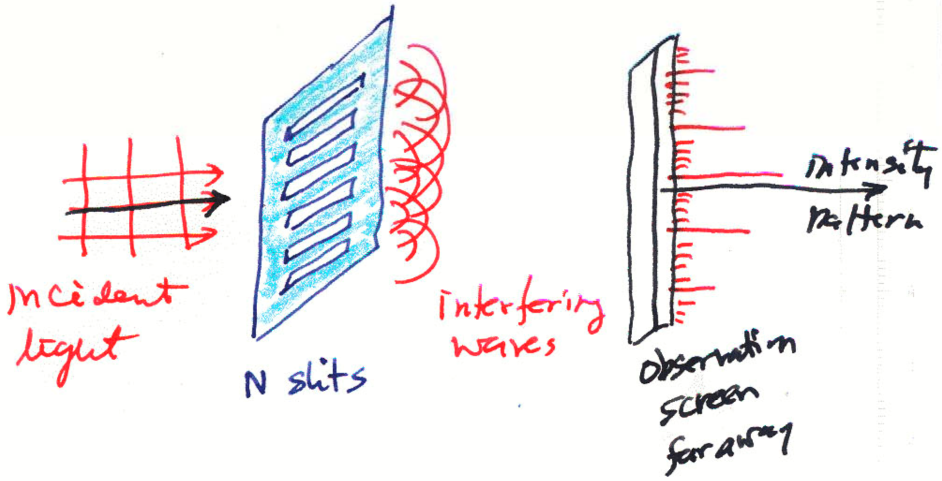
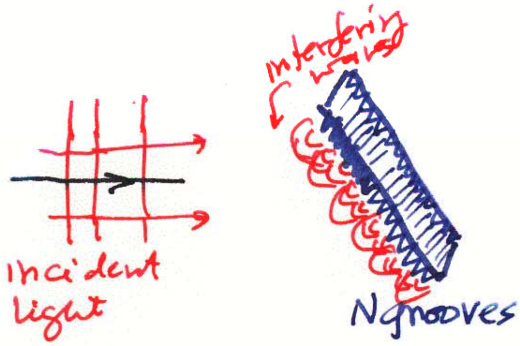
Diffraction – Applications

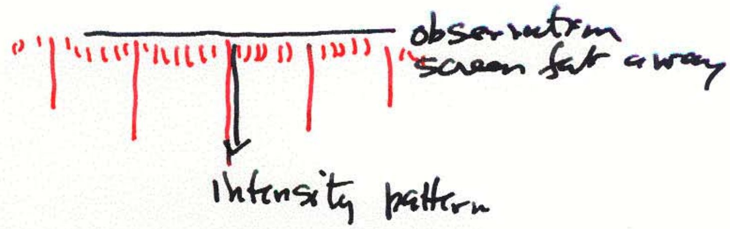
Diffraction Gratings 2 kinds

Transmission Grating



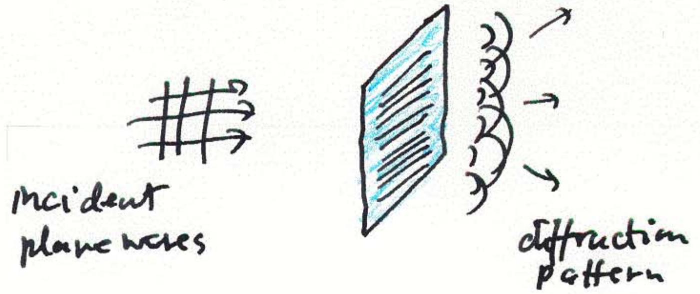
Reflection Grating





– an application of MULTIPLE BEAM INTERFERENCE

Parameters: N = # of lines (grooves or slits) ← LARGE
b = width of each line
s = distance (center to center) between lines
λ = wavelength of light



N, b, s, λ

The intensity I versus angle θ, in the Fraunhofer limit, is

$$I(\theta) = \frac{I(0)}{N^2}\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 \left(\frac{\sin\beta}{\beta}\right)^2$$

(Normal Incidence)

N slits — diffraction

$$\alpha = \frac{1}{2} k s \sin\theta$$

$$\beta = \frac{1}{2} k b \sin\theta$$

Check 2 familiar limits ....

- Limit $b \to 0$ $\qquad I(\theta) = \frac{I(0)}{N^2}\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$

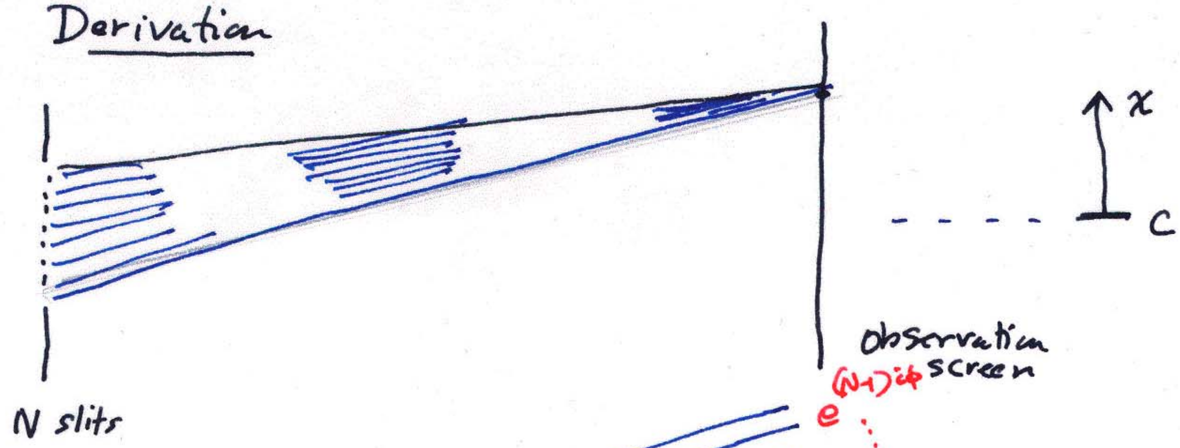
  e.g., $N=2$ $\qquad I(\theta) = I(0)\cos^2\alpha$

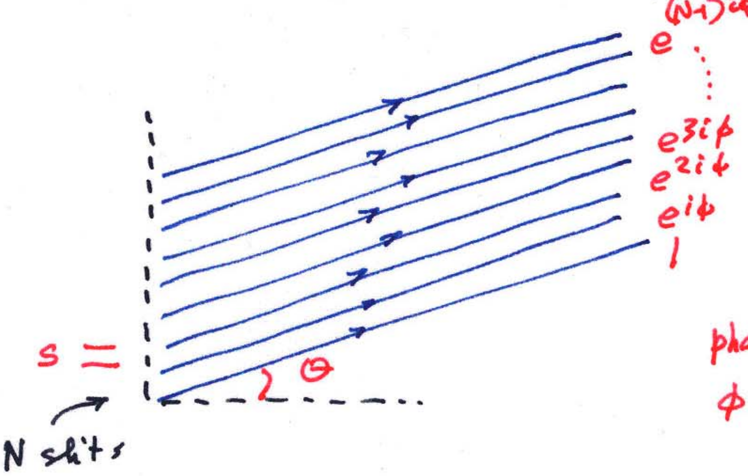
  Young's Experiment ✓

- Limit $s \to 0$ or $N=1$ $\qquad I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$

  Single-slit diffraction in Fraunhofer region ✓

Derivation





phase difference
$\phi = k s \sin\theta$

$$\text{Factor} = 1 + e^{i\phi} + e^{2i\phi} + e^{3i\phi} + \dots + e^{(N-1)i\phi}$$

$$= \sum_{n=0}^{N-1} p^n \quad \text{where } p = e^{i\phi}$$

$$= \frac{1-p^N}{1-p}$$

$$(1-p)(1+p+p^2+p^3+\dots+p^{N-1})$$
$$= 1 + p + p^2 + p^3 + \dots + p^{N-1}$$
$$\quad - p - p^2 - p^3 - \dots - p^N$$
$$= 1 - p^N$$

$$= \frac{p^{N/2}(p^{N/2} - p^{-N/2})}{p^{1/2}(p^{1/2} - p^{-1/2})}$$

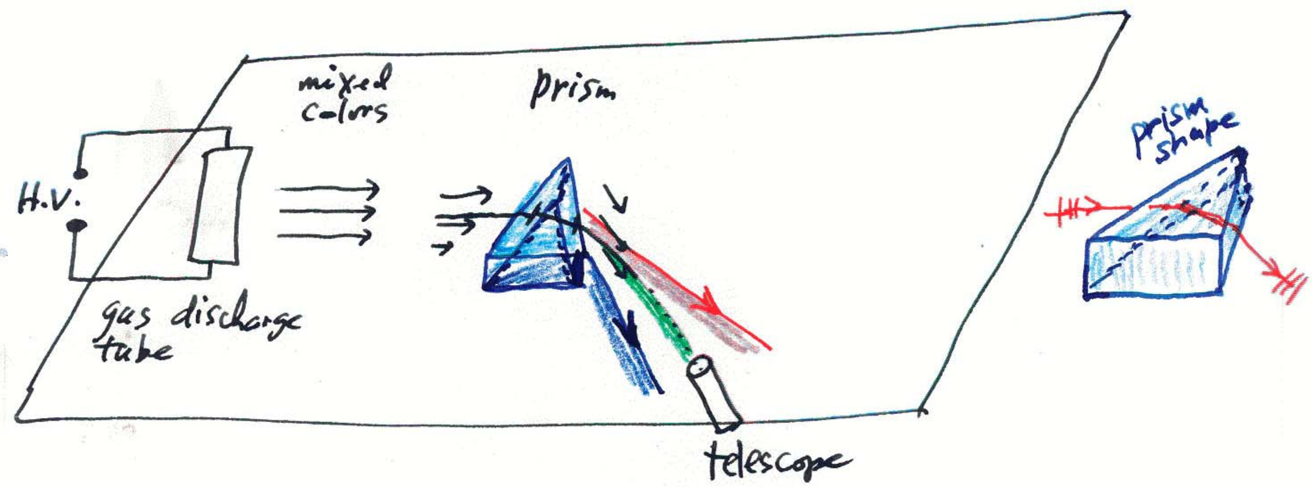
$$= e^{i\phi(N-1)/2} \frac{\sin N\phi/2}{\sin \phi/2}$$

$$|\text{Factor}|^2 = \frac{\sin^2 N\phi/2}{\sin^2 \phi/2} = \left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

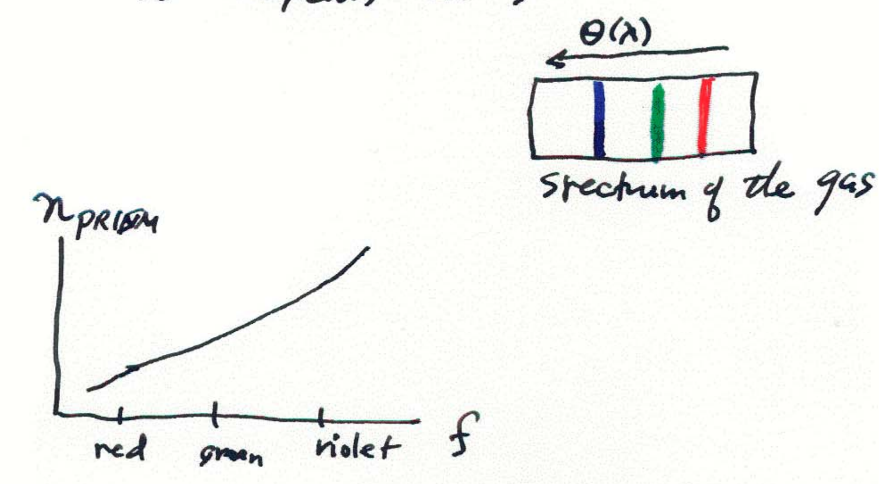
Diffraction Grating Spectroscopy B4/3

↳ separates colors of light (λ's) with a diffraction grating.

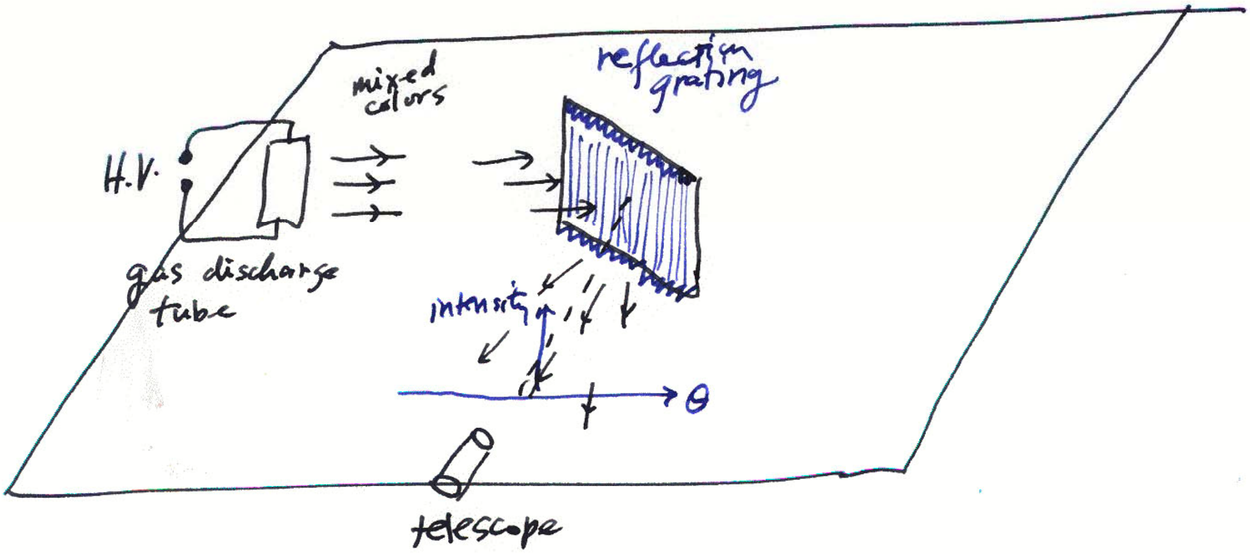
Prism Spectrometer



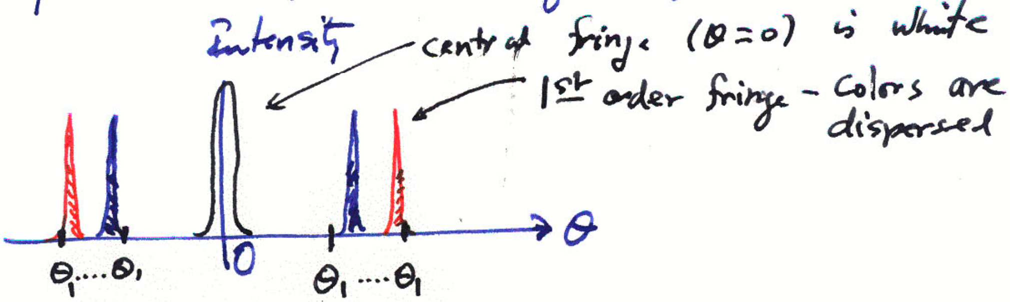
Dispersion: Colors are separated because n depends on f



Grating Spectrometer



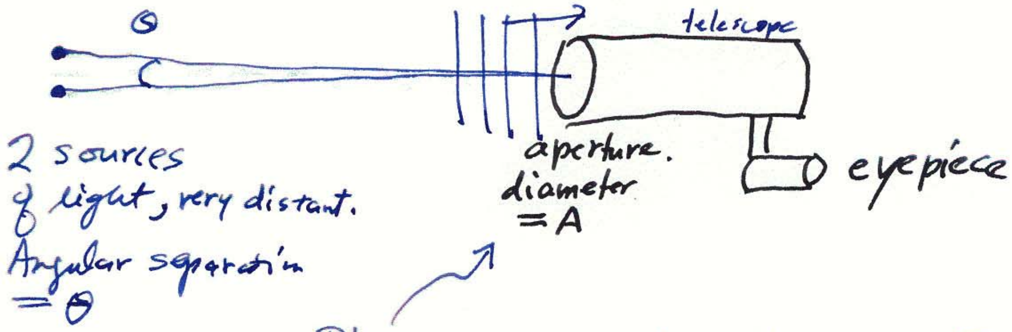
Intensity as a function of angle θ.



$$\frac{1}{N^2}\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2 \longrightarrow 1 \quad \text{for} \quad \alpha = \pi,\ 2\pi,\ 3\pi,\ \ldots$$

$$\alpha = \frac{1}{2}\,\frac{2\pi}{\lambda}\,s\,\sin\theta_1 = \pi \quad \Rightarrow \quad \sin\theta_1 = \frac{\lambda}{s}$$

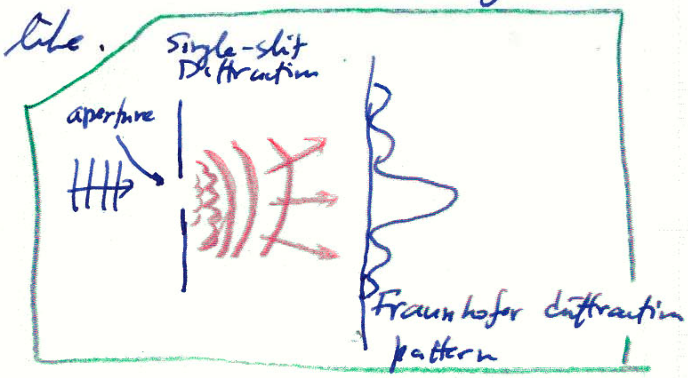
Resolving Power of a telescope



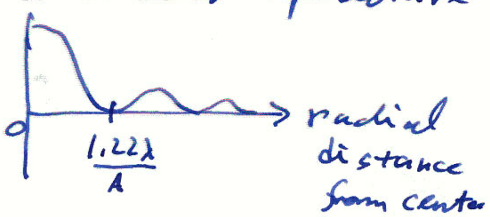
Plane waves enter the aperature.

Wave optics
Vs.
Ray optics

Diffraction at the aperature ⇒ the images are not point-like.



For a circular aperature



Diffraction Pattern of each Image

The two images can just barely be resolved if central maximum #2 coincides with first dark fringe #2

← just barely distinguishable from 1 point source

$\theta \gtrsim 1.22 \frac{\lambda}{A}$ for 2 sources to be resolvable ; A = aperture diameter