10. Properties of X-rays

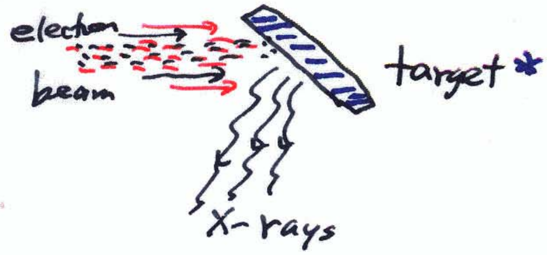
C2/1

— Production of X-rays

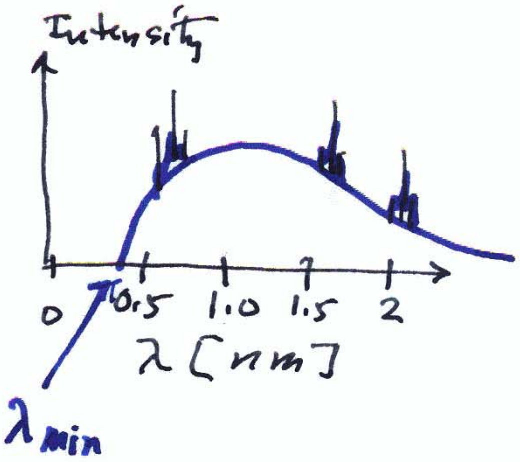
— Interaction of X-rays with matter
(scattering and absorption)

Production of X-rays

X-ray tube



* the anode in the X-ray tube



The X-ray line spectrum

* Energy levels of <u>inner</u> electron shells of target atoms

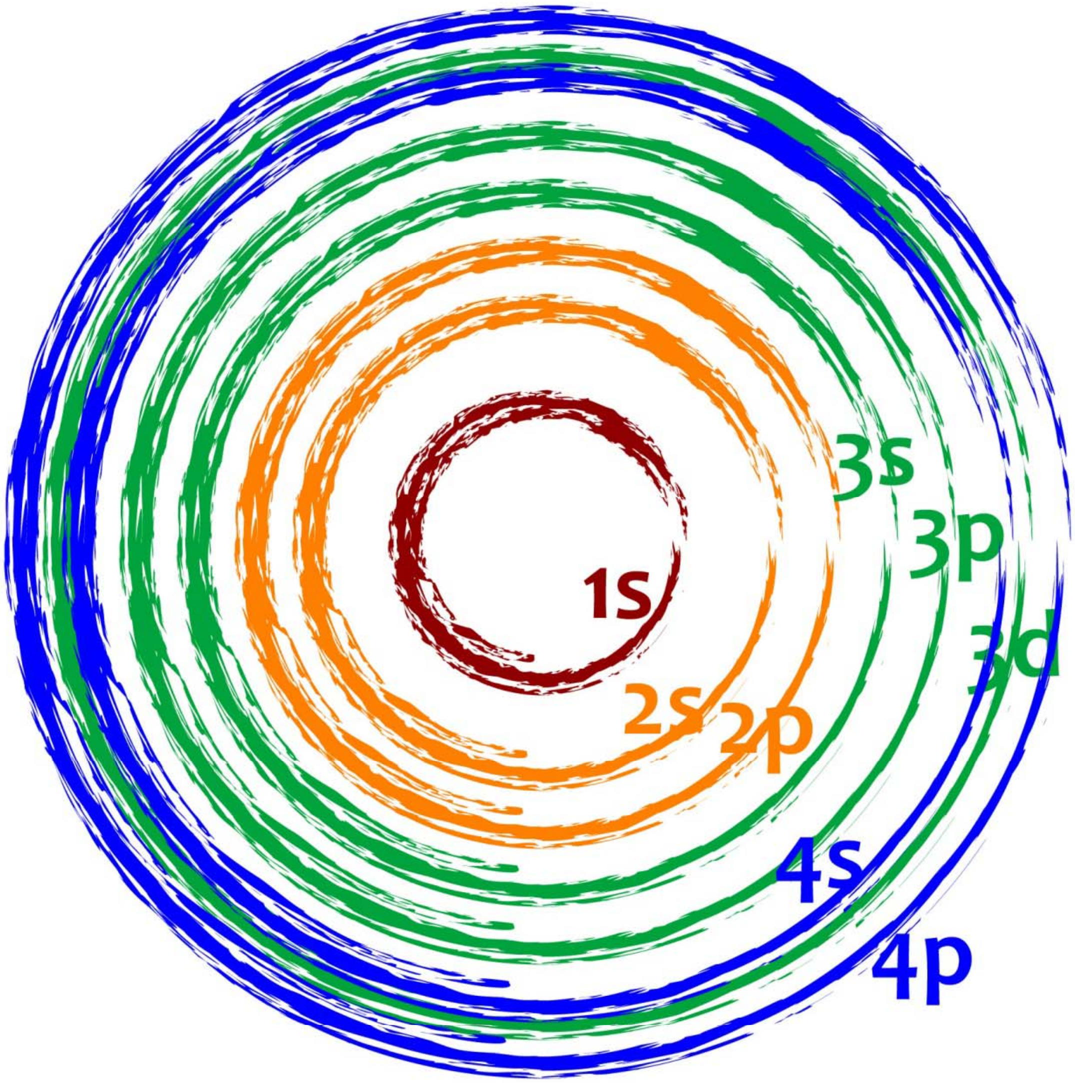
M ≡ $3s$, $3p_{1/2}$, $3p_{3/2}$; $3d_{3/2}$ $3d_{5/2}$

L ≡ $2s$; $2p_{1/2}$, $2p_{3/2}$

K — $1s$

X rays are produced when an electron falls from higher energy level E_1 to an empty lower energy level E_2 : $hf = \Delta E = E_1 - E_2$

Electron States in an Atom



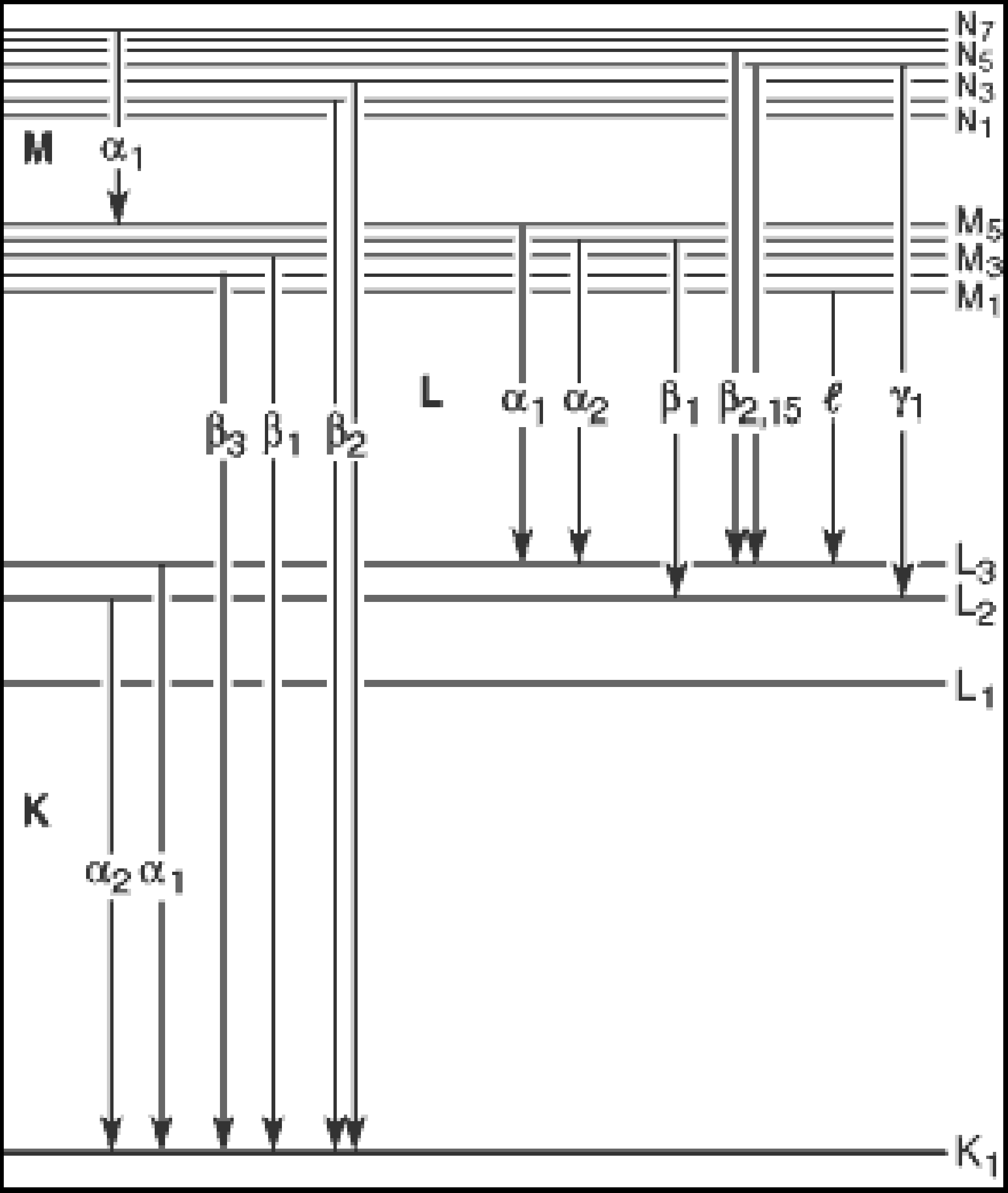
For example, Copper (Z=29):

$1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^1$

Optical Spectrum = outer orbits

X-ray spectrum = inner orbits

Inner electron shells and X-ray emission transitions



M shell
(3s, 3p1/2, 3p3/2
3d3/2, 3d5/2)

L shell
(2s, 2p1/2, 2p3/2)

K shell (1s)

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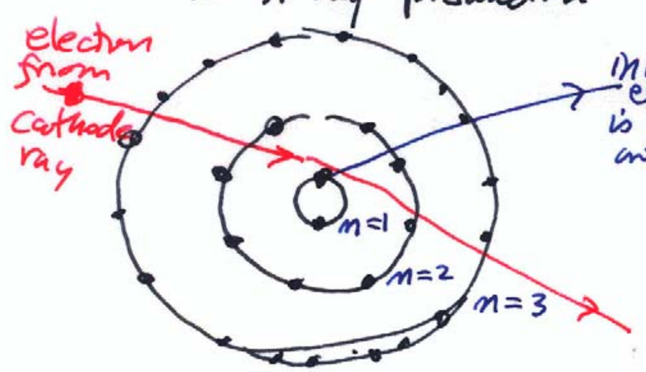
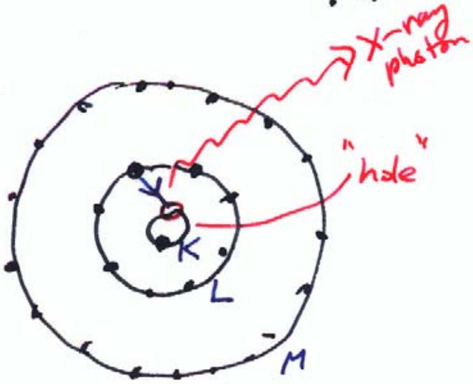
C2/2

★ X-ray production



(1) An inner shell electron is knocked out by energetic cathode-ray electron



(2) An outer electron falls into the hole in the inner shell and emits ~~an X-ray~~ a photon.

$$hf = E_L - E_K \quad (L \to K) \quad \text{or} \quad E_M - E_K \quad (M \to K)$$

$$\lambda = \frac{c}{f} = \frac{hc}{E_L - E_K} \; (L \to K) \quad \text{or} \quad \frac{hc}{E_M - E_K} \quad (M \to K)$$

★ Selection Rules

$$\Delta l = \pm 1 \quad \text{and} \quad \Delta j = 0, \pm 1$$

★ Approximate energy levels

Bohr model : $H = \frac{p^2}{2m} - \frac{Ze^2}{r}$

$$E_n = -\frac{Z^2 R_Y}{n^2} \quad \text{where} \quad R_Y = 13.6 \text{ eV}$$

This neglects the outer electrons, which is a pretty good approximation. (Why?) More accurately,

$$E_n = -\frac{(Z-\delta)^2 R_Y}{n^2} \quad \text{where} \quad \delta \approx 1$$ "screening approximation"

Examples

↳ from the X-ray Data Booklet

Carbon $Z = 6$ $K\alpha_1 = 277$ eV $2p_{3/2} \rightarrow 1s$

Bohr model $E_1 = -Z^2 R_y$ and $E_2 = -\frac{Z^2}{4} R_y$

$$E_2 - E_1 = \frac{3}{4} Z^2 R_y = 367 \text{ eV}$$

Screening model $E_1 = -(Z-1)^2 R_y$ and $E_2 = -\frac{(Z-1)^2}{4} R_y$

$$E_2 - E_1 = \frac{3}{4}(Z-1)^2 R_y = 255 \text{ eV}$$

not very accurate

Uranium $Z = 92$ $K\alpha_1 = 98.4$ keV $2p_{3/2} \rightarrow 1s$

Bohr model $E_2 - E_1 = \frac{3}{4} Z^2 R_y = 86.3$ keV

But the inner electrons of uranium are relativistic; $\frac{v}{c} \approx Z\alpha = \frac{92}{137} = 0.67$ so the Bohr model is not accurate.

Iron $Z = 26$ $K\alpha_1 = 6.40$ keV $2p_{3/2} \rightarrow 1s$

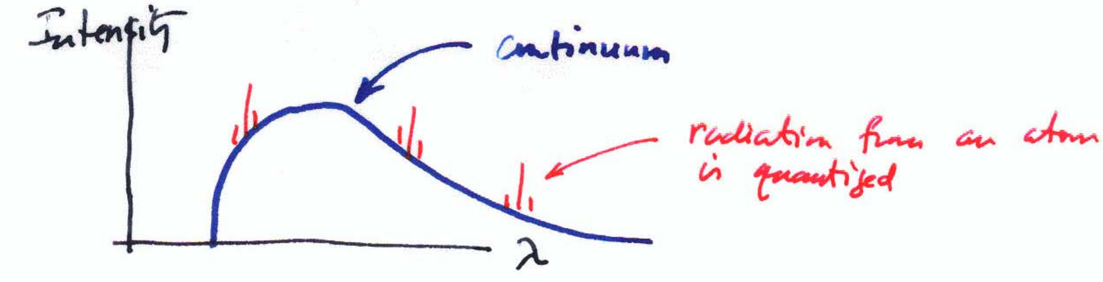
Bohr model $E_2 - E_1 = \frac{3}{4} Z^2 R_y = 6.90$ keV

Screening model $E_2 - E_1 = \frac{3}{4}(Z-1)^2 R_y = 6.38$ keV

for a wide range of intermediate Z values,
the Bohr model and screening model are quite accurate

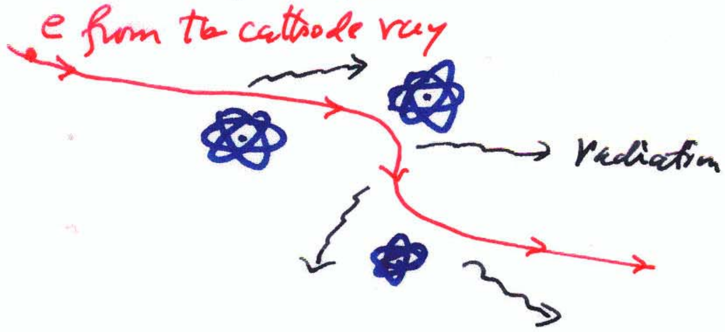
The Continuum Spectrum

C2/4



$$e + \text{atom} \longrightarrow e + \gamma + \text{atom}$$

In the classical theory of electromagnetic radiation, a charged particle radiates when it accelerates

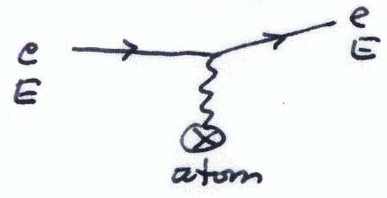


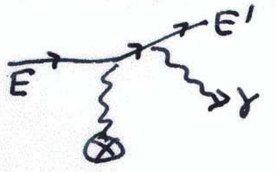
Classical e.m. radiation

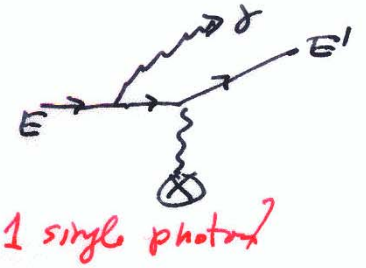
- continuous spectrum
- Larmor's Formula: $P_{radiated} = \dfrac{e^2 a^2}{6\pi\epsilon c^3}$

instantaneous power ; a = acceleration

Quantum electrodynamics (Feynman)

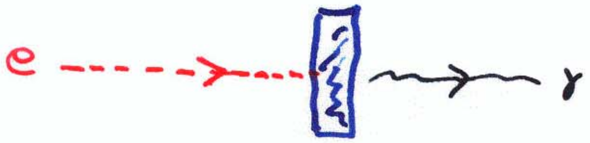






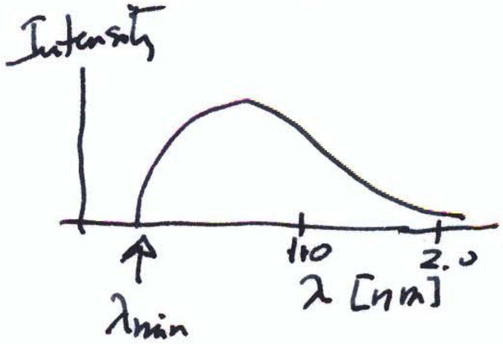
(emission of 1 single photon)

The X-ray spectrum is continuous, but radiation is probabilistic.



C2/5

As the electron slows down in the anode, about 1% of its kinetic energy is transferred to X-ray radiation (on average). The rest goes to heat.

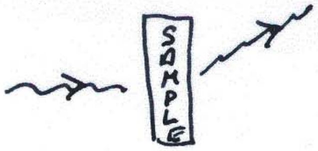


$$\lambda_{min} = \frac{hc}{E}$$

(why?)

Interaction of X-rays with matter

Scattering



- Thomson scattering

$$\gamma + e \rightarrow \gamma + e \qquad \text{with} \quad E_\gamma \ll mc^2$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 (1+\cos^2\theta) \quad \text{where} \quad r_e = \frac{e^2}{4\pi\epsilon_0 \, mc^2}$$

$$\sigma_T = \int \frac{d\sigma}{d\Omega} \sin\theta \, d\theta \, d\varphi = \frac{8\pi}{3} r_e^2$$

(J.J. Thomson, theory published ~1906)

$r_e = 2.8 \times 10^{-15}$ m

$\sigma_T = 6.6 \times 10^{-29}$ m^2 = 0.66 barns

- Compton scattering ($E_\gamma \gtrsim mc^2$)

The X-ray loses some energy

$$\frac{E_\gamma'}{E_\gamma} = \frac{mc^2}{mc^2 + E(1-\cos\theta)}$$

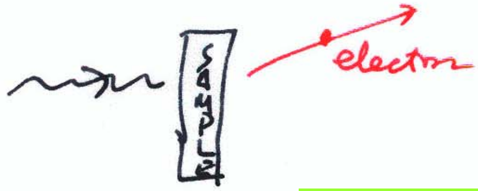
$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

$$E = hf = \frac{hc}{\lambda}$$

(Arthur Compton, 1923)

Interactions of X-rays with matter

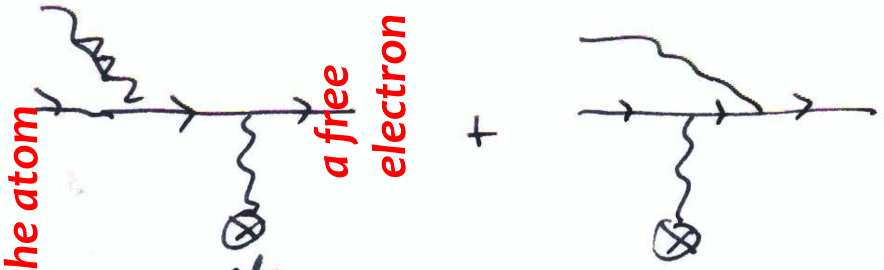
Absorption



… on an atomic scale

- Photo-electric effect

$$\gamma + \text{atom} \to e + \cancel{\text{atom}}\ \text{ion}$$

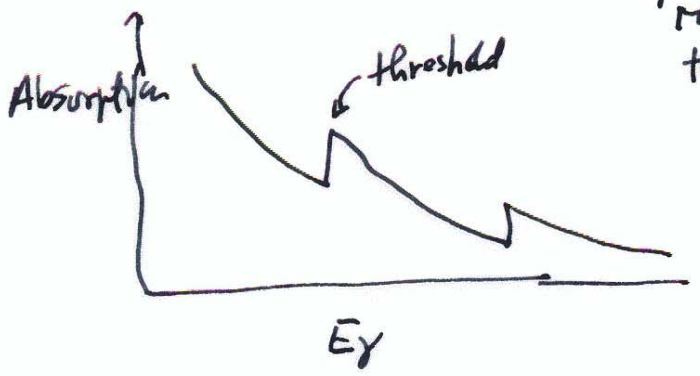


$$hf - B = \tfrac{1}{2}mv^2$$

$$hf = B + \tfrac{1}{2}mv^2 \geq B$$

(neglecting recoil energy of the atom)

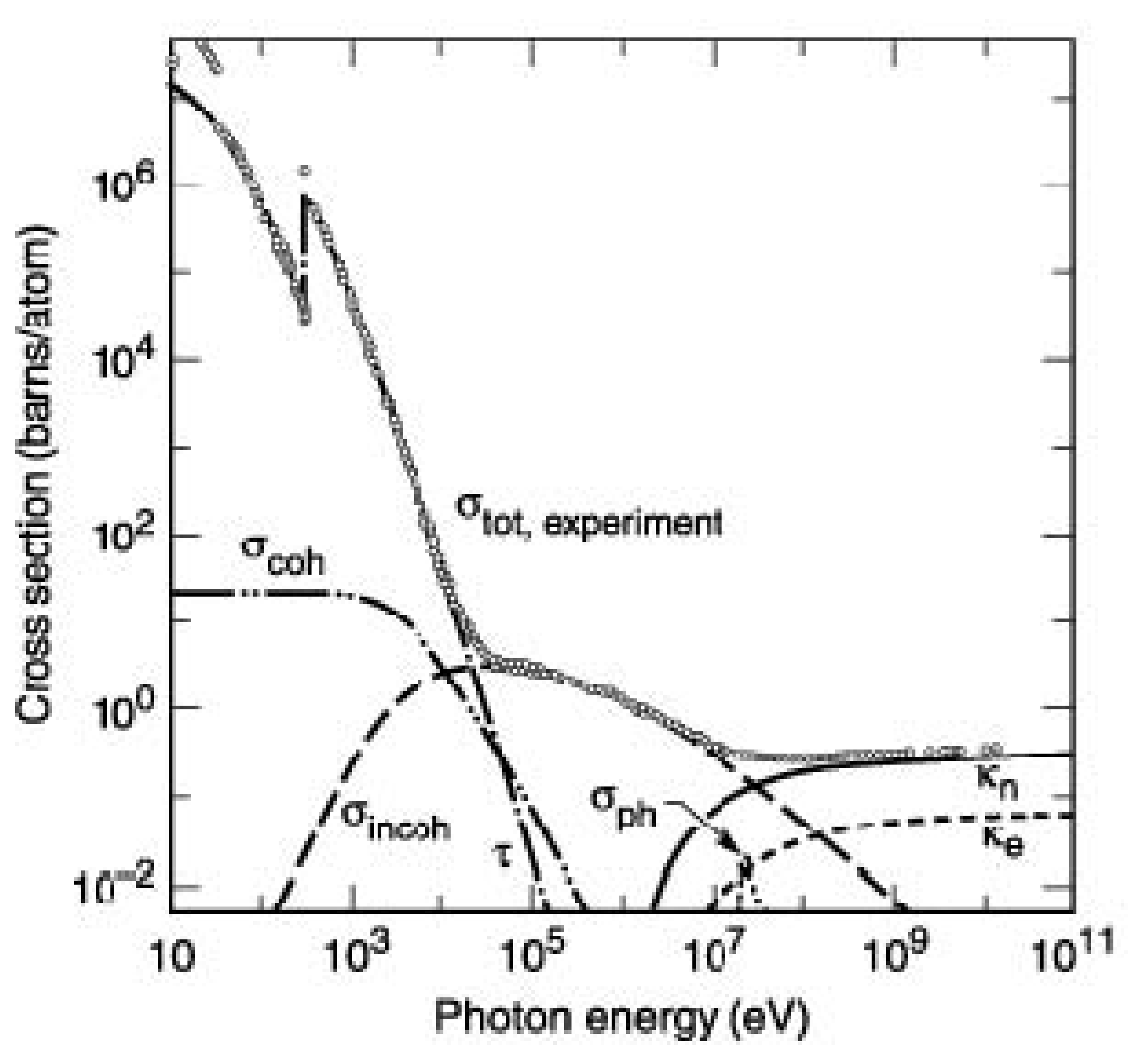
↑ minimum γ energy for this process.



- Pair Production: $\gamma + \text{atom} \to e^- e^+ + \text{atom}$

Can occur if $E_\gamma > 2mc^2 = 1.02$ MeV

Carbon cross sections



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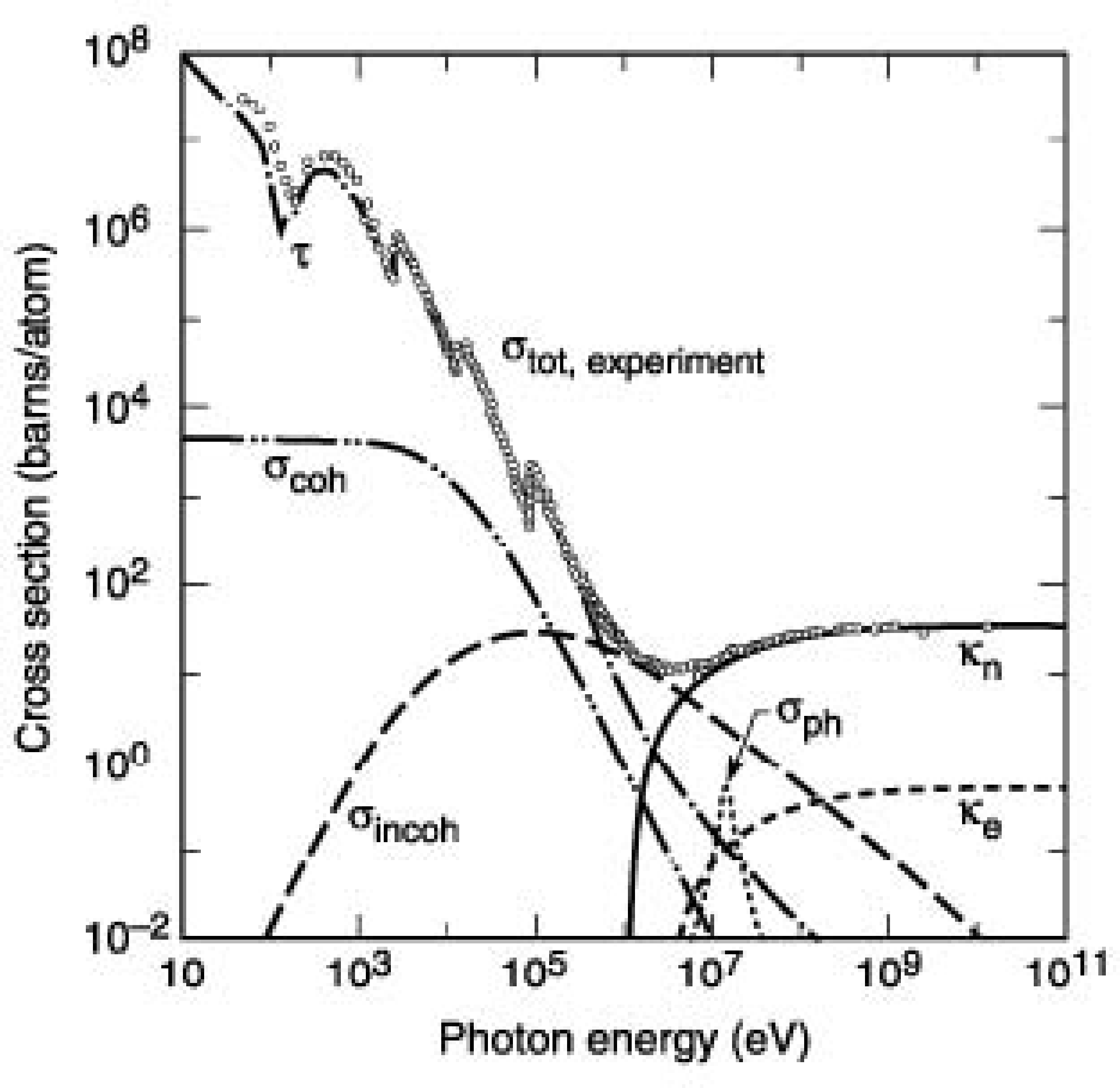
Lead cross sections



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