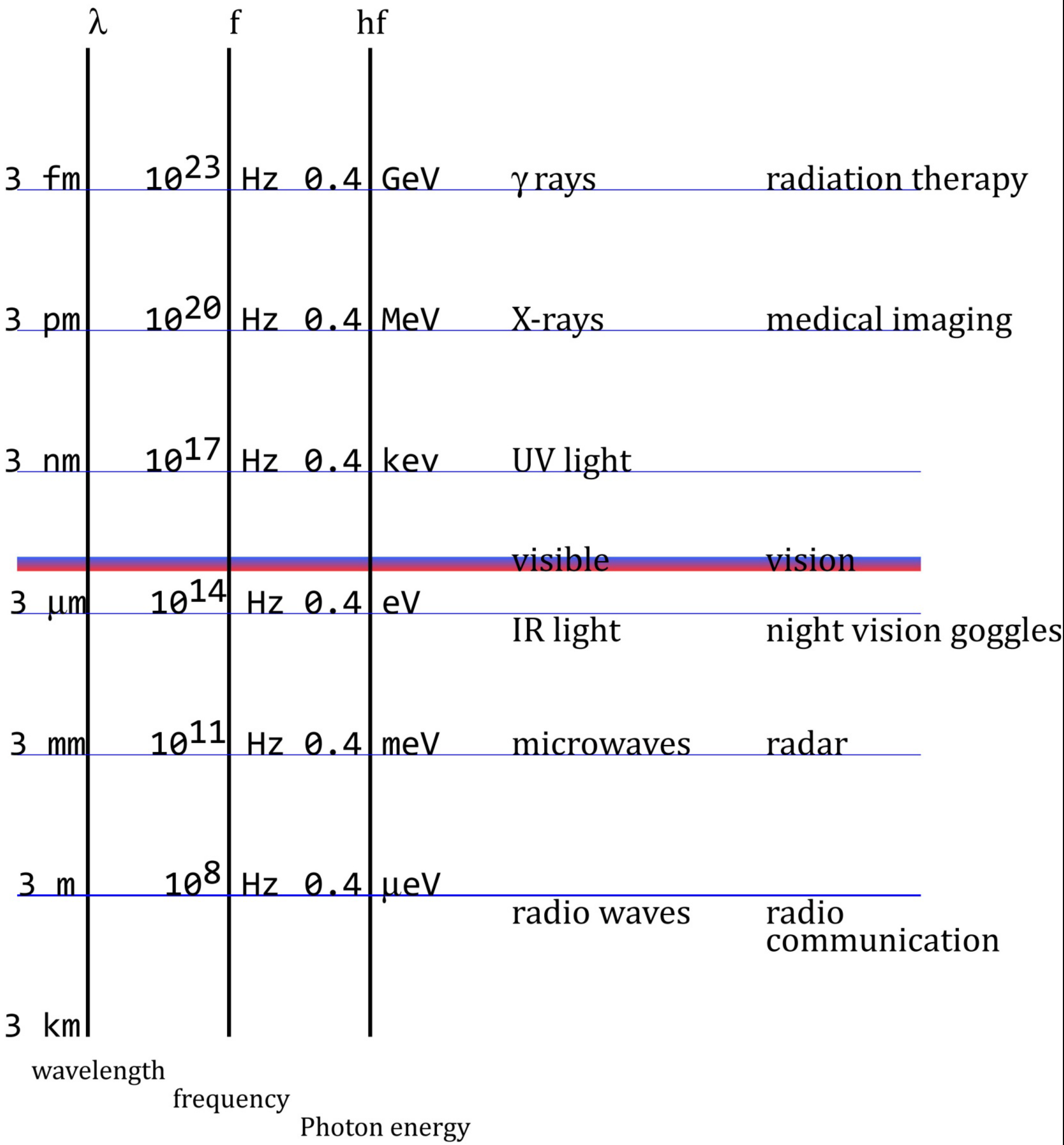
Reflection and Refraction

– the technology of visible light

The Electromagnetic Spectrum



Electric and Magnetic Fields in Matter

Conductors :

free charge will flow; i.e., electric current

$\vec{J}(\vec{x},t)$ = current density [A/m²]

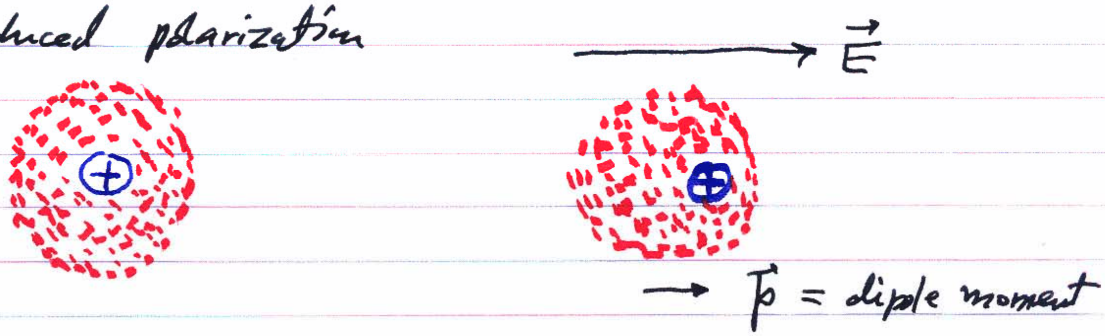
Usually, $\vec{J} = \sigma \vec{E}$ (Ohm's Law)

(Khare & Swarup, Chap10)

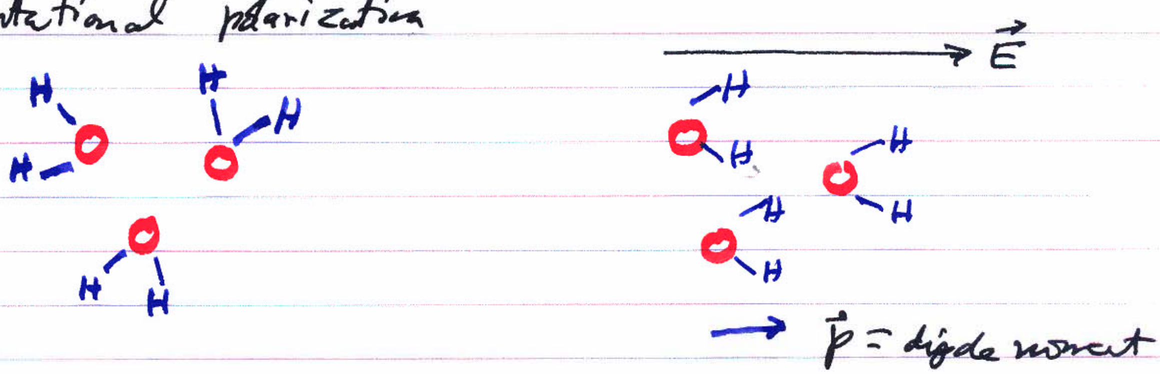
Insulators :

atoms or molecules will be polarized

- induced polarization



- orientational polarization



(We'll neglect magnetic dipole moments, magnetization)

Charge density $\rho = \rho_{free} + \rho_{polarization}$

ρ_{free} = applied charge

$\rho_{polarization}$ = charge density due to polarization $= -\nabla\cdot\vec{P}$

where P = polarization = dipole moment density

Define $\vec{D} = \epsilon_0\vec{E} + \vec{P}$

Then $\nabla\cdot\vec{D} = \rho - \rho_{pol.} = \rho_{free}$

Maxwell's Equations in a Dielectric

$\nabla \cdot \vec{D} = \rho_{free}$ where **$D=\varepsilon_0 E+P$**

$\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\nabla \times \vec{B} = \mu_0 (\vec{J}_{free} + \vec{J}_D)$ where $\vec{J}_D = \frac{\partial \vec{D}}{\partial t} = \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$

"Linear Materials" — polarizability and permittivity

For simple materials, $\vec{P} \propto \vec{E}$

We write $\vec{P} = \epsilon_0 \chi_e \vec{E}$

Then $\vec{D} = \epsilon_0 (1+\chi_e)\vec{E} = \epsilon \vec{E}$

We expect, especially for low frequencies,

$\chi_e > 0$ and $\epsilon > \epsilon_0$.

χ_e : susceptibility

ϵ : permittivity

$\epsilon = \epsilon_0 (1+\chi_e)$

Electromagnetic Waves in a dielectric

Important point : $\rho_{free} = 0$ and $\vec{J}_{free} = 0$

We seek solutions of these field equations:

$$\nabla \cdot \vec{D} = 0 \quad \text{and} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{E} = 0 \, *$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{and} \quad \nabla \times \vec{B} = \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t}$$

* uniform linear material

These have the same form as the field equations in empty space, except for one change : $\mu_0 \epsilon_0 \longrightarrow \mu_0 \epsilon$

Wave propagation in the dielectric

A2/3

$$\vec{E}(\vec{x},t) = \vec{E}_0 \sin(\vec{k}\cdot\vec{x} - \omega t)$$
$$\vec{B}(\vec{x},t) = \vec{B}_0 \sin(\vec{k}\cdot\vec{x} - \omega t)$$

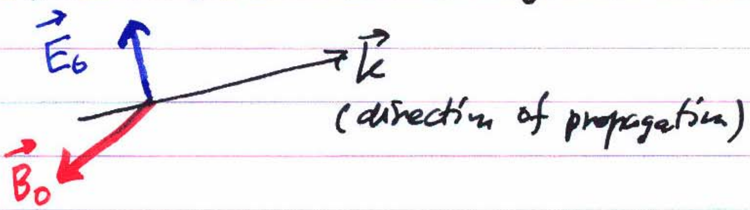
} harmonic, plane, polarized wave

where the wave propagates in the direction of $\vec{k}$

- Phase velocity

$$v_{phase} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \epsilon}}$$

- $\vec{k}, \vec{E}_0, \vec{B}_0$ form an orthogonal triad



- $$B_0 = \frac{E_0}{v_{phase}} = \sqrt{\mu_0 \epsilon}\, E_0$$

The index of refraction, n

Define $$n = \frac{c}{v_{phase}} = \frac{\sqrt{\mu_0 \epsilon}}{\sqrt{\mu_0 \epsilon_0}} = \sqrt{\frac{\epsilon}{\epsilon_0}}$$

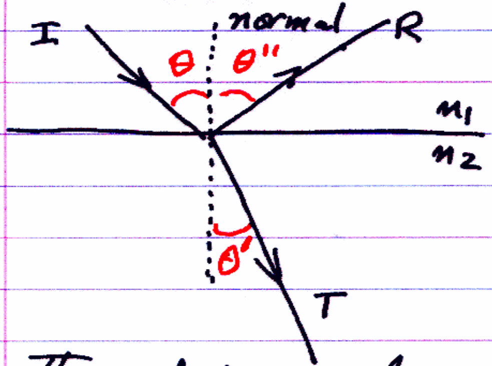
We expect $\epsilon > \epsilon_0$; then $n > 1$.

As the e.m. wave propagates in the dielectric, it travels slower than c.

... $v_{phase} = c/n$

Reflection and Refraction

What happens when light hits a dielectric surface?



I : incident ray
R : reflected ray
T : transmitted ray

Note 3 angles : θ θ' θ''

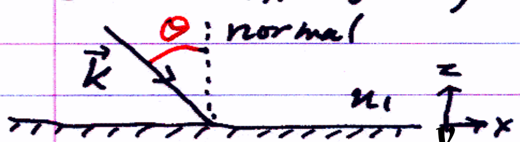
The electric and magnetic fields oscillate in $\perp$ directions.
(transverse waves)

The fields satisfy certain boundary conditions at the surface between the materials ("interface")

- $E_{\parallel}$ is continuous
- $D_{\perp}$ is continuous

$$\underbrace{\hat{n}\times(\vec{E}_I+\vec{E}_R)=\hat{n}\times\vec{E}_T \qquad \hat{n}\cdot(\epsilon_1\vec{E}_I+\epsilon_1\vec{E}_R)=\hat{n}\cdot(\epsilon_2\vec{E}_T)}_{\text{for all points on the interface at all times.}}$$

On the interface, each field $\propto \sin(k_{\parallel}x-\omega t)$



$$\vec{k}=k_{\parallel}\hat{x}+k_{\perp}\hat{z}$$

$$k_{\parallel}=k\sin\theta=\frac{\omega}{c}\sin\theta$$

The B.C. must hold for all t

$\Longrightarrow$ $\omega=\omega'=\omega''$

i.e., the frequencies must be equal.

The B.C. must hold for all x

$\Longrightarrow$ $k_{\parallel}=k_{\parallel}'=k_{\parallel}''$ i.e., rates of variation along the surface must be equal.

Thus $\frac{\omega}{v_1}\sin\theta=\frac{\omega}{v_2}\sin\theta'=\frac{\omega}{v_1}\sin\theta''$

$|||$ $\theta=\theta''$ (law of reflection)

$n_1\sin\theta=n_2\sin\theta'$ (law of refraction)