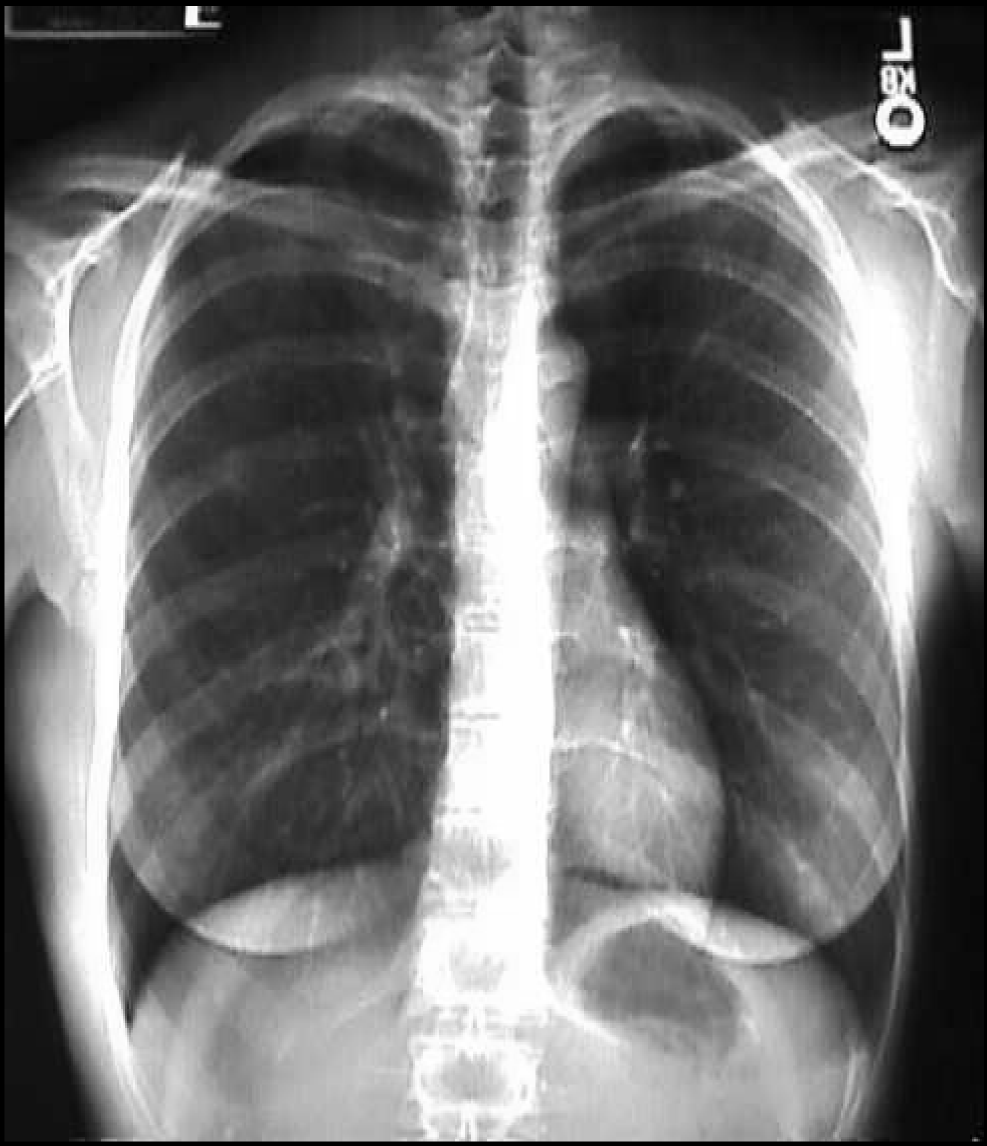
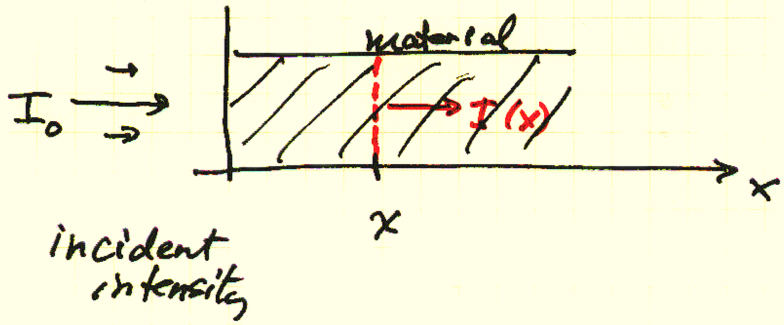
A normal chest X-ray



Interactions of X rays and matter

The linear absorption coefficient (μ)



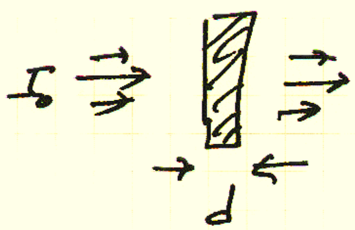
Intensity

$$I = \frac{dN_\gamma/dt}{\text{Area}}$$

$$dI = -\mu I \, dx \qquad \text{/absorption in slice } dx\text{/}$$

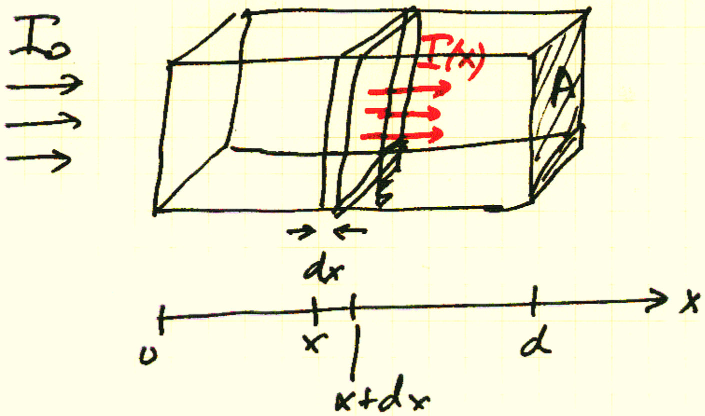
$$\frac{dI}{dx} = -\mu I \quad \Rightarrow \quad I(x) = I_0 e^{-\mu x}$$

For example



$$I_{out} = I_0 e^{-\mu d}$$

The mass absorption coefficient



$I(x)$ = # photons/m^2/sec at x

dN = # photons absorbed (and scattered) from x to $x+dx$ removed from beam

$dN = (\#\text{atoms})(dN)_{1\ \text{atom}}$

$\#\text{atoms} = \left(\frac{\rho}{M}\right) \times \text{volume} = \left(\frac{\rho}{M}\right)(A\,dx)$

$$(dN)_{\text{removed}} = \frac{\rho}{M} A\,dx\;\sigma\; I(x)\,dt$$

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$$I = \frac{dN_\gamma / dt}{\text{Area}}$$

$$I(x+dx) = \frac{1}{A}\left(\frac{dN_\gamma}{dt}\right)_{x+dx}$$

$$= \frac{1}{A} \quad \frac{dN_{\text{incident}} - dN_{\text{removed}}}{dt}$$

incident: at x

removed: from x to x+dx

$$= I(x) - \frac{1}{A} \frac{\frac{\rho}{M} A\, dx\, \sigma\, I\, dt}{dt}$$

$$= I(x) - \frac{\rho\sigma}{M} I\, dx$$

$$\frac{dI}{dx} = -\frac{\rho\sigma}{M} I \quad \text{so} \quad \boxed{\mu = \frac{\rho\sigma}{M}}$$

as we guessed by dimensional analysis

$$\boxed{I(x) = I_0 e^{-\mu x} \quad \text{where} \quad \mu = \frac{\rho\sigma}{M}}$$

Example X-ray screening by Aluminum

NIST X-ray mass attenuation coefficients

Consider 100 keV X-rays.

$E_\gamma = 10^{-1}$ MeV : $\frac{\mu}{\rho} = \underbrace{1.704 \times 10^{-1}}_{\text{table value}} = 0.1704 \ \frac{cm^2}{g}$

$\rho = 2.699 \ g/cm^3$ so $\mu = 0.4599 \ cm^{-1}$

x	$I(x)/I_0 = e^{-\mu x}$	% shielding (removed)
1 mm		
1 cm		
10 cm		
1 m		

That'

for

Calculate the cross section:

$$\sigma = M \frac{\mu}{\rho} = \frac{26.98\,\text{g}}{6.02 \times 10^{23}} \times 0.1704 \frac{\text{cm}^2}{\text{g}}$$

$$\sigma = 7.64 \times 10^{-24} \ \text{cm}^2$$

$$\sigma = \pi R^2 \qquad \text{implies} \qquad R = 1.56 \times 10^{-12} \ \text{m}$$