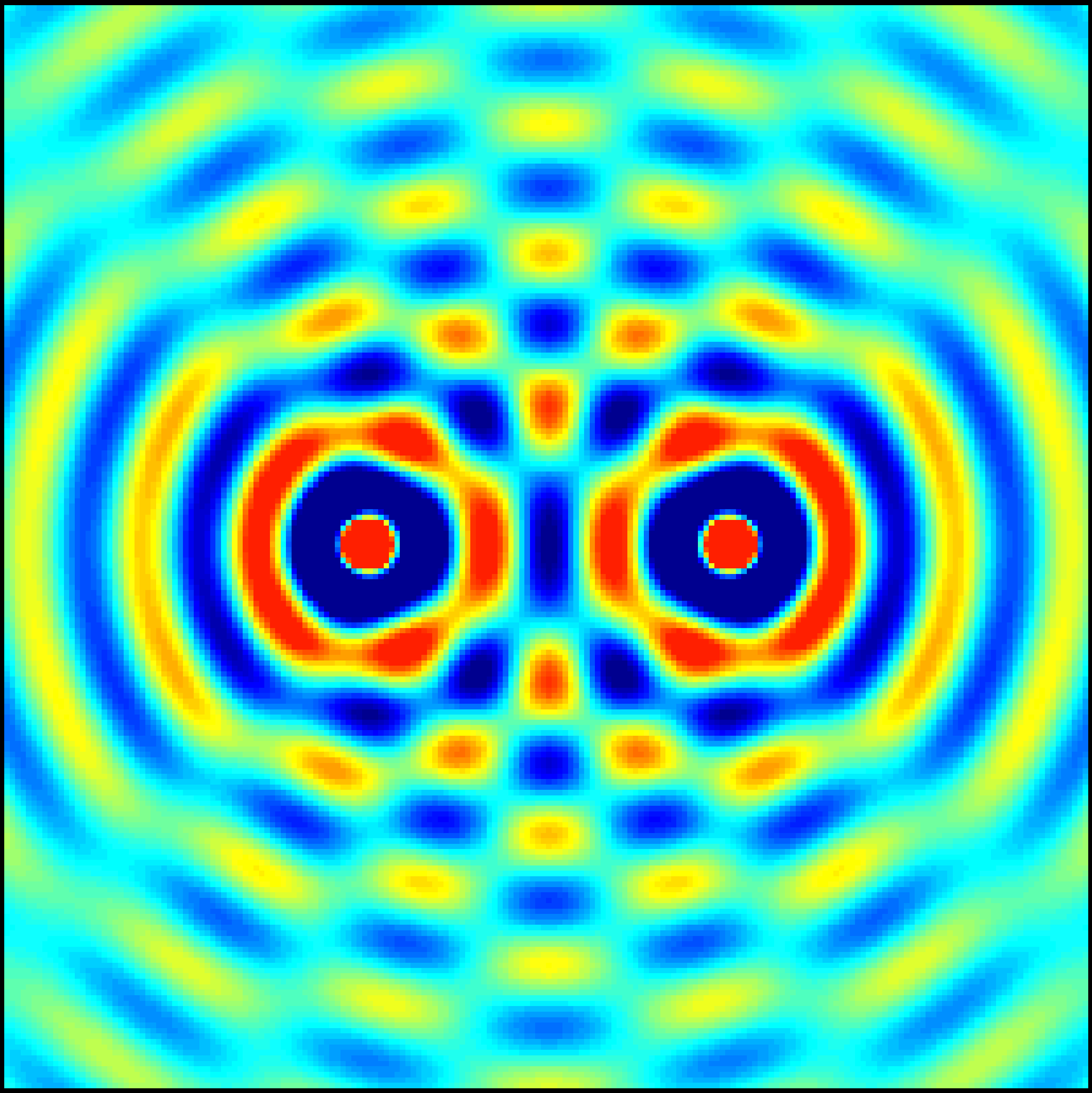
Interference of Light



Light …

… particles or waves?

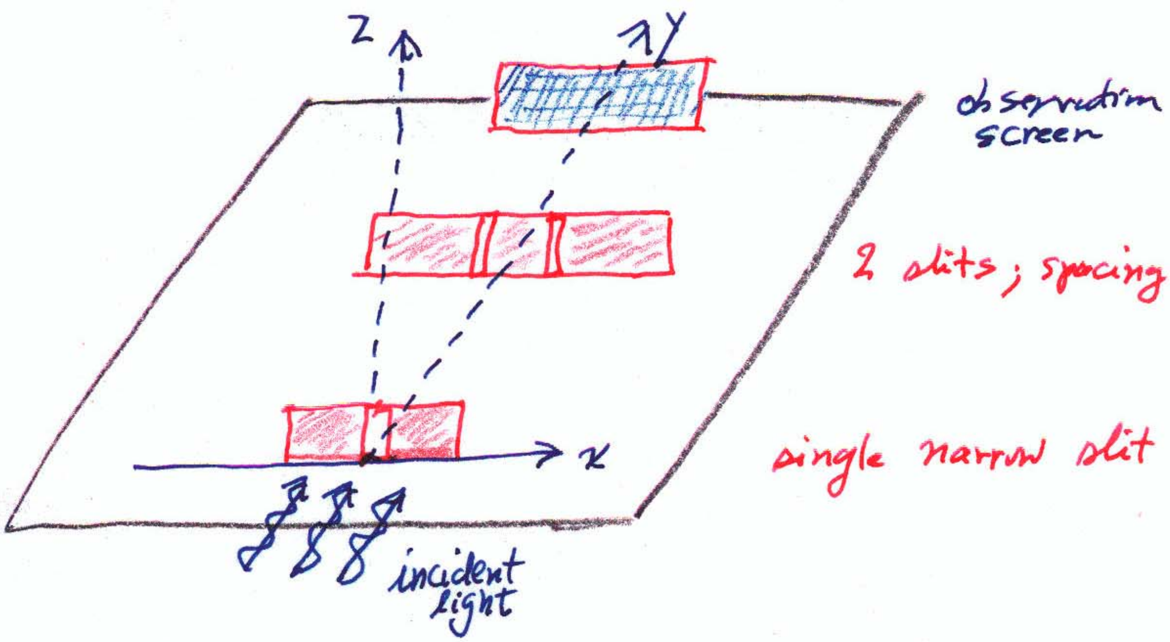
Christiaan Huygens – waves
Isaac Newton – particles

Wave phenomena of light
Thomas Young
Augustin Fresnel
The electromagnetic wave theory
James Clerk Maxwell
Quantum theory — the photon
Max Planck
Albert Einstein

Reading: Khare and Swarup, Chapter 4

Interference of Light – Young's Experiment

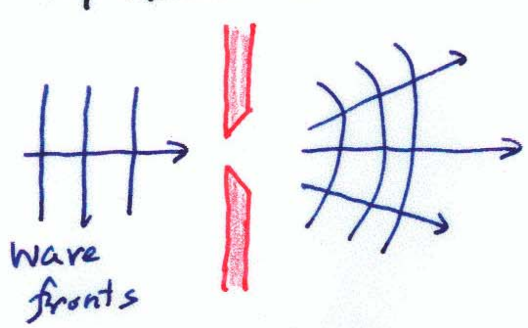
2-slit interference; Thomas Young, 1801



What would you expect to see on the observation screen?

- Particle theory of light:

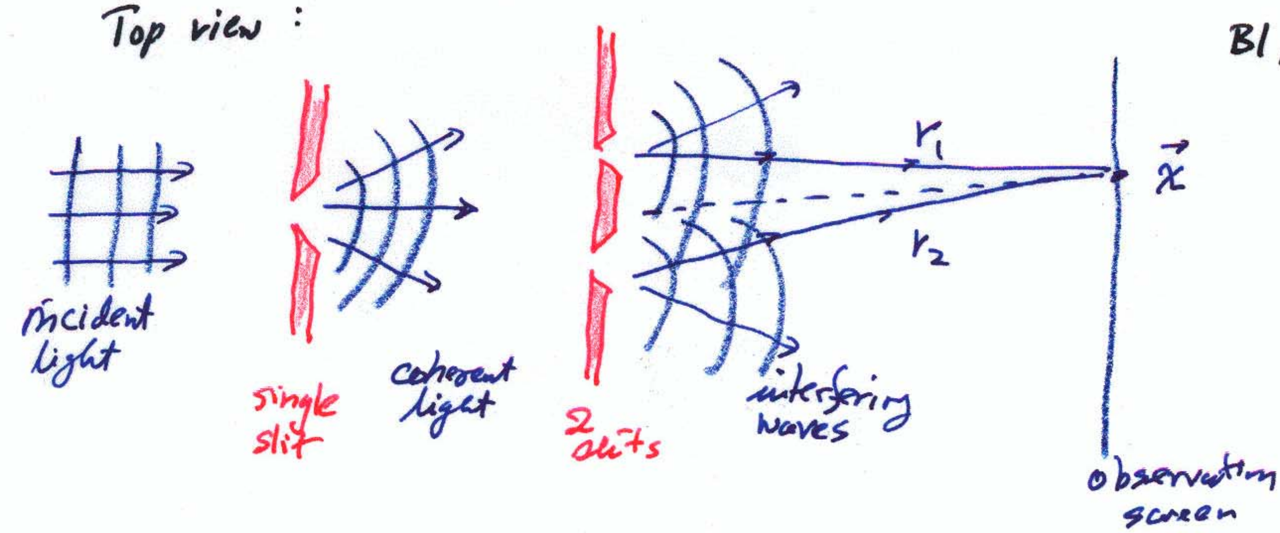
- Wave theory: As waves pass through an opening, they spread out



E.g., water waves passing through a narrow opening.

So, the TWO transmitted waves spread out, <u>overlap</u> and <u>interfere</u>.

Top view :



Huygens' Principle

Each slit acts as an independent source of waves. The two slits are coherent; i.e., Their emitted waves are <u>in phase</u> (crest to crest and trough to trough)

$$\phi_1(\vec{x},t) = A \cos(kr_1 - \omega t)$$

$$\phi_2(\vec{x},t) = A \cos(kr_2 - \omega t)$$

(in phase at $r_1 = r_2 = 0$)

The Superposition

$$\phi(\vec{x},t) = A\cos(kr_1 - \omega t) + A\cos(kr_2 - \omega t)$$

Let $r_1 = R - \frac{\delta r}{2}$ and $r_2 = R + \frac{\delta r}{2}$

I.e., $R = \frac{1}{2}(r_1 + r_2)$ and $\delta r = r_2 - r_1$

$$\phi(\vec{x},t) = 2A\cos\left(\tfrac{1}{2}k\,\delta r\right)\cos(kR - \omega t)$$

(amplitude of oscillations at $\vec{x}$) (oscillating in time)

Intensity on the observation screen

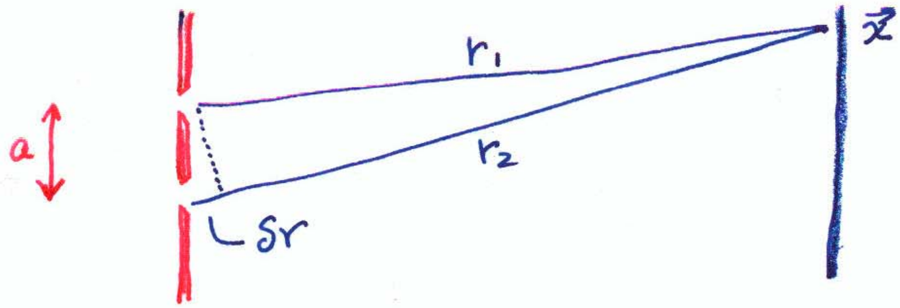
Intensity $= \langle p^2 \rangle$

$\langle \cdot \rangle$ means the time average

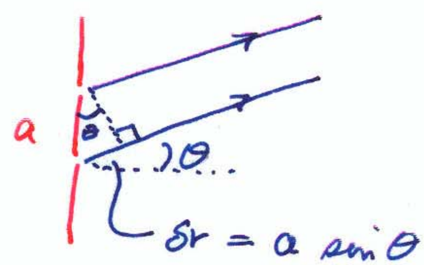
$$\langle \cos^2 (kR - \omega t) \rangle = 1/2$$

$$\text{Intensity} = 4A^2 \cos^2 \left(\tfrac{1}{2} k \,\delta r\right) \cdot 1/2$$

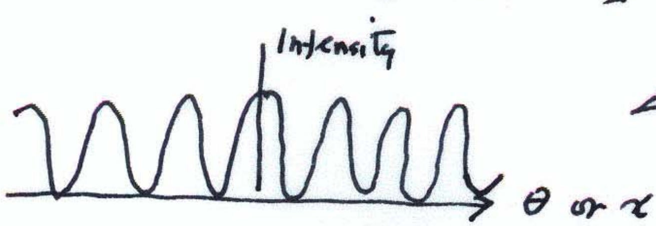
$$I(\vec{x}) = 2A^2 \cos^2 \left[\tfrac{1}{2} k (r_2 - r_1)\right]$$



For a distant screen,
$\vec{r}_1$ and $\vec{r}_2$ are nearly parallel
and $r_2 - r_1 \approx a \sin\theta$



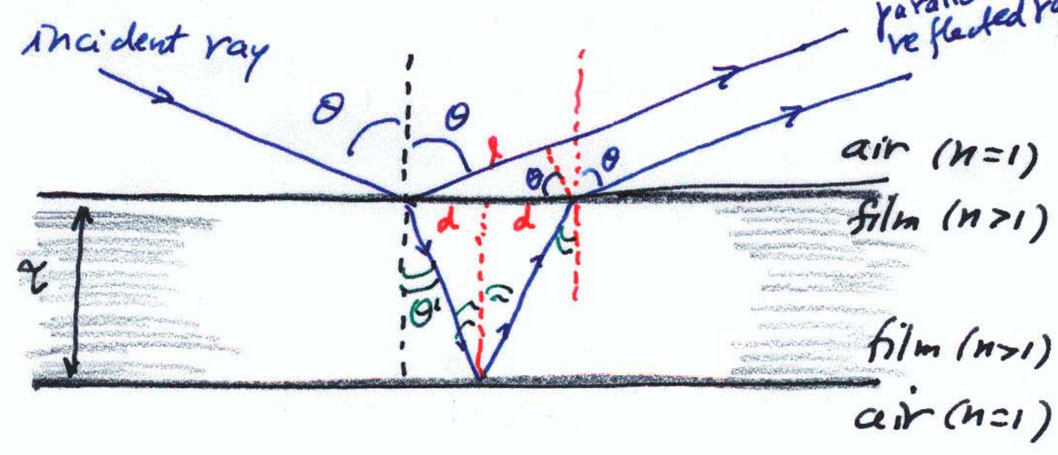
$$I(\vec{x}) = 2A^2 \cos^2 \left[\frac{\pi a}{\lambda} \sin\theta \right]$$



Interference pattern:
- alternating bright and dark fringes
- constructive and destructive interference

- Bright fringes: $\frac{\pi a}{\lambda} \sin\theta = n\pi \quad (n = 0, \pm 1, \pm 2, \pm 3, \ldots)$
  i.e. $a \sin\theta_n = n\lambda$
- Dark fringes: $a \sin\theta_n' = (n + 1/2)\lambda$

Thin film interference



Snell's law : $\sin\theta = n \sin\theta'$

- Path difference in the air $= l = 2d \sin\theta$
- Path difference in the film $= 2 \times$ hypotenuse

$$= 2 \frac{d}{\sin\theta'} = \frac{2dn}{\sin\theta}$$

The phase difference between the 2 outgoing waves is

$$\Delta\varphi = k' (\Delta r)_{film} - k(\Delta r)_{air} \pm \pi$$

phase shift by π radians at top surface!

$$\Delta\varphi = \frac{2\pi}{\lambda'} \frac{2dn}{\sin\theta} - \frac{2\pi}{\lambda} 2d\sin\theta + \pi$$

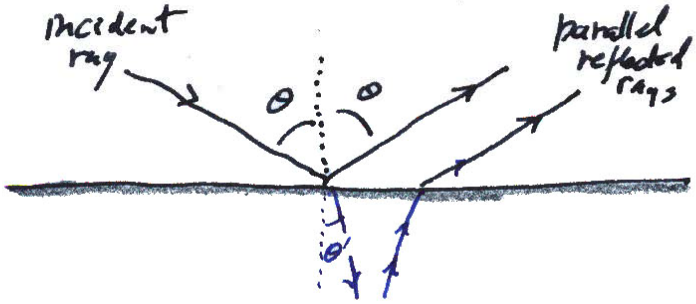
Note : $\lambda' = \frac{v}{f} = \frac{c/n}{f} = \frac{\lambda}{n}$

$$\Delta\varphi = \frac{4\pi d}{\lambda} \left[\frac{n^2}{\sin\theta} - \sin\theta \right] + \pi$$

Further simplify : $d = \tau \tan\theta' = \tau \frac{\sin\theta'}{\cos\theta'}$

$$d = \tau \frac{\sin\theta / n}{\sqrt{1 - \sin^2\theta / n^2}} = \frac{\tau \sin\theta}{\sqrt{n^2 - \sin^2\theta}}$$

$$\Delta\varphi = \frac{4\pi\tau}{\lambda} \sqrt{n^2 - \sin^2\theta} + \pi$$



$$\Delta\varphi = \frac{4\pi\tau}{\lambda}\sqrt{n^2 - \sin^2\theta} + \pi$$ ← phase difference of the two reflected waves

Constructive Interference i.e., bright fringes ....

$\Delta\varphi = 2\pi, 4\pi, 6\pi, \ldots$

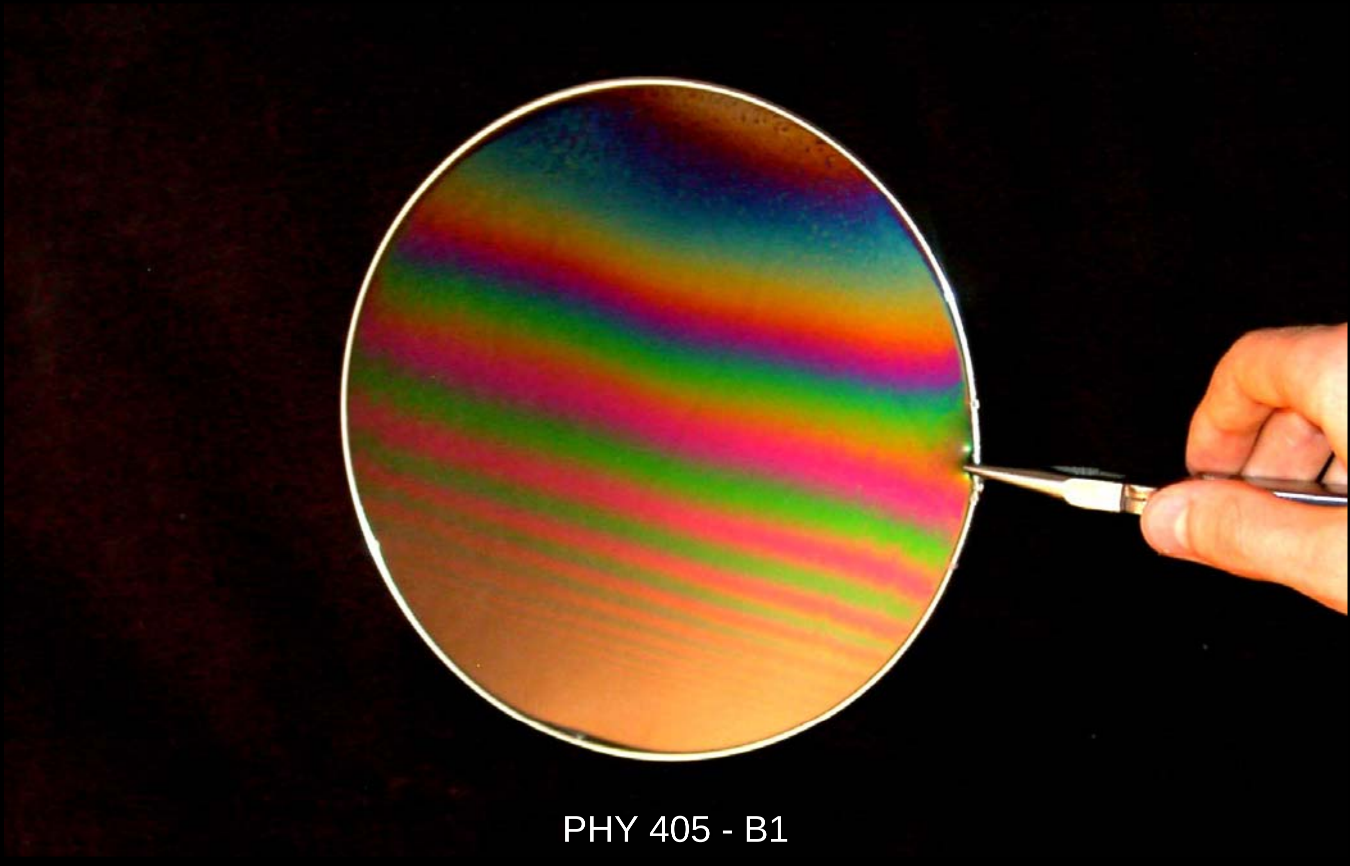
$\Delta\varphi = 2\pi m$ where m is an integer

$$\lambda = \frac{4\tau}{2m-1}\sqrt{n^2 - \sin^2\theta_m} \qquad (m = 1, 2, 3, \ldots)$$

Example Sun light reflecting from an oil slick.

White light has all colors (all λ's).

The angle for constructive interference depends on λ ⟹ observe bright colored rings.



__Iridescence__ is an example of interference of light waves.