The Electromagnetic Spectrum

λ	f	hf		
3 fm	10^{23} Hz	0.4 GeV	γ rays	radiation therapy
3 pm	10^{20} Hz	0.4 MeV	X-rays	medical imaging
3 nm	10^{17} Hz	0.4 kev	UV light	
			visible	vision
3 μm	10^{14} Hz	0.4 eV	IR light	night vision goggles
3 mm	10^{11} Hz	0.4 meV	microwaves	radar
3 m	10^{8} Hz	0.4 μeV	radio waves	radio communication
3 km				

wavelength

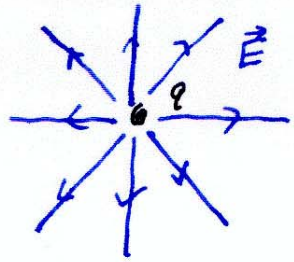
frequency

Photon energy

Maxwell's Equations

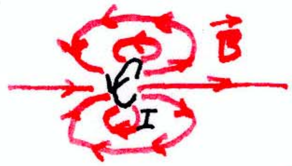
Pictorial

$$\nabla \cdot \vec{E} = \rho/\epsilon_0$$



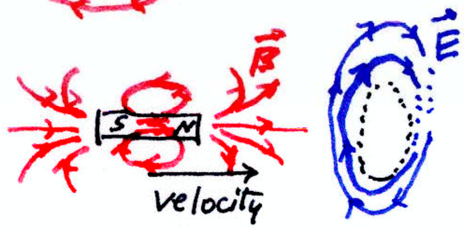
$$\vec{E} = \frac{q\hat{r}}{4\pi\epsilon_0 r^2}$$

$$\nabla \cdot \vec{B} = 0$$



The magnetic field is solenoidal.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$



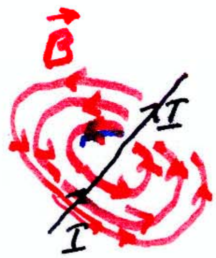
$$\mathcal{E}mf = -\frac{d\Phi}{dt}$$

Faraday, 1831

$$\nabla \times \vec{B} = \mu_0(\vec{J} + \vec{J}_D)$$

$$\vec{J}_D = \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

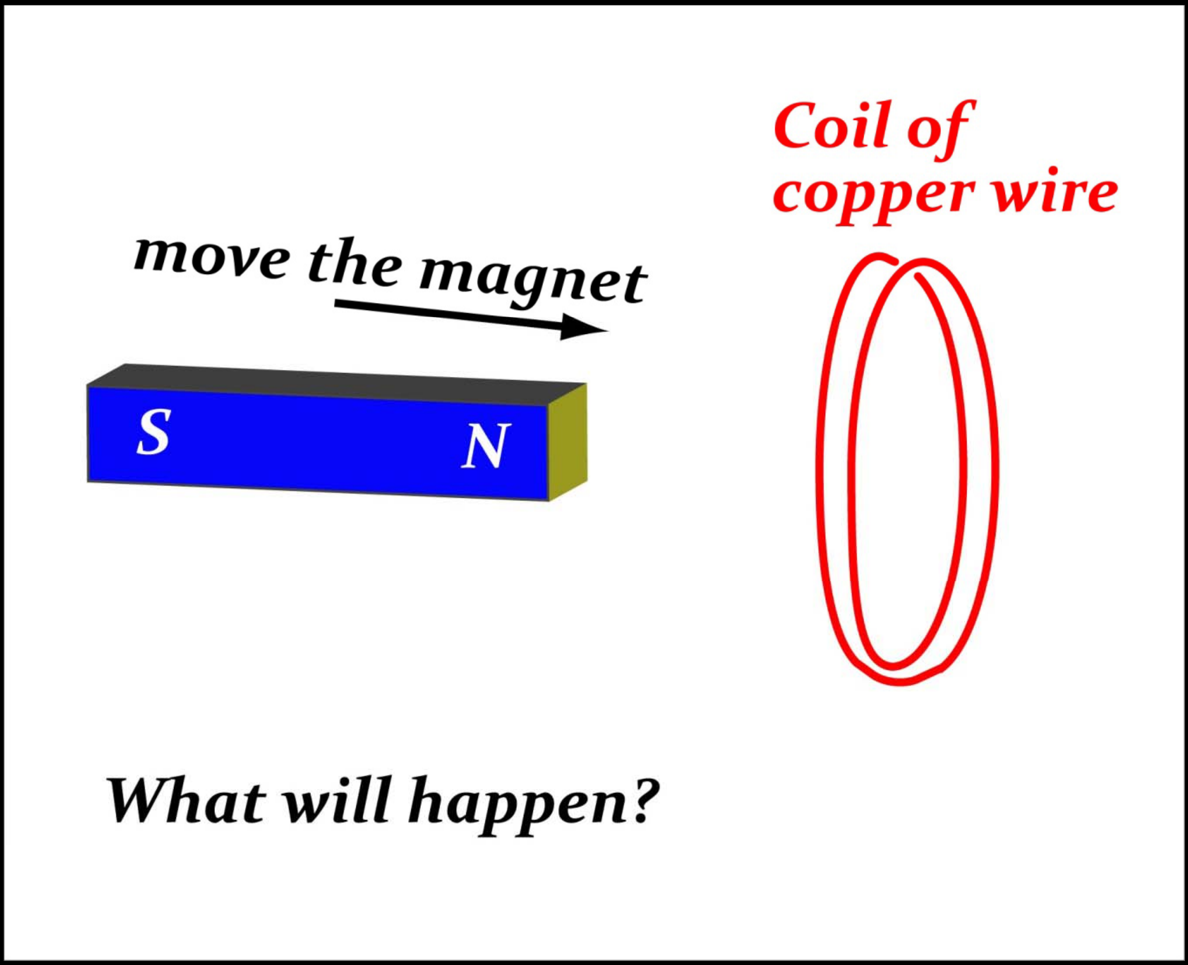
Displacement Current
Maxwell, 1864



$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$$

These field equations, published in 1864, are still used today. They describe electricity, magnetism, electromagnetism, and optics.

Electromagnetic Induction



$$\mathrm{EMF} = -\, d\Phi \,/\, dt$$

Electromagnetic Waves in Field Theory

We consider $\vec{E}$ and $\vec{B}$ in empty space

$$\nabla\cdot\vec{E} = 0 \qquad \text{and} \qquad \nabla\cdot\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad \text{and} \qquad \nabla\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

Now, construct solutions w/ $\vec{E}(\vec{x},t) = \hat{\imath}\,F(z,t)$

i.e., propagating in the z direction, polarized in the x direction.

- $\nabla\cdot\vec{E} = 0 \longrightarrow \quad 0 = 0 \quad \checkmark$

- $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \longrightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ F & 0 & 0 \end{vmatrix} = +\hat{\jmath}\frac{\partial F}{\partial z} = -\frac{\partial\vec{B}}{\partial t}$

$\therefore$ require $\vec{B}(\vec{x},t) = \hat{\jmath}\,G(z,t)$ and $\frac{\partial F}{\partial z} = -\frac{\partial G}{\partial t}$

- $\nabla\cdot\vec{B} = 0 \longrightarrow \quad 0 = 0 \quad \checkmark$

- $\nabla\times\vec{B} = \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t} \longrightarrow \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & G & 0 \end{vmatrix} = -\hat{\imath}\frac{\partial G}{\partial z} = \mu_0\epsilon_0\,\hat{\imath}\frac{\partial F}{\partial t}$

$\therefore$ requires $\frac{\partial G}{\partial z} = -\mu_0\epsilon_0\frac{\partial F}{\partial t}$

Electromagnetic Waves in Field Theory

$\vec{E}(\vec{x},t) = \hat{\imath}\, F(z,t)$ and $\vec{B}(\vec{x},t) = \hat{\jmath}\, G(z,t)$

$$\frac{\partial F}{\partial z} = -\frac{\partial G}{\partial t} \quad \text{and} \quad \frac{\partial G}{\partial z} = -\mu_0 \epsilon_0 \frac{\partial F}{\partial t}$$

Solutions

Consider $\frac{\partial^2 F}{\partial z^2} = -\frac{\partial^2 G}{\partial z \partial t} = -\frac{\partial^2 G}{\partial t \partial z} = \mu_0 \epsilon_0 \frac{\partial^2 F}{\partial t^2}$

$$\frac{\partial^2 F}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 F}{\partial t^2} \quad \text{(wave equation) w/ } c^2 = \frac{1}{\mu_0 \epsilon_0}.$$

wavespeed $= c$

A general solution is $F(z,t) = f(z-ct)$
where $f(\cdot)$ is any function.

$\left[\frac{\partial^2 F}{\partial z^2} = f''(z-ct) \text{ and } \frac{\partial^2 F}{\partial t^2} = c^2 f''(z-ct) \quad \checkmark \right]$

OK, now what is $G(z,t)$?

$$\frac{\partial G}{\partial t} = -\frac{\partial F}{\partial z} = -f'(z-ct)$$

$\therefore \; G(z,t) = \frac{1}{c} f(z-ct)$

$$\boxed{\begin{aligned} G(z,t) &= \frac{1}{c}F(z,t) \\ B &= \frac{1}{c}E \end{aligned}}$$

Harmonic Solutions

$f(\xi) = E_0 \sin(k\xi)$

$\vec{E}(\vec{x},t) = \hat{\imath}\, E_0 \sin(kz - \omega t)$

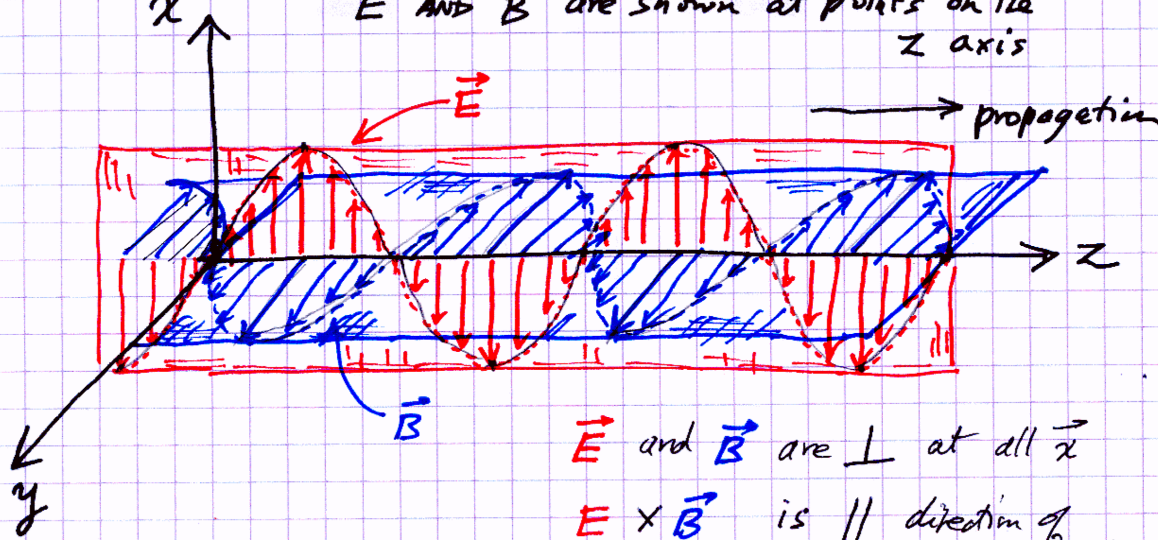
$\vec{B}(\vec{x},t) = \hat{\jmath}\, \frac{E_0}{c} \sin(kz - \omega t)$

$$\left\{ \begin{aligned} \omega &= ck \\ B_0 &= E_0/c \end{aligned} \right\}$$

The Harmonic Plane Wave

HARMONIC ELECTROMAGNETIC WAVE

$\vec{E}$ AND $\vec{B}$ are shown at points on the z axis



$\vec{E}$ and $\vec{B}$ are $\perp$ at all $\vec{x}$

$E \times \vec{B}$ is $\parallel$ direction of propagation

$$\vec{E}(\vec{x},t) = \hat{\imath}\, E_0 \sin(kz-\omega t)$$

$$\vec{B}(\vec{x},t) = \hat{\jmath}\, B_0 \sin(kz-\omega t)$$

<u>Wave properties</u>

λ : wavelength $\quad k\lambda = 2\pi$ or $\lambda = \frac{2\pi}{k}$

T : period $\quad \omega T = 2\pi$ or $T = 2\pi/\omega$

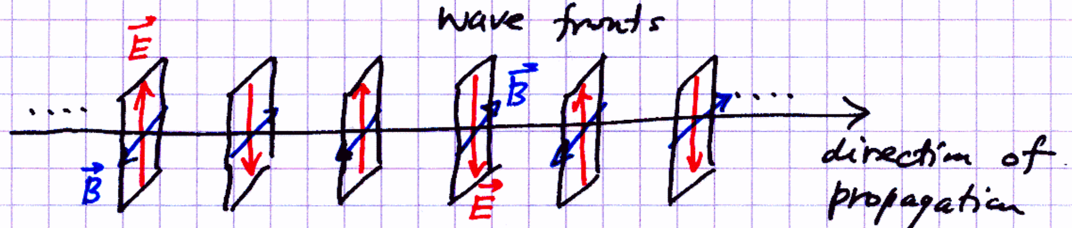
f : frequency $\quad f = 1/T = \frac{\omega}{2\pi}$

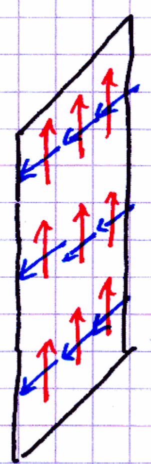
c : velocity $\quad c = \frac{\lambda}{T} = \frac{\omega}{k}$

<u>Maxwell's equations</u> require

- $B_0 = E_0/c$
- $\vec{E}, \vec{B}, \vec{k}$ form an orthogonal triad

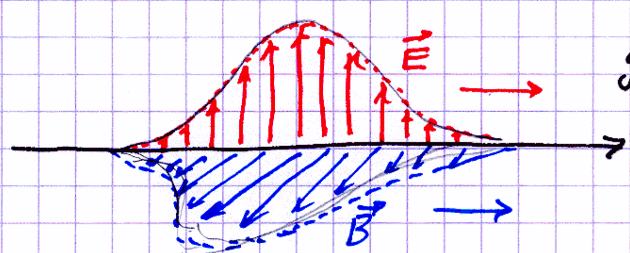
The plane wave is an idealization —
$\vec{E}$ and $\vec{B}$ are coherent on
infinite planar wave fronts



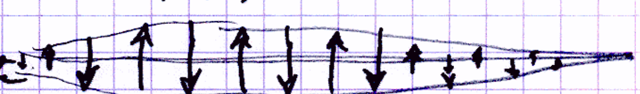


$\vec{E}$ and $\vec{B}$
are coherent on
infinite planar wave fronts
$\vec{E} \times \vec{B}$ is $\parallel$ to $\vec{k}$

A finite electromagnetic wave ; a "pulse"
e.g., $\vec{E}(\vec{x},t) = \hat{\imath}\, f(z-ct)$ and $\vec{B}(\vec{x},t) = \hat{\jmath}\, f(z-ct)/c$



But

If $\vec{E}$ and $\vec{B}$ oscillate 
with, then the plane wave is a
good approximation.