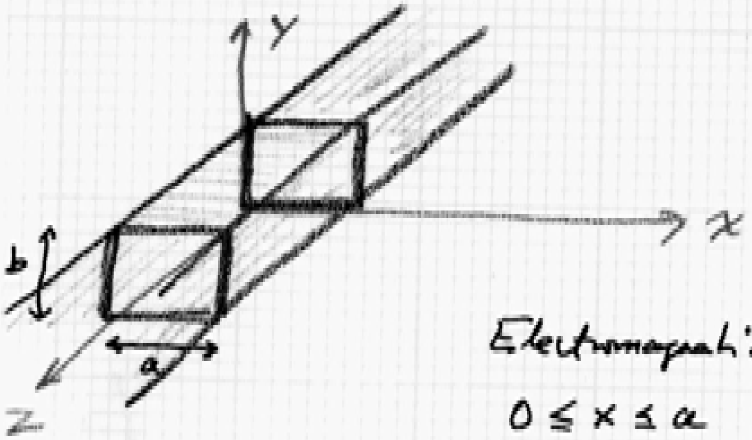
The Electromagnetic Spectrum

λ (wavelength)	f (frequency)	hf (Photon energy)		
3 fm	10^{23} Hz	0.4 GeV	γ rays	radiation therapy
3 pm	10^{20} Hz	0.4 MeV	X-rays	medical imaging
3 nm	10^{17} Hz	0.4 kev	UV light	
			visible	vision
3 μm	10^{14} Hz	0.4 eV	IR light	night vision goggles
3 mm	10^{11} Hz	0.4 meV	microwaves	radar
3 m	10^{8} Hz	0.4 μeV	radio waves	radio communication
3 km				

Wave Guides

Rectangular wave guide



Metal walls,
i.e., high conductivity

Electromagnetic waves in the space
$0 \le x \le a$ and $0 \le y \le b$.
The Z dimension is infinite;
more advanced — reflections at the ends

Field Equations

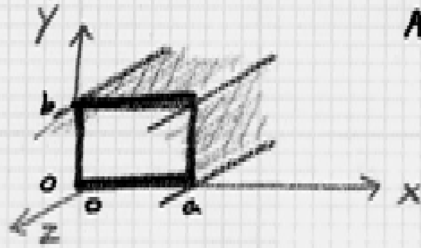
- Inside the wave guide

$$\nabla \cdot \vec{E} = 0 \quad \text{and} \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{and} \quad \nabla \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}$$

- Inside the metal, $\vec{E} = 0$.
  We could write $\vec{J} = \sigma \vec{E}$ and take the limit $\sigma \to \infty$ (high conductivity) which implies $\vec{E} = 0$.

- The boundary conditions
  $E_{\parallel}$ is continuous at the metal surfaces
  $\therefore E_{\parallel} = 0$ at the surfaces



TE waves

↳ We'll consider electromagnetic waves that have Transverse Electric polarization.

That is, the wave propagates in the z direction and is polarized in the x or y direction.

To be definite, take polarization direction = $\hat{\imath}$

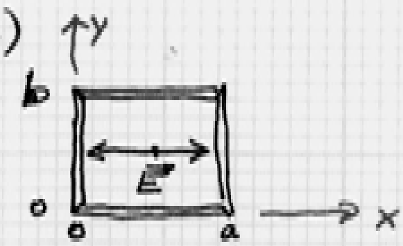
$$\vec{E}(\vec{x},t) = \hat{\imath}\ F(x,y)\ e^{i(kz-\omega t)}$$

$$i = \sqrt{-1}$$

Comment on Complex waves. The Real Part is Implied. We'll solve the equations for a complex wave, and take the Real Part at the end of the calculation.

Important: $e^{i\theta} = \cos\theta + i\sin\theta$

(Euler's equation)



The field equations

- $\nabla\cdot\vec{E} = 0$

$$= \frac{\partial E_x}{\partial x} = \frac{\partial F}{\partial x}\ e^{i(kz-\omega t)}$$

$$\frac{\partial F}{\partial x} = 0 \quad \text{so} \quad F = F(y)$$

- $\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

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$$\nabla \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & \partial/\partial y & ik \\ E_x & 0 & 0 \end{vmatrix}$$

$$= +\hat{j}\, ik E_x \quad + \hat{k}\left(-\frac{\partial E_x}{\partial y}\right)$$

$$= \left\{ \hat{j}\, ikF - \hat{k}\, \frac{dF}{dy} \right\} e^{i(kz-\omega t)} \qquad = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{B} = \left\{ \hat{j}\, \frac{k}{\omega} F + \hat{k}\, \frac{i}{\omega} \frac{dF}{dy} \right\} e^{i(kz-\omega t)}$$

Check : OK ✓
$\partial/\partial t$ same as $(-i\omega)\times$

Remember : the real part is implied.

- $\nabla \cdot \vec{B} = 0$

$$\longrightarrow \quad = \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z}$$

$$= \left[\frac{k}{\omega} \frac{dF}{dy} + \frac{i}{\omega} \frac{dF}{dy}\, ik \right] e^{ikz} e^{-i\omega t}$$

$$= 0 \quad ✓ \quad \text{OK!}$$

- $\nabla \times \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$

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$$\nabla \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & \partial/\partial y & ik \\ 0 & B_y & B_z \end{vmatrix}$$

$$= \hat{\imath}\left[\frac{\partial B_z}{\partial y} - ik B_y\right]$$

$$= \hat{\imath}\left[\frac{i}{\omega}\frac{d^2F}{dy^2} - \frac{ik^2}{\omega}F\right]e^{i(kz-\omega t)}$$

and

$$\frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \frac{-i\omega}{c^2}\,\hat{\imath}\,F(y)\,e^{i(kz-\omega t)}$$

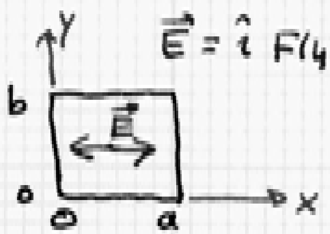
So, the Ampère-Maxwell equation requires

$$\frac{i}{\omega}\frac{d^2F}{dy^2} - \frac{ik^2}{\omega}F = -\frac{i\omega}{c^2}F$$

$$\frac{d^2F}{dy^2} + \left(\frac{\omega^2}{c^2} - k^2\right)F = 0.$$

All four field equations are satisfied if $F(y)$ satisfies this equation.

$\vec{E} = \hat{\imath}\,F(y)\,e^{i(kz-\omega t)}$



Boundary Conditions

$E_{\parallel} = 0$ on the boundaries.

For $y=0$, $E_x = 0$; i.e., $F(0) = 0$

For $y=b$, $E_x = 0$; i.e., $F(b) = 0$

For $x=0$ or $x=a$ we require $E_y = 0$ ✓ OK

Eigenfunctions $\quad \frac{d^2F}{dy^2} + K^2 F = 0$ where $K^2 = \frac{\omega^2}{c^2} - k^2$

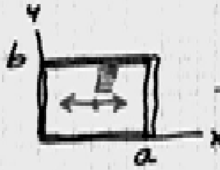
implies $\quad F(y) = A \sin Ky$ or $\underbrace{B \cos Ky}$ — violates the boundary condition $F(0) = 0$

and the boundary condition

$F(b) = 0$ implies $\sin Kb = 0$.

$K = \frac{n\pi}{b}$ w/ $n = 1, 2, 3, 4, \ldots$

Modes of Propagation



TE waves

$$\vec{E} = \hat{x} A \sin \frac{n\pi y}{b} \, e^{ikz} e^{-i\omega_n t}$$

where $\frac{\omega_n^2}{c^2} = \left(\frac{n\pi}{b}\right)^2 + k^2$

For a propagating wave, k must be real

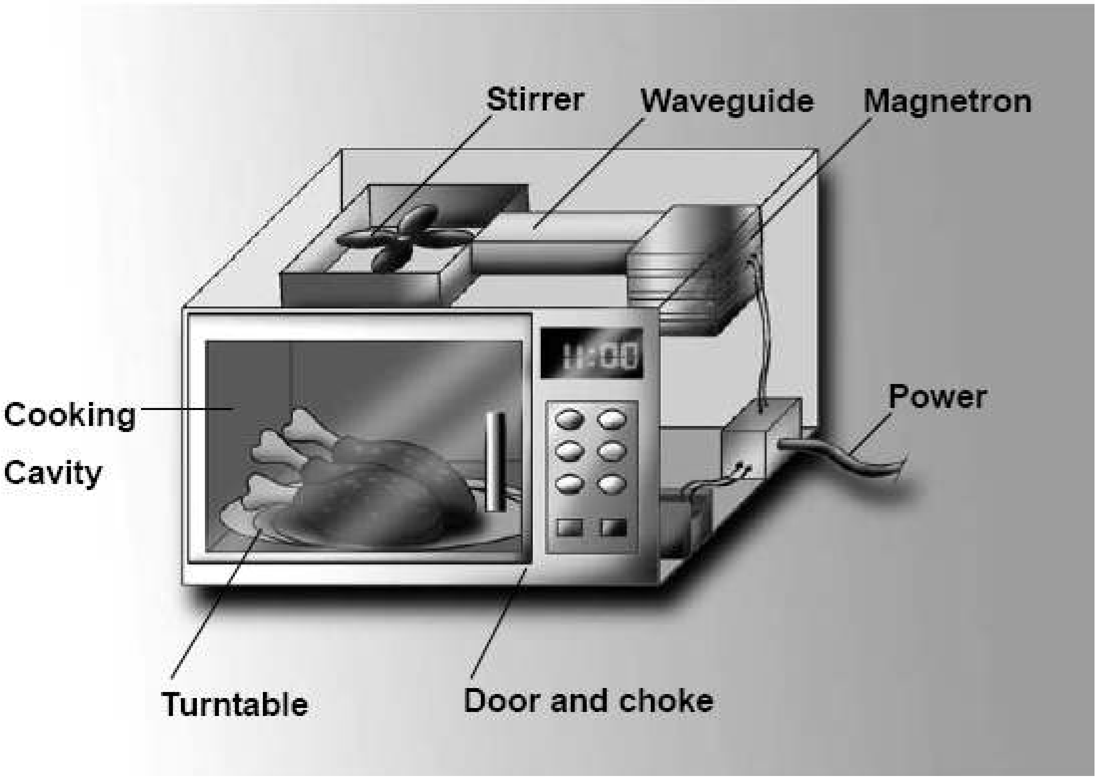
e^{ikz} is exponentially damped if k is not real

Thus $\quad \omega_n \geq \frac{n\pi c}{b}$ ← only these frequencies will propagate in the wave guide for mode n.

- If $\omega < \frac{\pi c}{b}$ there is no propagation mode
- If $\frac{\pi c}{b} < \omega < \frac{2\pi c}{b}$ only mode $n=1$ can propagate (TE)
- If $\frac{2\pi c}{b} < \omega < \frac{3\pi c}{b}$ only modes $n=1, 2$ can propagate (TE)

etc

Uses of Microwaves



Weather Radar