam_] := ArcSin[Sqrt[n^2 - (lam / 4 / tau) ^2]]
degrees[lam_] := theta1[lam] * 180 / 3.14
Plot[degrees[lam], {lam, 400, 700},
 PlotRange → {{400, 700}, {0, 90}},
 AxesLabel → {"wavelength [nm]", "angle [degrees]"}]
```

Out[5]= {1.4, 150}

Out[8]=



P.14 TWO SLIT INTERFERENCE



Specified: $D = 2\,m$, $a = 0.02\,mm$

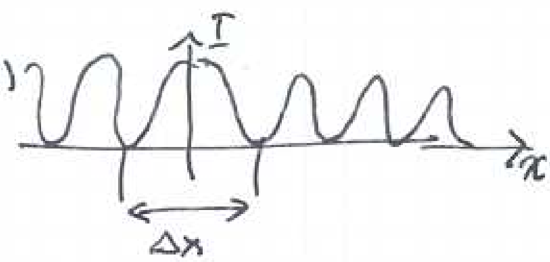
$\lambda = 550\,nm$

$$I(x) = 2A^2 \cos^2\left[\frac{\pi a}{\lambda}\sin\theta\right]$$

where $\sin\theta = \dfrac{x}{\sqrt{D^2+x^2}} \approx \dfrac{x}{D}$

(a) Plot a graph of $I(x)$ versus x. 2 points

(b)



$I = 0$ at $\dfrac{\pi a}{\lambda}\sin\theta = \pi/2$

i.e. $x = D\sin\theta = \dfrac{D\lambda}{2a}$

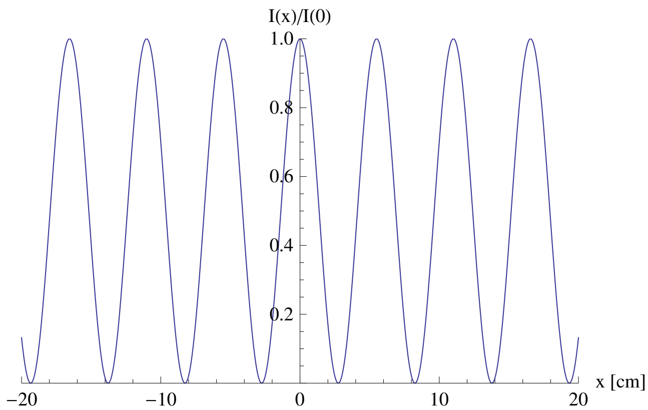
$$\Delta x = \frac{D\lambda}{a} = 0.055\ m = \underline{5.5\ cm}$$ 1 point

P. 14. Two slit interference

In[27]:=
```
{Dist, a, lam} = {2, 2.0*^-5, 550*^-9}
Intensity[x_] := (Cos[Pi * a / lam * x / Sqrt[Dist^2 + x^2]])^2
Plot[Intensity[0.01 * xcm], {xcm, -20, 20}, PlotRange → {{-20, 20}, {0, 1}},
 AxesLabel → {"x  [cm]", "I(x)/I(0)"}]
```

Out[27]= $\left\{2,\ 0.00002,\ \frac{11}{20\,000\,000}\right\}$

Out[29]=

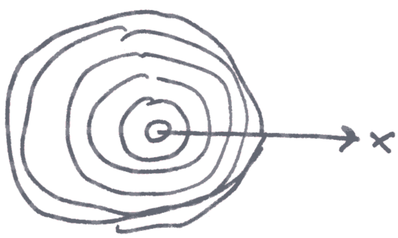


P.15. NEWTON'S RINGS

The radius of the bright ring of order m is x_m,

$$x_m^2 = \lambda R \, (m - 1/2).$$

The diameter of the ring is $2x_m$.



Given that $2x_5 = 5$ mm,

$$\therefore \lambda R = \frac{(2.5 \text{ mm})^2}{9/2} = 1.389 \text{ mm}^2$$

(a) For $m = 10$, the radius is

$$x_{10} = 3.63 \text{ mm}$$

1 point

and the diameter is $2x_{10} = 7.26$ mm

(b) The radial distance from ring 10 to ring 20 is

$$x_{20} - x_{10} = 1.572 \text{ mm}$$

1 point

P.16. NEWTON's RINGS Graphic

5 points

17 points

P. 16. Newton’ s Rings

In[38]:=
```
{lambda, Rc} = {550*^-9, 1}
Do[radius[m] = Sqrt[lambda * Rc * (m - 1 / 2)],
 {m, 1, 20}]
circles = Table[Circle[{0, 0}, radius[m]], {m, 1, 20}];
Show[Graphics[circles]]
```

Out[38]= $\left\{\frac{11}{20\,000\,000}, 1\right\}$

Out[41]=